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Martin Henson (Eds.)

Logics of Specification Languages

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Logics of Specification Languages

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For this we need resort to the proof system of the specification language — as well as to other means. We consider in this prelude three such means.

Verification

Verification, in general terms, is a wide and inclusive term covering all approaches which have the aim of establishing that a system meets certain properties. Even a simple *test case* demonstrates a, perhaps limited, fact: that in *this* case (though maybe no others) a given system achieves (or does not) a desirable outcome.

More specifically and usually, we use the term *verification* for more elaborate and systematic mathematical techniques for establishing that systems possess certain properties. Here, the *system* might be a more-or-less abstract description (a specification) or a concrete realisation in hardware or software. The *properties* may be specific emergent properties of abstract specifications; they include general statements of, say, *liveness*, *safety* and/or *termination*; and they cover the *correctness* of realisations or implementations of given system specifications. In all the cases of interest to us, the system description and the properties to be determined will be couched in a precise formal mathematical language. As a consequence, the results of such a verification will be correspondingly precise and formal.

There are three forms of formal verification that are relevant to the material covered in this book and that are, therefore, worth describing in just a little more detail.

Inferential Verification

This approach is often simply referred to as *verification* despite the fact that other approaches, such as model checking, are also such methods. Here, we have at our disposal logical principles, a logic or proof system, which correctly captures the framework within which the system is described. This framework might be a programming or specification language with a semantics which lays down, normatively, its meaning. The logical principles will (at the very least) be *sound* with respect to that semantics; thus ensuring that any conclusions drawn will be correct judgements of the language in question.

The logical principles, or fully-fledged logic, will provide means that are appropriate for reasoning about the techniques and mechanisms that are available in the language of description. For example, many frameworks provide a means for describing recursive systems, and appropriate induction principles are then available for reasoning about such systems.

Inference-based methods of verification allow us to make and support general claims about a system. These may demonstrate that an implementation is *always* guaranteed to meet its specification; that it *always* possesses certain characteristic properties (for example, that it is *deadlock-free* or maybe that it

terminates); or that an abstract specification will always possess certain implicit properties (which will, in turn, be inherited properties of *any* (correct) implementation).

Model Checking

This approach to verification (see, for example, [6]) aims to automatically establish (or provide a counterexample for) a property by direct inspection of a model of the system in question. The model may be represented (explicitly or implicitly) by a directed graph whose nodes are states and whose edges are legitimate state transitions; properties may be expressed in some form of temporal logic.

Two key issues are *finiteness* and the potential *combinatorial explosion* of the state space. Many techniques have been developed to minimise the search. In many cases it is not necessary to build the state graph but simply to represent it symbolically, for example by propositional formulae, and then, using techniques such as SAT-solvers, to mimic the graph search. Partial order reductions, which remove redundancies (in explicit graphs) arising from independent interleavings of concurrent events can also be employed to significantly reduce the size of the search space. It is also possible to simplify the system, through abstraction, and to investigate the simpler model as a surrogate for the original system. This, of course, requires that the original and abstracted systems are related (by refinement) and that the abstracted system is at least *sound* (if not *complete*) with respect to the original: that properties true of the abstracted system are also true of the original, even if the abstracted system does not capture *all* properties of the original.

Model checking has been a spectacularly successful technology by any measure; the model checker SPIN [23], for example, detected several crucial errors in the controller for a spacecraft [21]. Other important model checkers are SMV [31] and FDR, based on the standard *failures-divergencies* model of CSP [42].

Formal Testing

Dijkstra, in his ACM Turing Lecture in 1972, famously said: “... *program testing can be a very effective way to show the presence of bugs, but is hopelessly inadequate for showing their absence*” [9]. A correct contrast between informal testing (which might demonstrate a flaw in a system) and a formal verification (which might make a general correctness claim) was established by this remark. More recently, however, it has become clear that there is something to be gained by combining variations on the general theme of testing with formal specifications and verifications. Indeed, the failure of a *formal* test is a *counterexample*, which is as standard a mathematical result as could be wished for (and potentially as valuable too); the problem is that when testing

without a theoretical basis (informal testing), it is often simply unclear what conclusion can and should be drawn from such a methodology.

A portfolio approach, in which a variety of verification methods are used, brings benefits. In the case of *formal* testing, there is an interplay between test (creation, application and analysis) and system specification: a formal description of a system is an excellent basis for the generation (possibly automatically) of test cases which, themselves, have precise properties regarding coverage, correctness and so on. In addition, the creation of adequate test suites is expensive and time-consuming, not to say repetitious if requirements and specifications evolve; exploiting the precision implicit in formal specification to aid the creation of test suites is a major benefit of formal testing technologies.

1.3 Integration of Specification Languages

Domains, requirements or software being described, prescribed or designed, respectively, usually possess properties that cannot be suitably specified in one language only. Typically a variety, a composition, a “mix” of specification notations need be deployed. In addition to, for example, either of ASM, B, CafeOBJ, CASL, RAISE/RSL, VDM or Z, the specifier may resort to additionally using one or more (sometimes diagrammatic) notations such as Petri nets [27, 35, 37–39], message sequence charts [24–26], live sequence charts [7, 19, 28], statecharts [15–18, 20], and/or some textual notations such as temporal logics (Duration Calculus, TLA+, or LTL — for linear temporal logic [10, 29, 30, 34, 36]).

Using two or more notations, that is, two or more semantics, requires their integration: that an identifier a in one specification (expressed in one language) and “the same” identifier (a) in another specification (in another language) can be semantically related (i.e., that there is a ‘satisfaction relation’).

This issue of integrating formal tools and techniques is currently receiving high attention as witnessed by many papers and a series of conferences: [1, 3, 4, 13, 41]. The present book will basically not cover integration.¹

2 Structure of Book

The book is structured as follows: In the main part, Part II, we introduce, in alphabetic order, nine chapters on ASM, event-B, CafeOBJ, CASL, DC, RAISE, TLA⁺, VDM and Z. Each chapter is freestanding: It has its own list of references and its own pair of symbol and concept indexes. Part III introduces just one chapter, Review, in which eight “originators” of respective specification languages will comment briefly on the chapter on “that language”.

¹ TLA⁺ can be said to be an integration of a temporal logic of actions, TLA, with set-theoretical specification. The RAISE specification language has been “integrated” with both Duration Calculus and concrete timing.

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