

The background of the cover is a light purple color. On the left side, there is a vertical column of 3D cubes of varying heights, some with white tops. In the lower right, there is a large, stylized neuron with a central purple sphere and several branching processes. The overall design is modern and technical.

UNDERSTANDING NEURAL NETWORKS AND FUZZY LOGIC

BASIC CONCEPTS
AND APPLICATIONS

STAMATIOS V. KARTALOPOULOS

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UNDERSTANDING NEURAL NETWORKS AND FUZZY LOGIC

Basic Concepts and Applications



Stamatis V. Kartalopoulos, Ph.D.

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Carrie Briggs, *Administrative Assistant*
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PREFACE

The workings of the brain have fascinated me since childhood. I had observed with interest that whenever a question was asked in the classroom many different answers were given. Every classmate was thinking and perceiving the same question from an entirely different viewpoint and thus an answer was given according to their own particular perspective. This diversity in perspective is so profound. It adds even more dimension to world around us. We (humans) have a depth of visualization so powerful that we can close our eyes and . . . imagine. Imagination is timeless, boundless, unlimited and it happens right there in a few cubic centimeters of soft matter, the brain. Close your eyes and you can “see” faces you have not seen for years or “smell” summer fragrances in the middle of the winter; or “travel” through space, crossing distant galaxies with an incomprehensible speed that defies all laws of physics. Close your eyes and you can create ideas that never before existed. Someone “saw” a wheel for the first time and made a cart; another heard the first music before music was sung. Someone for the first time “saw” the benefit of the volcanic fire and used it to warm houses, to cook, to extract metals from rocks, and to make tools and weapons. And this inventiveness continues to this day. We “saw” the invisible forces of matter, controlled them, and produced electricity, we made radios and computers and we escaped into space. So, is it surprising that for many years this mind-boggling power of the brain has been the subject of research?

I have been compiling information about the biology of the brain and sifting through articles and studies on neural research for quite a few years now. As a physicist and an engineer, I wanted to understand the mechanics and innerworkings of the brain. As soon as I had an organized set of notes that I thought had pedagogical value, I decided to give a tutorial in neural

networks at the Globecom '91 communications conference. To the best of my knowledge, such a tutorial had not been presented previously at any communications conference and I thought this would be a good chance to find out how much interest there is in this area. We expected a relatively small audience. To our surprise, we had an overwhelming attendance—a full house. The feedback I received at the end of the tutorial was very enthusiastic. I therefore enhanced my tutorial notes, simplified certain math-intensive sections, and included fuzzy logic and fuzzy neural networks. I also organized conference sessions on neural networks and fuzzy logic. Although participation was small at first, it has steadily increased. The interest from the communications community alone has so increased that, in 1993, a conference was organized on neural networks in communications. I presented my tutorial a few more times and, every time, the audience suggested that I should publish my notes as a book.

The intention of this book is to provide an introduction to the subject of neural networks, fuzzy logic, and fuzzy neural networks; to provide, in a coherent and methodical manner, the concepts of neural networks and fuzzy logic with easy to understand examples that describe a number of applications in a nonmathematical way; to address a need of the scientific community that other books in neural networks and in fuzzy logic do not address; and to provide a linkage between neural networks and fuzzy logic. The majority of books I have seen on this subject require a level of expertise to understand the material. Some, however, are invaluable tools for the connoisseur. The material and depth of this book was prepared for those who want an introduction to neural networks and fuzzy logic but need more than a tutorial. For a more advanced textbook, IEEE PRESS, as well as other publishers, has a number of them available by catalog. I wish you happy and easy reading.

ACKNOWLEDGMENTS

Throughout history, few achievements of note have been the production of individual effort but, instead, have been accomplished through the efforts of many. Unfortunately, the many have been rarely recognized. Consider these contributions: of the anonymous scribes to the walls of the pyramids; of the stone cutters, polishers, and scribes to the creation of the Rosetta stone; and of the writers, illuminators, parchment makers and book binders to the manufacture of illuminated manuscripts. Durer's woodcuts are famous but surely they are the product of many laborers and not the achievement of just one, albeit gifted, individual. Therefore, to avoid repeating this regrettable habit of history, I wish to express my gratitude to all those that helped make this book a reality.

I would first like to acknowledge Dudley Kay, Director of Book Publishing at IEEE PRESS, who provided the stimulus and encouragement for transforming a tutorial manuscript into a volume in the IEEE PRESS Understanding Science & Technology Series. I would also like to thank the IEEE PRESS staff for their many contributions; in particular, Lisa Mizrahi for her enthusiasm and ability in obtaining constructive book reviews and Debbie Graffox for handling the production of this book. Deserved thanks also go to the reviewers who provided the valuable criticism that improved the scope and readability of the book, both those listed on the copyright page and those who prefer anonymity, and to the professionals at Beehive Production Services, Roaring Mountain Editorial Services, the Asterisk Group, Inc., and Techsetters, Inc., for their valued assistance.

I also thank my anonymous colleagues at AT&T who reviewed and approved the text, enabling me to publish my manuscript with IEEE PRESS. Finally, I would like to thank my family for the infinite patience and understanding they've demonstrated in allowing me to allocate family time to the writing of this book.

INTRODUCTION

A NEW BREED OF PROCESSOR: THE BRAIN

A new kind of arithmetic, called **Boolean logic**, was developed in the 19th century. The product of propositional logic, Boolean logic was based on binary rather than decimal arithmetic. Most people thought it useless, so it remained in obscurity for decades. However, Boolean logic was rediscovered and, along with integrated circuitry, brought to light the microprocessor and the modern computer.

The modern computer, based on binary arithmetic in conjunction with sophisticated programming, has changed the way we do business and exchange information. In many ways, it has changed our lifestyles and thinking. It is a primary tool in science especially in the development of intelligent machines, applications such as information processing (data, video, speech, etc.), intelligent communications networks, and control applications, from sophisticated research instruments to dishwashers. Despite the outstanding performance of today's computer, there is an increasing demand for higher speed, larger storage capacity, greater machine intelligence, and "ingenuity." Computer power keeps increasing while its size, as well as cost, keeps decreasing. In the computer industry, the pressure is on to have a "next generation" approximately every six to twelve months, with still lower cost and greater performance.

Advance in microelectronics continually shrink the size of the transistor, so that increasingly more circuitry (measured by many millions of transistors) is integrated into less silicon space. At the beginning of the 1970s the number of transistors integrated was around a few thousand, whereas in the beginning of the 1990s it was several million. In 1980,

the largest random access memory circuit was 64,000 bits; by the end of the decade it was 1 million bits and before the end of this decade it is expected to be 64 million bits. Moreover, transistor power consumption keeps decreasing, making it possible to use smaller, longer-lasting batteries (necessary for portable computers and communicators), thereby increasing the switching speed of the transistor and its performance. Presently, microprocessors with speeds above 100 MHz are supplied by several vendors and, at this rate of evolution, it won't be long before we have speeds of several hundred megahertz.

It is estimated that today's top-performance processor, with 100–150 million instructions per second (MIPS), will be the lower performer in just a few years. One of the metrics for evaluating the performance of a processor is the SPECint92 (for System Performance Evaluation Corporation integer calculations). Based on this metric, today's top performers have in excess of 100 SPECint92; before the end of the century they are expected to have more than 1000 SPECint92 (see Figure 1). As soon as greater speed is achieved, new applications emerge that demand even more.

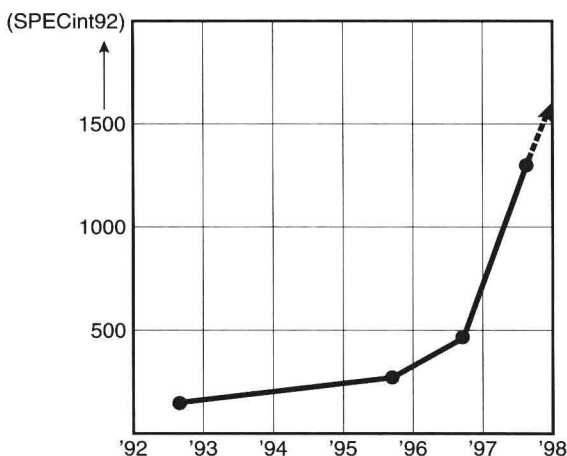


Figure 1 Expected processor performance increase.

With this gargantuan appetite for more computing power at lower cost, can current computer architecture satisfy us? Can present technology evolve at this rate ad infinitum? When will its limitations be reached?

Current technology is at an exponential rate of performance/cost increase, which in nature means, first, a rapid increase in exponential form and then, before an uncontrollable state is reached, saturation, i.e., no further increase. Thus, being in the exponential phase, advances in technology

should start slowing down at the start of the next millenium and should soon reach a plateau beyond which no significant performance/cost improvements can be achieved with the same processor architecture. Therefore, researchers are looking into new, more efficient, processor architectures. For example, the simple “pipeline” architecture of microprocessors used in personal computers has been replaced by the CISC (complex instruction set computer), which, in turn, is being replaced by the RISC (reduced instruction set computer) architecture, which itself will be soon replaced by superscalar and multiprocessor computer architectures. Each new architecture is the next optimum and, once all possible architectures have been explored, sooner or later a plateau of performance-to-cost will be reached, beyond which significant optimization cannot be achieved.

This forecasted plateau by no means implies the death of the microprocessor. The next century will demand high-performance processors coupled with sophisticated computing algorithms and techniques, such as genetic algorithms and evolutionary programming, and myriads of sophisticated applications will be seen. Therefore, the scientific community is searching not only the next generation of computing but also for the next **breed of processing machines**—small machines many times faster and more potent than those yet developed that can rapidly process massive amounts of data and that will figuratively learn, listen, and “think.” But to create this “brainlike machine,” revolutionary theories, technology, and architectures, such as the following, are required:

- Theories that explain what intelligence is, how it processes imprecise information, and stores, recalls, associates, correlates, infers, and extracts precise values
- Technology that, with a relatively small amount of circuitry, can process vast amounts of imprecise information in a very short time and provide precise results
- Architectures that encompass the new theories and technologies

THE ENGINEERING OF THE BRAIN

Biologists have studied **biological neural networks** for many years. The human brain is such a network. Discovering how the brain works has been an ongoing effort that started more than 2000 years ago with Aristotle and Heraclitus and has continued with the work of Ramon y Cajal, Colgi, Hebb,

and others. The better we understand the brain, the better we can emulate it and build artificial “thinking machines” and “repair” biological damage that leads to brain disorders.

As information about the functions of the brain was accumulated, a new technology emerged and the quest for an **artificial neural network** started. The brain processes information superquickly and superaccurately. It can be trained to recognize patterns and to identify incomplete patterns. Moreover, the trained network performs even if certain neurons fail. For example, even in a noisy football stadium with many thousands of people, we can still recognize a friend from afar or distinguish voices from the pandemonic noise. This ability of the brain (signal processing) to recognize information, literally buried in noise, and retrieve it correctly is one of the amazing processes that we wish could be duplicated by machine. Hence, if we manage to build a machine—an artificial neural network—that emulates the human brain, even at only 0.1% of its performance, we still have an extraordinary information processing and controlling machine. These training and learning features make neural networks suitable for applications in signal processing (image, speech, or data), control (robotics, power systems, communications systems, intelligent automotive vehicles), and many other fields.

Artificial neural networks made a rapid transition from the cognitive and neurobiology field to engineering with the pioneering work of McCulloch and Pitts, Rosenblatt, Widrow, Kohonen, Grossberg and Carpenter, Hopfield, Werbos, Anderson, and many others, who developed paradigms that are still applied today. Engineers from all disciplines (such as hardware, software, systems, and materials) are working on artificial neural networks.

A WORLD OF FUZZY THINKING

Parallel to the development of neural network theory, **fuzzy theory** or **fuzzy logic** emerged, with the pioneering work of Lotfi Zadeh, and immediately drew the attention of those technologists who had a special interest in artificial neural networks.

What is fuzzy theory? Why is the term *fuzzy* used? “Fuzziness” is found in our decisions, in our thinking, in the way we process information, and, particularly, in our language; statements can be unclear or subject to different interpretation. Phrases like “see you later,” “a little more,” or “I

don't feel very well" are fuzzy expressions. The fuzziness stems from the different interpretations or perceptions we give to "later," "a little more," and "very well." For example, "later" for fast-phenomena engineers may be on the order of nanoseconds, but for paleontologists it may be on the order of thousands of years. The order of magnitude is relative; therefore, if *some* fuzzy units are used, one should look at it within its context and find a point of reference and a measuring unit.

Occasionally, fuzzy statements indicate relative units and subunits that do not indicate absolute units. Consider this example: "Runner A is fast," "runner B is faster than A," and "runner C is slower than B." We make two observations: Fuzzy statements may establish *taxonomy* (B is faster than A, and C is slower than B) or *ambiguity* (it is not clear if A is faster than C) and there is no measure of the speed of A, B, or C. The statement "George is very tall" is fuzzy because there is no reference measurement. On a basketball team with an average height of 6 ft 2 in, "very tall" most likely means taller than 6 ft 2 in. To the average person, "very tall" often means taller than 5 ft 8 in, often but not necessarily 6 ft 2 in.

Fuzziness is often confused with probability. A statement is probabilistic if it expresses a likelihood or degree of certainty or if it is the outcome of clearly defined but randomly occurring events. For example, the statement "There is a 50/50 chance that I'll be there" is purely probabilistic. Probability itself can have some degree of fuzziness. In the statement "Most likely I'll be there," all odds have been mentally weighed and some degree of certainty or probability has been expressed. On the other hand, the statement "I may be there" expresses complete uncertainty, undecidability, and, hence, fuzziness.

CRISP VERSUS FUZZY LOGICS

You are probably familiar with logic that has well-defined decision levels or thresholds (binary, multivalued). *Boolean* or *binary logic* is based on two *crisp extremes*—yes–no or 1–0. Yes or no is an answer beyond doubt. *Trivalent logic* is a logic of three definite answers, such as empty–half full–full or 0–0.5–1. The binary numbers 1 or 0, or 1, 0.5, 0 in trivalent logic represent normalized thresholds. Similarly, the *multivalued logic* has many well-defined threshold levels.

Fuzzy logic, however, has *unclear* thresholds. For example, if we take the trivalent logic and *fuzzify* it (i.e., change the crisp thresholds to

obscure ones), then the values of the thresholds can be stated as a range of values. The crispness of the numbers 0, 0.5, and 1 may be replaced by “from 0 to about 0.4,” “from about 0.2 to about 0.8,” and “from about 0.6 to 1,” respectively. For example, if you look at three distinct dots through a well-focused camera lens, you will see the dots with crisp perimeters. If the image is out of focus, however, the dots become unclear and “fuzzy,” perhaps overlapping each other. This action is termed *fuzzification* and in fuzzy control systems is routinely done.

Fuzzy logic has been applied to military intelligence machines, the stock market, and even dishwashers. In communications, it has been used on the systems level and in signal processing. On the systems level, fuzzy logic applications determine the best parameter values for call switching, call routing, system reconfiguration, and so on. In signal processing, fuzzy logic applications determine the degree of the fuzzified received signal (distortions due to environmental variations, electrical interferences, medium mismatches, and other) and then “defuzzify” the signal. In a nutshell, fuzzy logic is a powerful tool for the intelligent retrieval of nonstatistical, ill-defined information in static, sequential, and real-time applications.

FUZZY AND NEURAL NETWORKS

Artificial neural networks and fuzzy logic work together, artificial neural networks classify and learn rules for fuzzy logic and fuzzy logic infers from unclear neural network parameters. The latter is a network with fast learning capabilities that produces intelligent, crisp output from fuzzy input and/or from fuzzy parameters and avoids time-consuming arithmetic manipulation.

Incorporating fuzzy principles in a neural network gives more user flexibility and a more robust system. Fuzziness in this case means more flexibility in the definition of the system; boundaries may be described more generally, not crisply; inputs may be described more vaguely, yet better control may be obtained. The network itself may be fuzzy, not well defined, and able to reconfigure itself for best performance. The power of such machines may be illustrated with the following “gedanken” examples.

Visualize a machine that has learned to analyze scenery, animals, other machines, and other items. A user describes a vague scene in terms of features such as “something like a tree, about here” and “something like an animal with four legs and a long tail and so tall, there,” and so on. Then

the machine draws a three-dimensional landscape with a tree and a dog nearby (and perhaps a mountain in the background, with a lake). Then the user may instruct the machine to make corrections to this scene, again in vague language, and the machine immediately projects a three-dimensional scene, very similar to the one the user had in mind. As all that is done, a train with a whistling sound may be crossing the scene (if the parameters are set right) and nearby a frightened bird flies away.

Imagine a machine that is instructed to design a new three-dimensional machine, based on some approximate specifications. Our *gedanken* machine designs a model from the vague specifications, simulates the created machine, makes corrections on the model, and, if the corrected one performs as expected, manufactures the first prototype—all in just a few minutes!

WHERE ARE FUZZY NEURAL NETWORKS HEADING?

Fuzzy logic follows the same path as Boolean and multiple value logic. Initially, binary logic started as a linguistic set of statements, such as if $A = B$, and if $B = C$, then is $A = C$? Then mathematical notation translated the linguistic statements into equations and theories were developed that are taught today. These theories have been applied successfully in the development of many logical applications.

Fuzzy logic also started as a linguistic set of statements. For example, if A is taller than B , B is shorter than C , what is A with respect to C ? A number of mathematical theories can be found in the literature. Thus, we may make a reasonable extrapolation and deduce that fuzzy logic will prove itself as binary logic did. The fusion of fuzzy logic and neural networks combines the best of each. Fuzzy concepts fused with “thinking” promise superior technology. These claims are validated by various integrated circuits, fuzzy controllers for general applicability, and applications for automobile engine control, robot control, cameras (film and video), appliances, and the military. In addition to hardware solutions, numerous “fuzzy algorithmic solutions” have been applied in communications, signal processing (speech, image), and other areas. The number of companies banking on fuzzy logic is growing rapidly. Many significant American, European, and Asian-Pacific companies have announced products or are exploring and advancing fuzzy logic for potential applicability in their own

products (see Chapter 6 for examples). In the near future we will see applications that encompass algorithmic fuzzy logic, fuzzy neural networks, and combinations of fuzzy and/or neural networks with high-performance microprocessors.

OBJECTIVES

The objectives of this text are simple and crisp: to provide a simplified yet comprehensive description of the concepts and potential applications of neural networks and fuzzy logic, to give an insight into fuzzy neural networks, and to demonstrate their applicability through examples.

Chapter 1 is an overview of biological neural networks. Specifically, I briefly describe the physiology of the neuron and neural networks. The intent is to sketch out the amazing microcosm of live neurons, including their function and organization. Chapter 2 describes concepts of artificial neural networks, most of which come from the biological and behavioral sciences. Chapter 3 provides a tutorial of the most popular paradigms and a brief description of several others. The first part of Chapter 4 provides a tutorial of fuzzy logic set forth by basic examples, and the second part is a more advanced treatment of temporal fuzzy logic, yet all are simplified to the greatest degree possible. Chapter 5 describes how fuzzy logic and neural networks are combined to build a fuzzy neural network, and Chapter 6 describes applications with neural networks, fuzzy logic, and fuzzy neural networks.

The material is organized so that it will serve both the reader who wants a simple introduction to the subject and the reader who is at an undergraduate level. To achieve this, we have followed four rules:

- The reader is not a biologist or a mathematician.
- The language has been simplified to eliminate unnecessary jargon yet retain necessary terminology.
- Mathematical description has been reduced to basics and those sections that involve extensive mathematics have been segregated in a description and math section. Thus, the reader may skip the math section at the first reading without any loss of understanding.
- Mathematical notation has been simplified. In a few cases, for simplicity, vectorial notation is used. Simple numerical examples demonstrate the math applicability and clarify any difficulties.

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