Quantum Electrodynamics

Walter Greiner Joachim Reinhardt

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With a Foreword by D. A. Bromley

With 146 Figures

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Professor Dr. Walter Greiner
Dr. Joachim Reinhardt

Institut für Theoretische Physik, Universität Frankfurt, Robert-Mayer-Str. 8–10
W-6000 Frankfurt am Main 11, Fed. Rep. of Germany

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Foreword

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics – with mathematics, the most fundamental of sciences – using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicists remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through use of consistent notation, logical ordering of topics and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

The present five volume set, *Theoretical Physics*, is in fact only that part of the complete set of textbooks developed by Greiner and his students that presents the quantum theory. I have long urged him to make the remaining volumes on classical mechanics and dynamics, on electromagnetism, on nuclear and particle physics, and on special topics available to an English-speaking audience as well, and we can hope for these companion volumes covering all of theoretical physics some time in the future.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that..." which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to a working physicist. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1798–1857) in his *Positive Philosophy* who noted, "To understand a science it is necessary to know its history". This is all too often forgotten in modern physics teaching and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner's lectures, which underlie these volumes, are internationally noted for their clarity, their completeness and for the effort that he has devoted to making physics an integral whole; his enthusiasm for his science is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again the reader will find that, after dipping into a particular volume to review a specific topic, he will end up browsing, caught up by often fascinating new insights and developments with which he had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

D. Allan Bromley
Henry Ford II Professor of Physics
Yale University
New Haven, CT USA

Preface

Theoretical physics has become a many-faceted science. For the young student it is difficult enough to cope with the overwhelming amount of new scientific material that has to be learned, let alone obtain an overview of the entire field, which ranges from mechanics through electrodynamics, quantum mechanics, field theory, nuclear and heavy-ion science, statistical mechanics, thermodynamics, and solid-state theory to elementary-particle physics. And this knowledge should be acquired in just 8–10 semesters, during which, in addition, a Diploma or Master's thesis has to be worked on or examinations prepared for. All this can be achieved only if the university teachers help to introduce the student to the new disciplines as early on as possible, in order to create interest and excitement that in turn set free essential new energy. Naturally, all inessential material must simply be eliminated.

At the Johann Wolfgang Goethe University in Frankfurt we therefore confront the student with theoretical physics immediately, in the first semester. Theoretical Mechanics I and II, Electrodynamics, and Quantum Mechanics I – An Introduction are the basic courses during the first two years. These lectures are supplemented with many mathematical explanations and much support material. After the fourth semester of studies, graduate work begins, and Quantum Mechanics II – Symmetries, Statistical Mechanics and Thermodynamics, Relativistic Quantum Mechanics, Quantum Electrodynamics, the Gauge Theory of Weak Interactions, and Quantum Chromodynamics are obligatory. Apart from these a number of supplementary courses on special topics are offered, such as Hydrodynamics, Classical Field Theory, Special and General Relativity, Many-Body Theories, Nuclear Models, Models of Elementary Particles, and Solid-State Theory. Some of them, for example the two-semester courses Theoretical Nuclear Physics or Theoretical Solid-State Physics, are also obligatory.

This volume of lectures deals with the subject of *Quantum Electrodynamics*. We have tried to present the subject in a manner which is both interesting to the student and easily accessible. The main text is therefore accompanied by many exercises and examples which have been worked out in great detail. This should make the book useful also for students wishing to study the subject on their own.

When lecturing on the topic of quantum electrodynamics, one has to choose between two approaches which are quite distinct. The first is based on the general methods of quantum field theory. Using classical Lagrangian field theory as a starting point one introduces noncommuting field operators, builds up the Fock space to describe systems of particles, and introduces techniques to construct and evaluate the scattering matrix and other physical observables. This program can be realized either by the method of canonical quantization or by the use of path integrals. The theory of quantum electrodynamics in this context emerges just as a particular example of the general formalism. In the present volume, however, we do not follow this general but lengthy path; rather we use a "short cut" which arrives at the same results with less effort, and which has the advantage of great intuitive appeal. This is the propagator formalism, which was introduced by R.P. Feynman (and, less well known, by E.C.G. Stückelberg) and makes heavy use of Green's functions to describe the propagation of electrons and photons in space-time.

It is clear that the student of physics has to be familiar with both approaches to quantum electrodynamics. (In the German edition of these lectures a special volume is dedicated to the subject of field quantization.) However, to gain quick access to the fascinating properties and processes of quantum electrodynamics and to its calculational techniques the use of the propagator formalism is ideal.

The first chapter of this volume contains an introduction to nonrelativistic propagator theory and the use of Green's functions in physics. In the second chapter this is generalized to the relativistic case, introducing the Stückelberg–Feynman propagator for electrons and positrons. This is the basic tool used to develop perturbative QED. The third chapter, which constitutes the largest part of the book, contains applications of the relativistic propagator formalism. These range from simple Coulomb scattering of electrons, scattering off extended nuclei (Rosenbluth's formula) to electron–electron (Møller) and electron–positron (Bhabha) scattering. Also, processes involving the emission or absorption of photons are treated, for instance, Compton scattering, bremsstrahlung, and electron–positron pair annihilation. The brief fourth chapter gives a summary of the Feynman rules, together with some notes on units of measurement in electrodynamics and the choice of gauges.

Chapter 5 contains an elementary discussion of renormalization, exemplified by the calculation of the lowest-order loop graphs of vacuum polarization, self-energy, and the vertex correction. This leads to a calculation of the anomalous magnetic moment of the electron and of the Lamb shift. In Chap. 6 the Bethe–Salpeter equation is introduced, which describes the relativistic two-particle system.

Chapter 7 should make the reader familiar with the subject of quantum electrodynamics of strong fields, which has received much interest in the last two decades. The subject of supercritical electron states and the decay of the neutral vacuum is treated in some detail, addressing both the mathematical description and the physical implications. Finally, in the last chapter, the theory of perturbative quantum electrodynamics is extended to the treatment of spinless charged bosons.

An appendix contains some guides to the literature, giving references both to books which contain more details on quantum electrodynamics and to modern treatises on quantum field theory which supplement our presentation. We should mention that in preparing the first chapters of our lectures we have relied heavily on the textbook *Relativistic Quantum Mechanics* by J.D. Bjorken and S.D. Drell (McGraw-Hill, New York 1964).

We enjoyed the help of several students and collaborators, in particular Jürgen Augustin, Volker Blum, Christian Borchert, Snježana Butorac, Christian Derreth, Bruno Ehrnsperger, Klaus Geiger, Mathias Grabiak, Oliver Graf, Carsten Greiner, Kordt Griepenkerl, Christoph Hartnack, Cesar Ionescu, André Jahns, Jens Konopka, Georg Peilert, Jochen Rau, Wolfgang Renner, Dirk-Hermann Rischke, Jürgen Schaffner, Alexander Scherdin, Dietmar Schnabel, Thomas Schönfeld, Stefan Schramm, Eckart Stein, Mario Vidovic, and Luke Winckelmann.

We are also grateful to Prof. A. Schäfer for his advice. The preparation of the manuscript was supervised by Dr. Béla Waldhauser and Dipl. Phys. Raffaele Mattiello, to whom we owe special thanks. The figures were drawn by Mrs. A. Steidl.

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Walter Greiner
Joachim Reinhardt

Contents

Contents of Examples and Exercises

1. Propagators and Scattering Theory

1.1 Introduction

In this course we will deal with quantum electrodynamics (QED), which is one of the most successful and most accurate theories known in physics. QED is the quantum field theory of electrons and positrons (the electron–positron field) and photons (the electromagnetic or radiation field). The theory also applies to the known heavy leptons (μ and τ) and, in general, can be used to describe the electromagnetic interaction of other charged elementary particles. However, these particles are also subject to nonelectromagnetic forces, i.e. the strong and the weak interactions. Strongly interacting particles (hadrons) are found to be composed of other particles, the quarks, so that new degrees of freedom become important (colour, flavour). It is believed that on this level the strong and weak interactions can be described by "non-Abelian" gauge theories modelled on QED, which is the prototype of an "Abelian" gauge theory. These are the theories of quantum chromodynamics (QCD) for the strong interaction and quantum flavourdynamics for the weak interaction. In this course we will concentrate purely on the theory of QED in its original form.

There are two approaches to QED. The more formal one relies on a general apparatus for the quantization of wave fields; the other, more illustrative, way originates from **Stückelberg** and **Feynman**, and uses the propagator formalism. Nowadays a student of physics has to know both, but it is better, both in terms of the physics and teaching, if it is obvious at an early stage why a formalism was developed and to what it can be applied. Almost everyone is keen to see as early as possible how different processes are actually calculated. Feynman's propagator formalism is the best way to achieve this. Consequently, it will be central to these lectures. References to the less intuitive but more systematic treatment of QED based on the formalism of quantum field theory are given in the appendix.

For the moment we turn to a more general discussion of scattering processes. The aim here is to calculate transition probabilities and scattering cross sections in the framework of Dirac's theory of electrons and positrons. These calculations will be exact in principle; practically, however, they will be carried out using perturbation theory, that is an expansion in terms of small interaction parameters. Because we have to describe the creation and annihilation processes of electron–positron pairs, the formalism has to be relativistic from the beginning.

In Feynman's propagator method, scattering processes are described by means of integral equations. The guiding idea is, that positrons are to be interpreted as electrons with negative energy which move in the reverse time direction. This idea was first formulated by E.C.G. Stueckelberg and was used extensively by R.Feynman.[1] Feynman was rewarded with the Nobel price for his formulation of quantum electrodynamics, together with J. Schwinger and S. Tomonaga in 1965. The latter gave alternative formulations of QED, that are mutually equivalent. In the following we want to convince ourselves of the power of Feynman's formulation of the theory. The more or less heuristic rules obtained in this way fully agree with the results that can be obtained with much more effort using the method of quantum field theory.

1.2 The Nonrelativistic Propagator

First it is useful to remember the definition of Green's functions in nonrelativistic quantum mechanics. The concepts and methods to be acquired here are then easily transferred to relativistic quantum mechanics.

We shall mainly consider quantum-mechanical scattering processes in three dimensions, where one particle collides with a fixed force field or with another particle. A scattering process develops according to the scheme outlined in Fig. 1.1. In practice, one arranges by means of collimators D that the incoming particles are focussed in a well-defined beam. Such a collimated beam is in general not a wave, which extends to infinity, e.g. of the form $\exp(\mathrm{i}kz)$, but a superposition of many plane waves with adjacent wave vectors $\boldsymbol{k}$, i.e. a wave packet. Nevertheless, for simplicity one often represents the incoming wave packet by a plane wave. Then one has only to ensure that interference between the incoming wave packet and the scattered wave is impossible at the position of the detector which is far removed from the scattering centre. If plane waves are used in calculations, therefore one has to exclude this interference explicitly.[2]

In scattering processes we consider wave packets, which develop in time from initial conditions, which were fixed in the distant past. So in general, one does not consider stationary eigenstates of energy (i.e. stationary waves). A typical question for a scattering problem is then: What happens to a wave packet that represents a particle in the distant past and approaches a centre of scattering (a potential or another particle)? What does this wave look like in the distant future?

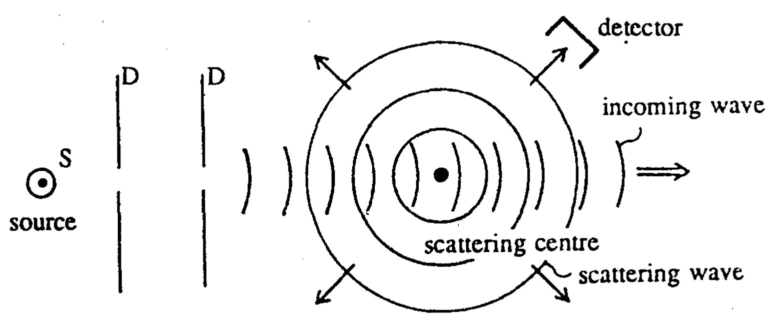

Fig. 1.1. Schematic representation of an experimental arrangement to measure a scattering process. Collimators D ensure that, at the position of the detector no interference occurs between incoming and scattered waves

[1] See for example R.P. Feynman: Phys. Rev. 76, 749 (1949).

[2] For a more detailed discussion of the wave-packet description see for example M.L. Goldberger and K.M. Watson: *Collision Theory* (Wiley, New York 1964), Chap. 3, or R.G. Newton: *Scattering Theory of Waves and Particles* (McGraw-Hill, New York 1966), Chap 6.

Here the generalized Huygens principle helps us to answer these questions. If a wave function $\psi(\boldsymbol{x},t)$ is known at a certain time t, then its shape at a later time t' can be deduced by regarding every spatial point $\boldsymbol{x}$ at time t as a source of a spherical wave that emerges from $\boldsymbol{x}$. It is plausible to assume that the intensity of the wave, which emerges from $\boldsymbol{x}$ and arrives at $\boldsymbol{x}'$ at time t', is proportional to the initial exciting wave amplitude $\psi(\boldsymbol{x},t)$. Let us call the constant of proportionality

$$\mathrm{i}G(\boldsymbol{x}',t';\boldsymbol{x},t) \quad . \tag{1.1}$$

The generalized Huygens principle can thus be expressed in the following terms:

$$\psi(\boldsymbol{x}',t') = \mathrm{i}\int d^3x\, G(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \quad , \quad t' > t \quad . \tag{1.2}$$

Here $\psi(\boldsymbol{x}',t')$ is the wave that arrives at $\boldsymbol{x}'$ at time t'. The quantity $G(\boldsymbol{x}',t';\boldsymbol{x},t)$ is known as the Green's function or propagator. It describes the effect of the wave $\psi(\boldsymbol{x},t)$, which was at point $\boldsymbol{x}$ in the past (at time $t < t'$), on the wave $\psi(\boldsymbol{x}',t')$, which is at point $\boldsymbol{x}'$ at the later time t'. If the Green's function $G(\boldsymbol{x}',t';\boldsymbol{x},t)$ is known, the final physical state $\psi(\boldsymbol{x}',t')$, which develops from a given initial state $\psi(\boldsymbol{x},t)$, can be calculated using (1.2). Knowing G therefore solves the complete scattering problem. Or, in other words: Knowing G is equivalent to the complete solution of Schrödinger's equation. First, however, we want to gain some mathematical insight and discuss the various ways of defining Green's functions.

1.3 Green's Function and Propagator

To explain the mathematical concepts it is best to start with Schrödinger's equation,

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\psi(\boldsymbol{x},t)}{\partial t} &= \hat{H}\psi(\boldsymbol{x},t) = \left(\hat{H}_0 + V(\boldsymbol{x},t)\right)\psi(\boldsymbol{x},t) \quad , \\ \hat{H}_0 &= -\frac{\hbar^2}{2m}\nabla^2 \quad , \end{aligned} \tag{1.3}$$

which describes the interaction of a particle of mass m with a potential source fixed in space. If we replace m by the reduced mass $\mu = m_1m_2/(m_1+m_2)$, (1.3) remains valid for the (nonrelativistic) two-body problem. The differential equation (1.3) is of first order in time, i.e., there are no higher-order time derivatives. Therefore, the first derivative with respect to time, $\partial\psi(\boldsymbol{x},t)/\partial t$, can always be expressed by $\psi(\boldsymbol{x},t)$, which is obviously the meaning of (1.3). From this, in turn, it follows that, if the value of $\psi(\boldsymbol{x},t)$ is known at one certain time (e.g. t_0) and at all spatial points $\boldsymbol{x}$, i.e. if $\psi(\boldsymbol{x},t_0)$ is known, one can calculate the wave function $\psi(\boldsymbol{x},t)$ at any point and any times (at earlier times $(t < t_0)$ as well as at later times $(t > t_0)$). Furthermore, since Schrödinger's equation is linear in ψ, the superposition principle is valid, i.e. solutions can be linearly superposed and the relation between wave functions at different times ($\psi(\boldsymbol{x},t)$ and $\psi(\boldsymbol{x},t_0)$) has to be linear. This means that $\psi(\boldsymbol{x},t)$ has to satisfy a linear homogenous integral equation of the form

$$\psi(\boldsymbol{x}',t') = \mathrm{i}\int d^3x\, G(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \quad , \tag{1.4}$$

where the integration extends over the whole space. This relation also defines the function G, which is called the Green's function corresponding to the Hamiltonian $\hat{H}$. It is important to note that relation (1.4) – in contrast to (1.2) – makes no difference between a propagation of ψ forward in time ($t' > t$) or backward in time ($t' < t$). However, in most cases it is desirable to distinguish clearly between these two cases. For forward propagation one therefore defines the *retarded Green's function* or propagator by

$$G^{+}(\boldsymbol{x}',t';\boldsymbol{x},t)=\begin{cases}G(\boldsymbol{x}',t';\boldsymbol{x},t) & \text{for} \quad t'>t\\ 0 & \text{for} \quad t'<t\end{cases} \quad . \tag{1.5}$$

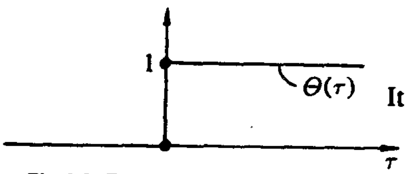

Fig. 1.2. The unit step function

It is now useful to introduce the step function $\Theta(\tau)$ (Fig. 1.2):

$$\Theta(\tau)=\begin{cases}1 & \text{for} \quad \tau>0\\ 0 & \text{for} \quad \tau<0\end{cases} \quad . \tag{1.6}$$

With this the causal evolution of $\psi(\boldsymbol{x}',t')$ from $\psi(\boldsymbol{x},t)$, with $t'>t$, can be formulated as follows:

$$\Theta(t'-t)\psi(\boldsymbol{x}',t')=\mathrm{i}\int d^3x\, G^{+}(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \quad . \tag{1.7}$$

For $t'<t$ this relation is trivial because of (1.5) and (1.6), which together give $0=0$, and for $t'>t$ it is identical with (1.4). Equation (1.7) ensures that the original wave packet $\psi(\boldsymbol{x},t)$ develops into a later $\psi(\boldsymbol{x}',t')$ with $t'>t$. Hence there exists a causal connection between $\psi(\boldsymbol{x}',t')$ and $\psi(\boldsymbol{x},t)$. We will return to this question in Sect. 1.6 and Exercise 1.1. If one wants to describe the evolution backwards in time, it is useful to introduce the *advanced Green's function* G^-:

$$G^{-}(\boldsymbol{x}',t';\boldsymbol{x},t)=\begin{cases}-G(\boldsymbol{x}',t';\boldsymbol{x},t) & \text{for} \quad t'<t\\ 0 & \text{for} \quad t'>t\end{cases} \quad . \tag{1.8}$$

Then the determination of the former wave packet $\psi(\boldsymbol{x}',t')$ from the present one $\psi(\boldsymbol{x},t)$, with $t'<t$, proceeds according to the relation

$$\Theta(t-t')\psi(\boldsymbol{x}',t')=-\mathrm{i}\int d^3x\, G^{-}(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \quad , \tag{1.9}$$

which is again trivial for $t'>t$ because of (1.6) and (1.8) and is identical with (1.4) for $t'<t$.

EXERCISE

1.1 Properties of G

Problem. Show the validity of the following relations:

a) if $t'>t_1>t$:

$$G^{+}(\boldsymbol{x}',t';\boldsymbol{x},t)=\mathrm{i}\int d^3x_1 G^{+}(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1)\times G^{+}(\boldsymbol{x}_1,t_1;\boldsymbol{x},t) \quad ,$$

b) if $t'<t_1<t$:

$$G^{-}(\boldsymbol{x}',t';\boldsymbol{x},t)=-\mathrm{i}\int d^3x_1 G^{-}(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1)\times G^{-}(\boldsymbol{x}_1,t_1;\boldsymbol{x},t) \quad ,$$

c) if $t>t_1$:

$$\delta^3(\boldsymbol{x}-\boldsymbol{x}')=\int d^3x_1 G^{+}(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1)\times G^{-}(\boldsymbol{x}_1,t_1;\boldsymbol{x},t) \quad ,$$

d) if $t<t_1$:

$$\delta^3(\boldsymbol{x}-\boldsymbol{x}') = \int d^3x_1 G^-(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1) \times G^+(\boldsymbol{x}_1,t_1;\boldsymbol{x},t) \quad .$$

Solution. a) The first two assertions (a) and (b) are readily understood because of relations (1.7) and (1.9), respectively. If we consider the propagation of an arbitrary wave packet $\psi(\boldsymbol{x},t)$ into the future, we are able to conclude that

$$\psi(\boldsymbol{x}',t') = \mathrm{i}\int d^3x\, G^+(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \tag{1}$$

if $t' > t$. $\psi(\boldsymbol{x},t)$ can be chosen at any arbitrary time t. Thus we can also insert an intermediate step:

$$\begin{aligned}\psi(\boldsymbol{x}',t') &= \mathrm{i}\int d^3x_1 G^+(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1)\psi(\boldsymbol{x}_1,t_1) \\ &= \mathrm{i}\int d^3x_1 G^+(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1) \\ &\quad \times \mathrm{i}\int d^3x\, G^+(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t) \\ &= \mathrm{i}\int d^3x\, \mathrm{i}\int d^3x_1 G^+(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1) \\ &\quad \times G^+(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\,\psi(\boldsymbol{x},t)\end{aligned} \tag{2}$$

If we compare relations (1) and (2), assertion (a) follows.

b) The proof of case (b) proceeds along similar lines:

$$\psi(\boldsymbol{x}',t') = -\mathrm{i}\int d^3x\, G^-(\boldsymbol{x}',t';\boldsymbol{x},t)\psi(\boldsymbol{x},t) \tag{3}$$

if $t' < t$. Again we insert an intermediate step:

$$\begin{aligned}\psi(\boldsymbol{x}',t') &= -\mathrm{i}\int d^3x_1 G^-(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1)\psi(\boldsymbol{x}_1,t_1) \\ &= -\mathrm{i}\int d^3x_1 G^-(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1) \\ &\quad \times (-\mathrm{i})\int d^3x\, G^-(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t) \\ &= -\mathrm{i}\int d^3x\,(-\mathrm{i})\int d^3x_1 G^-(\boldsymbol{x}',t';\boldsymbol{x}_1,t_1) \\ &\quad \times G^-(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t)\end{aligned} \tag{4}$$

if $t' < t_1 < t$. Comparing relations (3) and (4) assertion (b) follows.

c) The proof of relations (c) and (d) proceeds similarly. We first write

$$\begin{aligned}\psi(\boldsymbol{x}',t) &= \mathrm{i}\int d^3x_1 G^+(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1)\psi(\boldsymbol{x}_1,t_1) \\ &= \mathrm{i}\int d^3x_1 G^+(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1) \\ &\quad \times (-\mathrm{i})\int d^3x\, G^-(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t) \\ &= \int d^3x \int d^3x_1 G^+(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1) \\ &\quad \times G^-(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t)\end{aligned} \tag{5}$$

if $t > t_1$. For a constant time t, $\psi(\boldsymbol{x},t)$ can be expressed with the help of the δ function as

$$\psi(\boldsymbol{x}',t) = \int d^3x\,\delta(\boldsymbol{x}-\boldsymbol{x}')\psi(\boldsymbol{x},t) \quad . \tag{6}$$

The comparison of relations (5) and (6) yields assertion (c).

d) The proof of (c) can be exactly copied

$$\begin{aligned}\psi(\boldsymbol{x}',t) &= -\mathrm{i}\int d^3x_1 G^-(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1)\psi(\boldsymbol{x}_1,t_1) \\ &= \int d^3x \int d^3x_1 G^-(\boldsymbol{x}',t;\boldsymbol{x}_1,t_1) \\ &\quad \times G^+(\boldsymbol{x}_1,t_1;\boldsymbol{x},t)\psi(\boldsymbol{x},t)\end{aligned} \tag{7}$$

if $t < t_1$. Comparing (7) with the integral representation (6) proves (d).