THE
ELEMENTS OF AEROFOIL
AND AIRSCREW THEORY

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H. GLAUERT

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PREFACE

The aim of aerofoil theory is to explain and to predict the force experienced by an aerofoil, and a satisfactory theory has been developed in recent years for the lift force in the ordinary working range below the critical angle and for that part of the drag force which is independent of the viscosity of the air. Considerable insight has also been obtained into the nature of the viscous drag and into the behaviour of an aerofoil at and above the critical angle, but the theory remains at present in an incomplete state. The problem of the airscrew is essentially a part of aerofoil theory, since the blades of an airscrew are aerofoils which describe helical paths, and a satisfactory theory of the propulsive airscrew has been developed by extending the fundamental principles of aerofoil theory.

The object of this book is to give an account of aerofoil and airscrew theory in a form suitable for students who do not possess a previous knowledge of hydrodynamics. The first five chapters give a brief introduction to those aspects of hydrodynamics which are required for the development of aerofoil theory. The following chapters deal successively with the lift of an aerofoil in two dimensional motion, with the effect of viscosity and its bearing on aerofoil theory, and with the theory of aerofoils of finite span. The last three chapters are devoted to the development of airscrew theory.

In accordance with the object of the book, complex mathematical analysis has been avoided as far as possible and in a few cases results have been quoted without proof, the reader being referred for further details to standard text-books or to original papers on the subject.

My thanks are due to my wife for her assistance in preparing a number of the figures and in reading the proof sheets, and to the Cambridge University Press for their care and vigilance in passing the book through the proof stage.

<div align="right">H. G.</div>

Farnborough, April 1926.

PREFACE TO SECOND EDITION

Great advances in the theory of aeronautics have taken place since the first edition of this book by my late husband appeared in 1926, but the more fundamental parts of the theory, which are the subject of this book, remain in large measure unchanged. Particularly important advances have been made in the theory of viscous motion and of the flow in the boundary layer. At my request Mr H. B. Squire of the Royal Aircraft Establishment, Farnborough, who was a colleague of my husband, has prepared a set of notes which appear as an Appendix to the present edition and these notes indicate where important developments have taken place and where further information on the subject matter can be found. I am most grateful to Mr Squire for his assistance and desire to tender him my sincere thanks.

In preparing this second edition the opportunity has been taken to replace the non-dimensional k coefficients by the now more generally accepted C coefficients and my son, M. B. Glauert, has undertaken the necessary revision. One or two other minor changes have been made and a bibliography of some of the more important modern books on aerodynamics has been added.

M. G.

Cambridge 1946

REFERENCES

The following abbreviations are used:

RM   = *Reports and Memoranda of the Aeronautical Research Committee.*

NACA = *Reports of the National Advisory Committee for Aeronautics* (U.S.A.).

ZFM   = *Zeitschrift für Flugtechnik und Motorluftschiffahrt.*

ZAMM = *Zeitschrift für angewandte Mathematik und Mechanik.*

FD   = *Modern Developments in Fluid Dynamics.*

CONTENTS

INTRODUCTION

1·1. It is a fact of common experience that a body in motion through a fluid experiences a resultant force which, in most cases, is mainly a resistance to the motion. A class of body exists, however, for which the component of the resultant force normal to the direction of motion is many times greater than the component resisting the motion, and the possibility of the flight of an aeroplane depends on the use of a body of this class for the wing structure.

A wing or aerofoil has a plane of symmetry passing through the mid-point of its span, and the direction of motion and the line of action of the resultant force usually lie in this plane. The section of an aerofoil by a plane parallel to the plane of symmetry is of an elongated shape with a rounded leading edge and a fairly sharp trailing edge. The chord line of an aerofoil is defined as the line joining the centres of curvature of the leading and trailing edges and the projection of the aerofoil section on this line is defined as the chord length. Aerofoil sections which are used on airscrews are flat over most of the lower surface and the chord line of these sections is usually taken along the flat under-surface of the
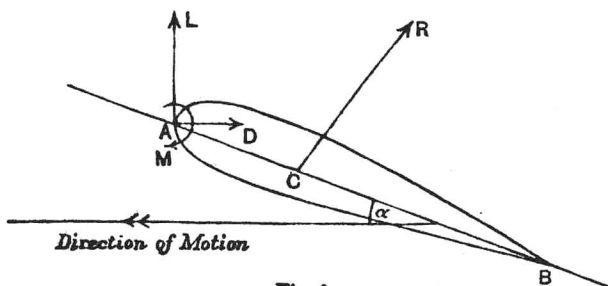

Fig. 1.

aerofoil. The angle of incidence α of an aerofoil is defined as the angle between the chord and the direction of motion

relative to the fluid, and the centre of pressure C of an aerofoil is defined as the point in which the line of action of the resultant force R intersects the chord AB (fig. 1). The resultant force is resolved into two components, the lift L at right angles to the direction of motion and the drag D parallel to that direction but opposing the motion. It is customary to use the leading edge A of the chord as a point of reference and the resultant force has a moment M about this point, whose sense is such that a positive moment tends to increase the angle of incidence*. The magnitude of this moment is

$$M = -AC\,(L\cos\alpha + D\sin\alpha),$$

where AC is the distance of the centre of pressure behind the leading edge of the chord.

The resultant force on an aerofoil of a given shape at a definite angle of incidence depends mainly on the density ρ of the fluid, the relative velocity V of the aerofoil and the fluid, and some typical length l of the aerofoil. These three quantities can be combined in the unique form $l^2\rho V^2$ to give the dimensions of a force, and non-dimensional coefficients of lift and drag may be defined by dividing the force components by this product. The standard lift and drag coefficients of an aerofoil are defined by the equations

$$L = C_L \cdot \tfrac{1}{2}\rho V^2 S,$$
$$D = C_D \cdot \tfrac{1}{2}\rho V^2 S,$$

where S is the maximum projected area of the aerofoil which, in the case of a rectangular aerofoil, is the product of the chord and the span. The corresponding definition for the moment coefficient is

$$M = C_M \cdot \tfrac{1}{2}\rho V^2 Sc,$$

where c is the chord of the aerofoil. These definitions are not unique and the older British practice is to use ρV^2 instead of the dynamic pressure $\tfrac{1}{2}\rho V^2$. This gives coefficients k_L, k_D and k_m half as large as those above.

The lift and drag coefficients of an aerofoil are functions of the angle of incidence and fig. 2 shows the curves for a typical aerofoil, the drag being drawn to five times the scale

* See Note 1 of Appendix.

of the lift. The lift coefficient varies linearly with the angle
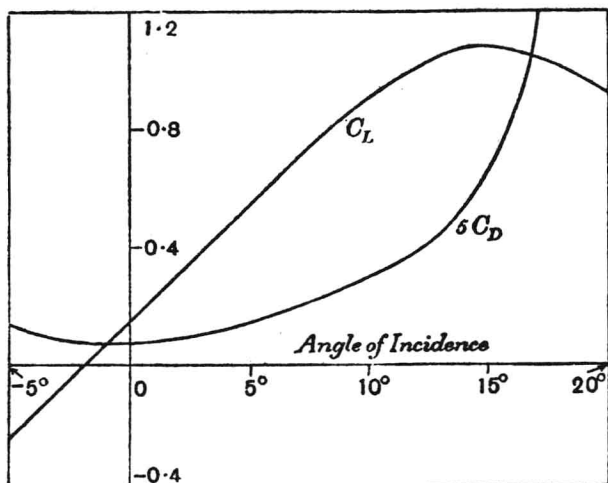of incidence for a certain range and then attains a maximum

Fig. 2.

value at the critical angle of incidence. The important work-
ing range of an aerofoil is represented by the linear part of
the lift curve and in this range the drag is small compared
with the lift, but on approaching the critical angle the drag
increases rapidly.

Fig. 3 shows the variation of the position of the centre of
pressure, the distance of the centre of pressure behind the
leading edge of the aerofoil being expressed as a fraction of
the chord. Analytically this centre of pressure coefficient is

$$\frac{AC}{AB} = -\frac{C_M}{C_L \cos \alpha + C_D \sin \alpha} = -\frac{C_M}{C_L} \text{ (approximately),}$$

and theory and experiment agree in showing that the moment
coefficient varies in a linear manner with the lift coefficient
below the critical angle. The centre of pressure of an aerofoil
section normally moves backwards as the angle of incidence
decreases and tends to infinity at the negative angle of in-
cidence for which $(C_L \cos \alpha + C_D \sin \alpha)$ vanishes, i.e. when

the resultant force on the aerofoil is parallel to the chord. This angle of incidence is approximately equal to the angle at which the lift vanishes.
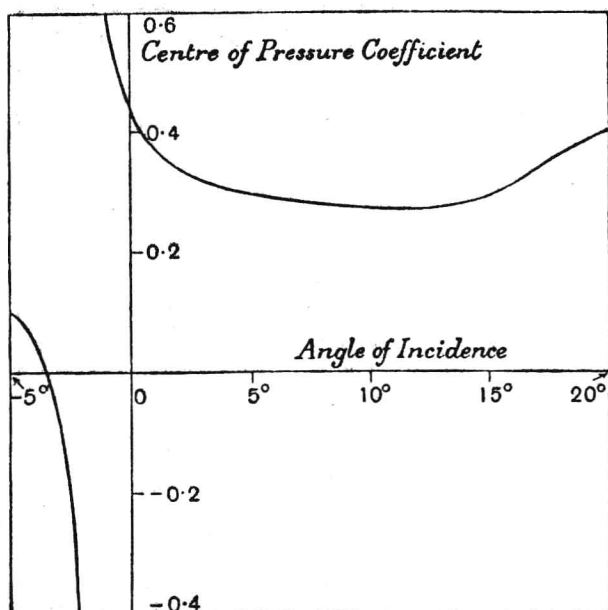
Fig. 3.

The main object of aerofoil theory is to explain and to predict the lift and drag experienced by an aerofoil, and a satisfactory theory has been developed in recent years for the ordinary working range below the critical angle. The determination of the maximum lift of an aerofoil and of the critical angle at which it occurs is not yet possible, although some insight has been obtained into the cause of the phenomenon.

1·2. *The development of aerofoil theory.*

The explanation of the lift force of an aerofoil depends essentially on the nature of the fluid, and the difficulty of obtaining a satisfactory theory is associated with the difficulty

of defining the essential characteristics of the fluid in a simple
and reliable manner.

An early attempt to develop a theory of the force on an
inclined flat plate is due to Newton, who assumed that the
fluid consisted of a large number of solid corpuscles. These
corpuscles were assumed to be inelastic and, on striking the
plate, the component of their velocity normal to the plate
would be destroyed. The mass of fluid meeting a plate of
area S at an angle of incidence α in unit time is $S\rho V \sin \alpha$ and
the velocity normal to the plate is $V \sin \alpha$. Hence the plate
would experience a force normal to its surface of magnitude

$$R = S\rho V^2 \sin^2 \alpha.$$

If the corpuscles are assumed to be perfectly elastic, this
force is doubled, but in either case the force given by this
theory at small angles of incidence is too small. The estimate
of the drag of a flat plate set normal to the direction of
motion is more satisfactory and is of the correct order of
magnitude.

A better definition of the characteristics of a fluid was
obtained by regarding the fluid as a continuous homogeneous
medium. An essential characteristic of a fluid is that it
cannot support tangential stresses in a state of equilibrium,
but when adjacent layers of the fluid are in relative motion
tangential stresses exist and oppose the motion. This charac-
teristic is due to the internal friction or viscosity of the fluid.
The viscosity of the air is small and may be neglected in a
large number of problems, but at times the viscosity is of
fundamental importance and in all cases it appears to exert
a determining influence on the type of motion which occurs,
even when the motion proceeds exactly as in a non-viscous
fluid. Another characteristic of a fluid is its compressibility,
which is negligible for a liquid but important for a gas. The
density of the air must be regarded in general as a function
of the pressure and temperature, but the variations of the
pressure in the flow past a body are sufficiently small to
justify the assumption that the density of the air is constant.
This assumption, however, ceases to be valid when the

velocity of the flow becomes comparable in magnitude with the velocity of sound and allowance must then be made for the compressibility of the air.

These considerations led to the conception of the air as a perfect fluid, i.e. as a continuous incompressible non-viscous medium. The development of the theory of fluid motion has been based on this conception and the results deduced from the theory are of great value in many cases. Unfortunately the theory led to the astonishing conclusion that a body in motion through a perfect fluid does not experience any resultant force.

An attempt to surmount this discrepancy between theory and fact was made by Helmholtz and Kirchhoff by assuming that the flow past a body, instead of passing round the whole surface, leaves a wake or dead-water region behind the body. This method of discontinuous flow* has been applied to an inclined flat plate in two dimensional motion, which is equivalent to an aerofoil of infinite span, and gives a resultant force normal to the surface of magnitude

$$R = \frac{\pi \sin \alpha}{4 + \pi \sin \alpha} S\rho V^2.$$

This force is of the correct order of magnitude for small angles of incidence and also for a flat plate set normal to the direction of motion, but the actual numerical values are not in good agreement with experimental results.

A lift force can also be obtained in a perfect fluid if the flow is assumed to have a tendency to circulate round the body, and modern aerofoil and airscrew theory is based on this conception. The development of the theory for an aerofoil of infinite span, which corresponds to motion in two dimensions, is due in the first place to Kutta† and Joukowski‡, and

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* For the development of the theory see Lamb, *Hydrodynamics*, § 73 *et seq.*

† "Auftriebskräfte in strömenden Flüssigkeiten," *Illust. aeronaut. Mitteilungen*, 1902. "Über eine mit den Grundlagen des Flugproblems in Beziehung stehende zwei dimensionale Strömung," *Sitzb. d. k. Bayr. Akad. d. Wiss.* 1910.

‡ "Über die Konturen der Tragflächen der Drachenflieger," *ZFM*, 1910.

the extension to the general case in three dimensions, which follows the general lines suggested by Lanchester[*], is due to Prandtl[†]. The theory gives results in close agreement with experiment but there remains the difficulty of explaining the origin of the circulation. In a perfect fluid this circulation could not develop and it must be ascribed to the action of the viscosity in the initial stages of the motion.

The general aerofoil theory indicates that there is a drag force (induced drag) associated with the lift of an aerofoil, but for motion in two dimensions this induced drag becomes zero and it is again necessary to turn to the viscosity of the fluid for the explanation of the small drag force (profile drag) which actually exists. The development of the theory of an aerofoil is therefore based in the first place on the assumption that the air is a perfect fluid, and the viscosity is introduced at a later stage to explain the origin of the circulation and the existence of the profile drag.

1·3. *Atmospheric relationships.*

Although the compressibility of the air can be neglected in most problems of the flow past a body, the density of the air cannot be regarded as an absolute constant but must be determined as a function of the pressure and temperature of the undisturbed air according to the physical law

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} \frac{\theta}{\theta_0},$$

where p is the pressure, ρ the density and θ the absolute temperature.

In the atmosphere the pressure and density are connected with the height above the ground by the equation

$$\frac{dp}{dh} = -g\rho,$$

but to determine the conditions at any height it is necessary to know also the relationship between the temperature and

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[*] *Aerodynamics*, 1907. An account of his theory in a less developed form was given by Lanchester to the Birmingham Natural History and Philosophical Society in 1894.

[†] "Tragflügeltheorie," *Göttingen Nachrichten*, 1918 and 1919.

the height. This relationship will vary at different places and at different times, but a standard atmosphere has been adopted by many countries as a basis of comparison. The standard atmosphere is defined by a pressure of 760 mm. of mercury (14·7 lb. per sq. in.) at ground level and by the temperature law

$$T = 15 - 0 \cdot 0065z,$$

where T is the temperature in degrees centigrade and z is the height in metres. This law represents the average conditions in western Europe and is valid up to the height where the temperature ceases to fall on approaching the isothermal layer. The variation of pressure and density with height for the standard atmosphere is given in table 1.

When a change of pressure occurs so rapidly that there is no exchange of heat between adjacent fluid elements, the pressure and density are related by the adiabatic law

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

where γ is the ratio of the two specific heats of the gas and has the numerical value 1·4 for air. The adiabatic law would be satisfied in the atmosphere if the temperature gradient were 3° C. per 1000 ft., and whenever the temperature gradient rises above this value the atmosphere is in an unstable condition which gives rise to convection currents.

Table 1.

Standard Atmosphere.

Height ft.	Pressure p/p_0	Density ρ/ρ_0	Temperature ° C.
0	1·000	1·000	15·0
5,000	0·832	0·862	5·1
10,000	0·688	0·738	− 4·8
15,000	0·565	0·630	− 14·7
20,000	0·460	0·534	− 24·6
25,000	0·372	0·449	− 34·5
30,000	0·298	0·375	− 44·4

1·4. *Units.*

It is customary in aeronautics to express numerical values in British Engineering units and to take the second as the unit of time, the foot as the unit of length and the pound as the unit of force. A new unit of mass becomes necessary, defined by the condition that unit force acting upon unit mass produces unit acceleration. This unit of mass is called the *slug* and is such that a body which weighs W lb. has a mass of W/g slugs ($g = 32\cdot2$, approx.).

Continental writers use a similar engineering system in which the second is the unit of time, the metre is the unit of length and the kilogram is the unit of force. The name *newton* has been proposed for the corresponding unit of mass.

The principal relationships between the two systems of units are as follows:

Length	1 m.	$= 3\cdot281$ ft.,
Force	1 kg.	$= 2\cdot204$ lb.,
Mass	1 newton	$= 0\cdot672$ slug,

and the standard density of the air at ground level is $0\cdot00238$ slug per cubic foot or $0\cdot125$ newton per cubic metre.