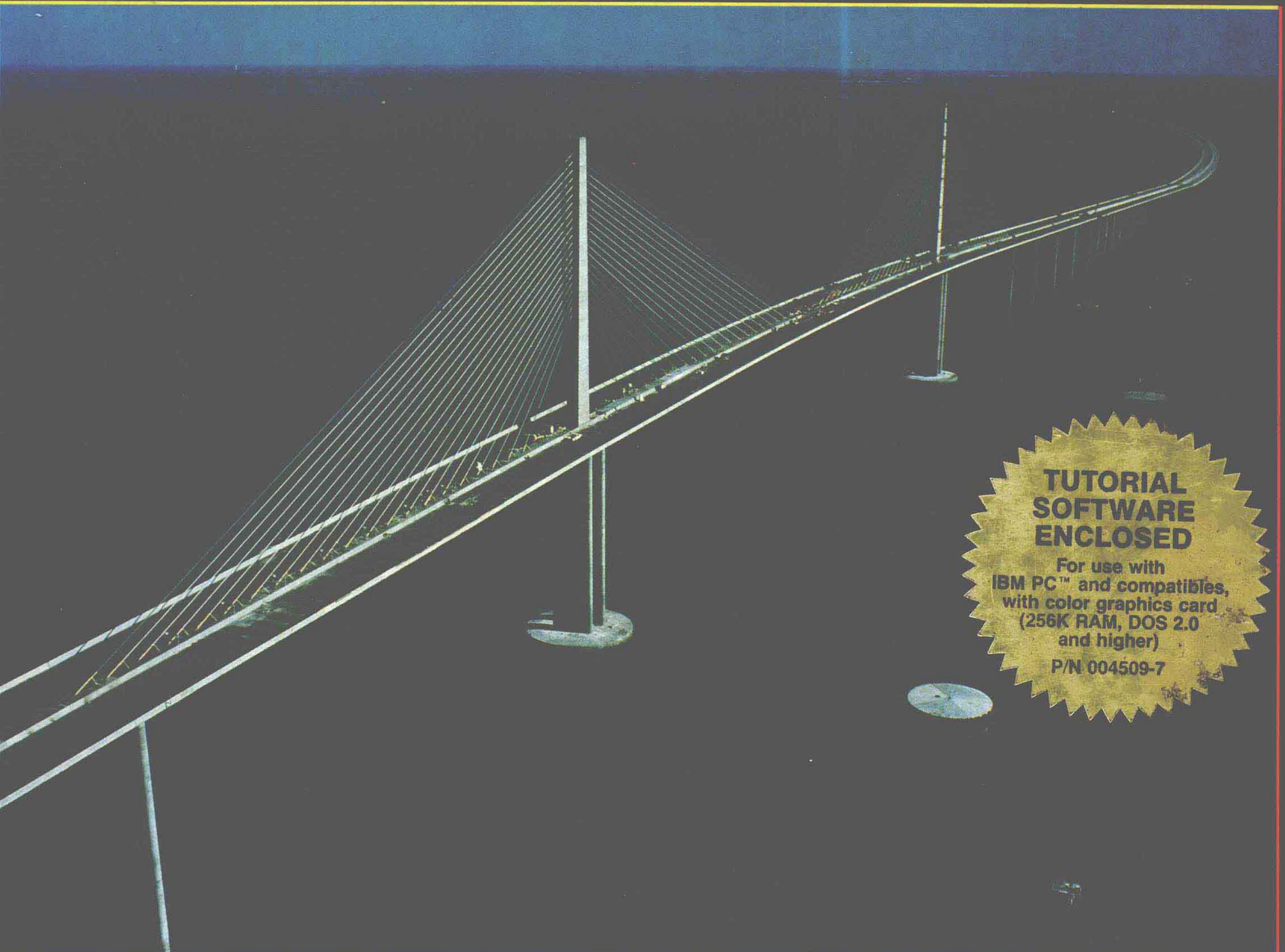
# Vector Mechanics for Engineers

Fifth Edition

Ferdinand P. Beer
E. Russell Johnston, Jr.

## Statics



# Vector Mechanics for Engineers

# Statics

**Fifth Edition**

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**McGraw-Hill Book Company**
New York St. Louis San Francisco Auckland Bogotá Hamburg
London Madrid Mexico Milan Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

The cover photograph is of Florida's Sunshine Skyway Bridge as it was nearing completion. This bridge, which rises 175 feet above Tampa Bay, is Florida's first cable-stayed bridge, and its 1200-foot main span is the longest concrete span in the Americas. The Sunshine Skyway Bridge was designed by Figg and Muller Engineers, Inc., for the Florida Department of Transportation. *(Photograph by Fred Fox.)*

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VECTOR MECHANICS FOR ENGINEERS: **Statics**



90 KGPKGP 92

P/N 004507-0
Part of
ISBN 0-07-079946-6

This book was set in Laurel by York Graphic Services, Inc.
The editors were Anne Duffy and David A. Damstra;
the designer was Merrill Haber;
the production supervisor was Joe Campanella.
The drawings were done by Felix Cooper.
Arcata Graphics/Kingsport was printer and binder.

**Library of Congress Cataloging-in-Publication Data**

Beer, Ferdinand Pierre
Vector mechanics for engineers/Ferdinand P. Beer, E. Russell Johnston, Jr.—5th ed.
p. cm.
Bibliography: v. [1], p.
Includes index.
Contents: [1] Statics
ISBN: 0-07-079946-6
1. Mechanics, Applied. 2. Vector analysis. 3. Mechanics, Applied—Problems, exercises, etc. I. Johnston, E. Russell (Elwood Russell) (date). II. Title.
TA350.B3552 1988
620. 1—dc19 87-18633

# ABOUT THE AUTHORS

"How did you happen to write your books together, with one of you at Lehigh and the other at UConn, and how do you manage to keep collaborating on their successive revisions?" These are the two questions most often asked of our two authors.

The answer to the first question is simple. Russ Johnston's first teaching appointment was in the department of civil engineering and mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in statics and dynamics. Born in France and educated in France and Switzerland (he holds an M.S. degree from the Sorbonne and a Sc.D. degree in the field of theoretical mechanics from the University of Geneva), Ferd had come to the United States after serving in the French army during the early part of World War II and had taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Born in Philadelphia, Russ had obtained a B.S. degree in civil engineering from the University of Delaware and a Sc.D. degree in the field of structural engineering from MIT.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the courses in statics and dynamics. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented in a graphic way. Together they wrote lecture notes, to which they later added problems they felt would appeal to future engineers, and soon they had produced the manuscript of the first edition of *Mechanics for Engineers.*

The second edition of their text found Russ Johnston at Worcester Polytechnic Institute and the third at the University of Connecticut. In the meantime, both Ferd and Russ had assumed administrative responsibilities in their departments, and both were involved in research, consulting, and the supervision of graduate students, Ferd in the area of stochastic processes and random vibrations, and Russ in the area of elastic stability and structural analysis and design. However, their interest in improving the teaching of the basic mechanics courses had not subsided and they both taught sections of these courses as they kept revising their texts.

This brings us to the second question: How did the authors manage to work together so effectively after Russ Johnston had left Lehigh? Part of the answer may be provided by their phone bills and the money they spend on postage. As the publication date of a new edition approaches, they call each other daily and rush to the post office with express-mail packages in order to double-check their work. There are also frequent visits between the two families. At one time there were even joint camping trips, with both families pitching their tents next to each other. The Beers were the first to graduate to a trailer, which was used to illustrate a problem in one of the early editions of their text, but was replaced by the Johnstons' trailer in the next one. Now this trailer has also been replaced, both authors preferring the comforts of a motel and its dining room to those of a camping ground and its fireplaces.

# Preface

The main objective of a first course in mechanics should be to develop in the engineering student the ability to analyze any problem in a simple and logical manner and to apply to its solution a few, well-understood basic principles. It is hoped that this text, designed for the first course in statics offered in the sophomore year, and the volume that follows, *Vector Mechanics for Engineers: Dynamics*, will help the instructor achieve this goal.†

Vector algebra is introduced early in the text and used in the presentation and the discussion of the fundamental principles of mechanics. Vector methods are also used to solve many problems, particularly three-dimensional problems where their application results in a simpler and more concise solution. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector algebra is presented chiefly as a convenient tool.‡

One of the characteristics of the approach used in these volumes is that the mechanics of *particles* has been clearly separated from the mechanics of *rigid bodies*. This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of more difficult concepts. In this volume, for example, the statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors have been introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems. In the volume on dynamics, the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are

† Both texts are also available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics*, fifth edition.

‡ In a parallel text, *Mechanics for Engineers: Statics*, fourth edition, the use of vector algebra is limited to the addition and subtraction of vectors.

introduced and first applied to problems involving only particles. Thus students may familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

Since this text is designed for a first course in statics, new concepts have been presented in simple terms and every step explained in detail. On the other hand, by discussing the broader aspects of the problems considered, a definite maturity of approach has been achieved. For example, the concepts of partial constraints and of static indeterminacy are introduced early in the text and used throughout.

The fact that mechanics is essentially a *deductive* science based on a few fundamental principles has been stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive*, simple applications have been considered first. Thus the statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6. Also, in Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.

Free-body diagrams are introduced early, and their importance is emphasized throughout the text. Color has been used to distinguish forces from other elements of the free-body diagrams. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text. Free-body diagrams are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. This approach is particularly useful as a preparation for the study of the dynamics of rigid bodies. As will be shown in the volume on dynamics, by placing the emphasis on "free-body-diagram equations" rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics may be achieved.

Because of the current trend among American engineers to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics have been introduced in Chap. 1 and are used throughout the text. Approximately half the sample problems and 60 percent of the problems to be assigned have been stated in these units, while the remainder retain U.S. customary units. The authors believe that this approach will best serve the needs of the students, who will be entering the engineering profession during the period of transition from one system of units to the other. It also should be recognized that the passage from one system to the other entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or $\text{lb} \cdot \text{sec}^2/\text{ft}$). The authors, therefore, believe that problem assignments should include both systems of units. The actual distribution of assigned problems between the two systems of units, however, has been left to the instructor, and a sufficient number of problems of each type have been provided so that four complete lists of assignments may be selected with the proportion of problems stated in SI units set anywhere between 50 and 75 percent. If so desired, two complete lists of assignments may also be selected from problems stated in SI units only and two others from problems stated in U.S. customary units.

A large number of optional sections have been included. These sections are indicated by asterisks and may thus easily be distinguished from those which form the core of the basic statics course. They may be omitted without prejudice to the understanding of the rest of the text. Among the topics covered in these additional sections are the reduction of a system of forces to a wrench, applications to hydrostatics, shear and bending-moment diagrams for beams, equilibrium of cables, products of inertia and Mohr's circle, mass products of inertia and principal axes of inertia for three-dimensional bodies, and the method of virtual work. The sections on beams are especially useful when the course in statics is immediately followed by a course in mechanics of materials, while the sections on the inertia properties of three-dimensional bodies are primarily intended for the students who will later study in dynamics the motion of rigid bodies in three dimensions.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus, and all the elements of vector algebra necessary to the understanding of the text have been carefully presented in Chaps. 2 and 3. In general, a greater emphasis has been placed on the correct understanding of the basic mathematical concepts involved than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of moment of area firmly before introducing the use of integration. The presentation of numerical solutions takes into account the universal use of calculators by engineering students and instructions on the proper use of calculators for the solution of typical statics problems have been included in Chap. 2.

Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems. The body of the text has been divided into units, each consisting of one or several theory sections, one or several sample problems, and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally may be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes have been added to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

The sample problems have been set up in much the same form that students will use in solving the assigned problems. They thus serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. Most of the problems to be assigned are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the basic principles of mechanics. The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty. Problems requiring special attention have been indicated by asterisks. Answers to all even-numbered problems are given at the end of the book.

The introduction in the engineering curriculum of instruction in computer programming and the increasing availability of personal computers or mainframe terminals on most campuses make it now possible for engineering students to solve a number of challenging statics and dynamics problems. Only a few years ago these problems would have been considered inappropriate for an undergraduate course because of the large number of computations their solutions require. In this new edition of *Vector Mechanics for Engineers*, a group of four problems designed to be solved with a computer has been added to the review problems at the end of each chapter. In *Statics*, these problems may involve the analysis of a structure for various configurations or loadings of the structure, or the determination of the equilibrium positions of a given mechanism. In *Dynamics*, they may involve the determination of the motion of a particle under various initial conditions, the kinematic or kinetic analysis of mechanisms in successive positions, or the numerical integration of various equations of motion. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem.

A diskette containing interactive software designed for the IBM PC or any compatible computer has been placed in the pocket inside the rear cover of this text. This software includes twelve "Tutorials," each consisting of a problem in statics for which students are more likely to require help; thus, the software plays a role similar to that of the Sample Problems. However, being interactive, the Tutorials allow the students to vary the data, let them participate in the solution of the problem, and check the results obtained. By allowing the students to vary the data, the Tutorials provide them with an inexhaustible source of problems of the same type. They also allow the more inquisitive students to test the "sensitivity" of a given problem to a change in data or to find the conditions which lead to special or critical situations. The authors wish to thank Professor Raymond P. Canale of the University of Michigan for helping them prepare the Tutorials and Mr. Tad Slawecki of EnginComp Software, Inc., for programming them.

The authors gratefully acknowledge the many helpful comments and suggestions offered by the users of the previous editions of *Mechanics for Engineers* and of *Vector Mechanics for Engineers*.

Ferdinand P. Beer
E. Russell Johnston, Jr.

# List of Symbols

| | |
|---|---|
| $a$ | Constant; radius; distance |
| $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ | Reactions at supports and connections |
| $A, B, C, \ldots$ | Points |
| $A$ | Area |
| $b$ | Width; distance |
| $c$ | Constant |
| $C$ | Centroid |
| $d$ | Distance |
| $e$ | Base of natural logarithms |
| $\mathbf{F}$ | Force; friction force |
| $g$ | Acceleration of gravity |
| $G$ | Center of gravity; constant of gravitation |
| $h$ | Height; sag of cable |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | Unit vectors along coordinate axes |
| $I, I_x, \ldots$ | Moment of inertia |
| $\bar{I}$ | Centroidal moment of inertia |
| $I_{xy}, \ldots$ | Product of inertia |
| $J$ | Polar moment of inertia |
| $k$ | Spring constant |
| $k_x, k_y, k_O$ | Radius of gyration |
| $k$ | Centroidal radius of gyration |
| $l$ | Length |
| $L$ | Length; span |
| $m$ | Mass |
| $\mathbf{M}$ | Couple; moment |
| $\mathbf{M}_O$ | Moment about point $O$ |
| $\mathbf{M}_O^R$ | Moment resultant about point $O$ |
| $M$ | Magnitude of couple or moment; mass of earth |
| $M_{OL}$ | Moment about axis $OL$ |
| $\mathbf{N}$ | Normal component of reaction |
| $O$ | Origin of coordinates |
| $p$ | Pressure |
| $\mathbf{P}$ | Force; vector |
| $\mathbf{Q}$ | Force; vector |
| $\mathbf{r}$ | Position vector |

| | |
|---|---|
| $r$ | Radius; distance; polar coordinate |
| $\mathbf{R}$ | Resultant force; resultant vector; reaction |
| $R$ | Radius of earth |
| $\mathbf{s}$ | Position vector |
| $s$ | Length of arc; length of cable |
| $\mathbf{S}$ | Force; vector |
| $t$ | Thickness |
| $\mathbf{T}$ | Force |
| $T$ | Tension |
| $U$ | Work |
| $\mathbf{V}$ | Vector product; shearing force |
| $V$ | Volume; potential energy; shear |
| $w$ | Load per unit length |
| $\mathbf{W}$, $W$ | Weight; load |
| $x, y, z$ | Rectangular coordinates; distances |
| $\bar{x}, \bar{y}, \bar{z}$ | Rectangular coordinates of centroid or center of gravity |
| $\alpha, \beta, \gamma$ | Angles |
| $\gamma$ | Specific weight |
| $\delta$ | Elongation |
| $\delta\mathbf{r}$ | Virtual displacement |
| $\delta U$ | Virtual work |
| $\boldsymbol{\lambda}$ | Unit vector along a line |
| $\eta$ | Efficiency |
| $\theta$ | Angular coordinate; angle; polar coordinate |
| $\mu$ | Coefficient of friction |
| $\rho$ | Density |
| $\phi$ | Angle of friction; angle |

# Contents