# Nonlinear Optics and Optical Computing

Edited by S. Martellucci
and A. N. Chester

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# Nonlinear Optics and Optical Computing

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# PREFACE

The conference "Nonlinear Optics and Optical Computing" was held May 11-19, 1988 in Erice, Sicily. This was the 13th conference organized by the International School of Quantum Electronics, under the auspices of the "Ettore Majorana" Center for Scientific Culture. This volume contains both the invited and contributed papers presented at the conference, providing tutorial background, the latest research results, and future directions for the devices, structures and architectures of optical computing.

The invention of the transistor and the integrated circuit were followed by an explosion of application as ever faster and more complex microelectronics chips became available. The information revolution occasioned by digital computers and optical communications is now reaching the limits of silicon semiconductor technology, but the demand for faster computation is still accelerating. The fundamental limitations of information processing today derive from the performance and cost of three technical factors: speed, density, and software. Optical computation offers the potential for improvements in all three of these critical areas:

Speed is provided by the transmission of impulses at optical velocities, without the delays caused by parasitic capacitance in the case of conventional electrical interconnects. Speed can also be achieved through the massive parallelism characteristic of many optical computing architectures;

Density can be provided in optical computers in two ways: by high spatial resolution, on the order of wavelengths of light, and by computation or interconnection in three dimensions. In general, optical computer architectures avoid the yield and rework difficulties often associated with densely packaged electronic devices; and,

Software is facilitated in optical computers, as in some electronic computers, through the use of highly parallel architectures, and through the use of adaptive, self-programming configurations analogous to networks.

Before optical computer benefits can be realized in practice, considerable development is needed of the devices, structures, and architectures which only exist in research laboratories today. Fortunately, a strong foundation already exists in these areas, and this book treats both the fundamental devices and the computing architectures which will make possible the advanced computers of the future. Due to the peculiar characteristics of this rapidly developing field, we did not interfere with the original manuscripts in editing this material and wanted only to arrange it without reference to the chronology of the conference into five categories:

1) "Optical Nonlinearities and Bistability", a group of five papers emphasizing nonlinear Fabry-Perot resonators and other bistable structures, which could serve as basic logic and memory elements;
2) "Quantum Wells and Fast Nonlinearities", four papers describing quantum well structures and the fast nonlinear effects they exhibit;
3) "Optical Computing, Neural Networks, and Interconnects", a set of four papers covering optical computer architectures, optical interconnects, and their practical implementation;
4) "Materials and Devices", three papers treating lasers, nonlinear fibers, and nonlinear effects at surfaces, as possible elements for optical computation; and
5) "Suggestion for Further Reading", two papers containing an extensive annotated bibliography on nonlinear optical activity and nonlinear eigenpolarizations, and additional selected references on optical computing using phase conjugation.

These papers, and the further references therein, should form a useful guide to today's research results, and the basis for future advances in optical computers.

Before concluding, the Editors acknowledge Miss R. Colussi, who volunteered to retype and revise the entire volume, as well as the continuous assistance of the Plenum Editor (J. Curtis); they wish to express sincere appreciation to Prof. A. M. Scheggi, the Scientific Secretary of the conference, and to Mrs. V. Cammelli for the very specialized assistance in the successfull organization of the conference. Thanks are also due to the organizations who sponsored the conference; among them, the Ettore Majorana Centre for Scientific Culture, whose support made the conference possible.

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CONTENTS

## OPTICAL NONLINEARITIES AND BISTABILITY



## QUANTUM WELLS AND FAST NONLINEARITIES

## OPTICAL COMPUTING, NEURAL NETWORKS, AND INTERCONNECTS



## MATERIALS AND DEVICES



## SUGGESTION FOR FURTHER READING

OPTICAL NONLINEARITIES AND BISTABILITY

# THEORY OF OPTICAL BISTABILITY AND OPTICAL MEMORY

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## I. INRODUCTION

Optical bistability (OB) characterises an optical system with two possible state outputs for a single input[1-3]. This phenomenon, and its possible applications to optical information processing, was proposed nearly two decades ago[4]. It has since been demonstrated in a wide variety of systems and media, including vacuum and malt whisky!

Two objective of this presentation is to review the basic physics of OB with emphasis on those features and systems most relevant to information processing. There will thus be a bias towards small systems exploiting electronic nonlinearities, especially semiconductors. The seminal theoretical and experimental work in two level systems such as Na vapour[1-3], and the important field of instabilities in OB systems[5], will be largely neglected: see Lugiato for a detailed review[5].

This short review begins with perhaps the simplest model of optical bistability[6] which displays rather general features[7] in a direct manner. Most OB systems involve a resonant cavity, and the basic theory of absorptive and dispersive OB in cavities is presented, leading on to questions of optimisation[8] and the mapping of OB output states on to logic functions[9]. I then analyse switching dynamics, which leads to slowing-down phenomena[10] and gain-bandwidth considerations[11]. The possibility of competing nonlinearities (e.g. electronic and thermal) is considered, which can lead to self-oscillation[12] but also has important implications for design of optical processors[13].

The minimum size of OB devices is determined by the transverse coupling mechanisms: both diffractive and diffusive mechanisms are analysed, and hysteresis loops and beam profiles obtained numerically are discussed[3,14]. Diffusion-dominated OB presents an interesting and practical limit in which key features of optical memory devices can be analysed. In particular switching-wave phenomena imply that large-area memory and image processing

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devices must be "pixellated" into arrays of OB elements[15,16], while minimum pixel spacings in such arrays can be inferred by analogy with nonlinear dynamical systems[17]. Similar considerations arise for diffractive coupling, with the extra feature that a self-focussing type of nonlinearity leads to the spontaneous formation of soliton-like structures[18-20]. Both as "intrinsic pixels" and in their own right as nonlinear wave phenomena, these are among the most significant current developments in the theory of optical bistability.

## II. SIMPLE MODEL

As a simple and instructive model for OB, consider a slice of material, thickness L, on which a plane wave of intensity $I_0$ is incident. If the absorption coefficient $\alpha$ depends on temperature $\phi$, then

$$\frac{dI}{dz} = -\alpha(\phi)I \tag{1}$$

Clearly $\phi$ will in turn depend on the absorbed energy, and may be crudely modelled thus:

$$\frac{d\phi}{dt} + \Gamma(\phi-\phi_0) = I_0 f(\phi) \tag{2}$$

where $\phi_0$ is the ambient temperature, while $f(\phi)$ is proportional to the bulk absorption rate $\sim(1-e^{-\alpha L})$ and $\Gamma^{-1}$ is the thermal time constant. In steady state, (2) leads to

$$(\Gamma/I_0)\ (\phi-\phi_0) = f(\phi) \tag{3}$$

Similar equations to (3) govern most OB systems ($\phi$, not T, is used so that these equations can be used below in other contexts), and graphical solution of such equations is instructive (Fig. 1). Each possible steady state is represented by the intersection of a line through $(\phi_0,0)$ of slope $\sim I_0^{-1}$, with the response function $f(\phi)$. There are evidently multiple intersections for $I_0$ large enough provided $f(\phi)$ is sufficiently steeply increasing over a suitable range of $\phi$. For bounded $f(\phi)$, there are an odd number of intersections: linearisation of (2) around these roots enables their stability to be determined. Suppose that $\phi_s$ solves (2), and let

$$\phi = \phi_s + \varepsilon e^{\lambda t}$$

Substituting into (2):

$$(\lambda+\Gamma)\varepsilon e^{\lambda t} = I_0 f'(\phi_s)\varepsilon e^{\lambda t} + 0(\varepsilon^2)$$

and hence

$$\lambda = I_0(f'(\phi_s) - \Gamma/I_0).$$

If $\lambda<0$, the perturbation damps out, and $\phi_s$ is stable, and vice versa. By

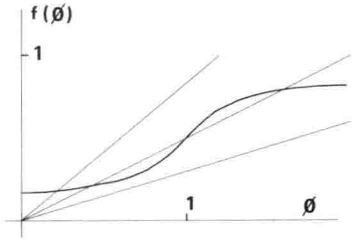


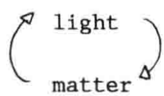

Fig. 1. Graphical solution of Eq. 3. The straight line's intersections with the response function give one, three or more solutions.

comparison with (3) and Fig. 1 it can be seen that when the line cuts $f(\phi)$ "from below" the corresponding state is dynamically stable, and vice versa[2]. Thus the middle intersection in Fig. 1 belongs to an unstable state, while the other two are stable, and we indeed have optical bistability.

While not necessarily very practical, this simple model illustrates key features of OB:

(i) nonlinearity - of the light-matter coupling function $f(\phi)$

(ii) feedback - symbolycally (1) and (2) represent a feedback loop:




Both features seem necessary for OB: in fact the feedback is commonly explicit, as when the nonlinear medium is enclosed in a cavity structure. In such a structure the response is sensitive to the phase, as well as the intensity, of the optical field. OB can then arise from nonlinear refraction (Kerr effect), with the cavity Airy function as the appropriate response function, and also from saturable absorption[4]. These schemes are known as dispersive and absorptive OB respectively[5].

## III. CAVITY OPTICAL BISTABILITY

Cavity OB has been analysed many times[1-5] so a complete treatment is not necessary here, but it is worthwhile to develop the usual model.

Consider a two-level atomic system enclosed in a ring cavity. On the assumption that the polarisation may be adiabatically eliminated (certainly valid in condensed media) the Maxwell-Bloch equations reduce to a pair of coupled delay-differential equations[2].

$$E(t+t_r) = A + Re^{i\theta}E(t)\exp\left[-\frac{\alpha L}{2}(1-i\Delta)D(t)\right] \qquad (4)$$

$$\tau\dot{D}(t) = 1-D(t) - |E(t)|^2 [1-\exp(-\alpha LD(t)]/\alpha L \quad (5)$$

E(t) is the scaled field amplitude on entrance to the nonlinear medium, which has length L and small-signal absorption coefficient $\alpha$. E is scaled to the saturation intensity $I_s$. D(t) is the spatially-averaged population inversion which is seen from (5) to be normalised to unity for $E\to 0$, and $\tau$ is its decay time. The cavity has round trip time $t_R$, mistuning parameter $\theta$ and $R=(R_1R_2)^{\frac{1}{2}}$ where $R_1, R_2$ are the input and output mirror reflectivities. A is the scaled input field entering the cavity, and thus equal to $(1- R_1)^{\frac{1}{2}}E_{in}$. Finally $\Delta$ is the scaled detuning from the atomic resonance: $\Delta=0$ gives a purely absorptive effect, while $|\Delta|>>1$ makes dispersive effects dominant.

Other nonlinear media give qualitatively similar equations, perhaps with $\alpha$ and $\Delta$ empirically determined, as in semi-conductors[21].

For condensed media in short cavities, $t_R<<\tau$ is normal, so that the left side of (4) may be approximated by E(t). E(t) may then be eliminated from (5) to yield, in terms of the convenient new variable $\phi = (1-D(t))$

$$\dot{\phi} + \tau^{-1}\phi = I_0 f(X) \quad (6)$$

where $I_0 = |E_{in}|^2 I_s$, and
$$f(\phi) = \frac{(1-R_1)(1-e^{-\alpha L}\, e^{\alpha L\phi})/\alpha L}{\tau I_s \left|1-b\exp[\alpha L(1-i\Delta)\phi/2]\right|^2} \quad (7)$$

where $b = Re^{i\theta} \exp\,[-\alpha L(1-i\Delta)/2]$.

Equation (6) is evidently formally identical to (2), and thus OB will exist provided $f(\phi)$ has a suitable form. Since the numerator of (7) decreases monotonically with $\phi$, and is zero at $\phi=1$ (complete saturation), OB relies on the denominator more than compensating this decrease. The denominator is, physically, the ratio of the input to the internal intensities, and exhibits a cavity resonance structure. Fig. 2 shows $f(\phi)$ for two important special cases, $\Delta=0$ (purely absorptive OB) and $\Delta=15$ (largely dispersive OB). In the latter case $f(\phi)$ is oscillatory, leading to multiple intersections for $I_0$ large enough. In fact, in the <u>dispersive</u> or <u>Kerr - limit</u>: $|\alpha L|\to 0$, $|\Delta|\to\infty$, $|\alpha L\Delta|\sim 1$, $f(\phi)$ becomes simply the Airy function governing the spectral response of an optical cavity, while the nonlinearity can be described by an intensity-dependent refractive index: $n=n_0+n_2 I$. This is a favourable and practically-important limit.

The above analysis assumes a ring cavity configuration, whereas OB experiments, especially those in solids, generally use a Fabry-Perot or folded cavity. To a large extent, this leads to the same OB phenomena as the ring cavity, but with rescaled parameters: in particular the inversion is driven by both forward and backward propagating fields, while the fields accumulate nonlinear phase and amplitude changes over 2L instead of L: both favourable effects. Standing-wave effects in principle lead to a spatial modulation of the inversion with period $\lambda/2$: this seriously complicates the analysis, but fortunately in many cases the inversion population is sufficiently mobile that diffusion "washes out" the population grating, leading back to quasi-ring-cavity behaviour. A more subtle change is that the space-time averaging implicit in D[5] is no longer valid, and both field and population have to be considered as function of z, the longitudinal coordinate, necessitating approximations or numerical solution or both.

Finally, it should be noted that the above analysis works just as well for $\alpha L$ negative, i.e. an amplifying medium, as for an absorbing medium, provided that $|b|<1; b = 1$ is just the laser threshold. This is important for a number of practical reasons, not least of which is that OB in semiconductor laser amplifiers has extremely low switching powers and energies[21-23].

Bistability evidently implies a binary memory capability: other logic functions can be performed in OB or closely-related states. For example, the reflected and transmitted beams from a Kerr cavity have the basic response features sketched in Fig. 3, whose shape clearly allows digital optical logic to be performed[9].

Sequences of input-output curves such as those in Fig. 3 are obtained experimentally by tilting, heating or otherwise varying the cavity mistuning $\theta$. One finds a maximum loop size at fixed $I_0$, and a minimun $I_0$ below which OB vanishes, corresponding to coalescence of the three intersections in Fig. 1. It is then of interest to optimise this minimum with respect of other parameters. Miller[8] and Wherrett[24] have analysed this problem for Kerr cavities and find that it is possible to decouple the microscopic and macroscopic aspects, so that the material nonlinearity and cavity can be separately optimised. Miller finds that $R\ e^{-\alpha L}$ is optimum for passive cavities, while Wherret emphasises that the contrast of OB loops is much larger in reflection than transmission if $\alpha L$ is appreciable. Adams[22] considers the behaviour of OB amplifiers, which have lower thresholds for small R, in contrast with passive cavities, where large R and low absorption gives lowest thresholds[24].

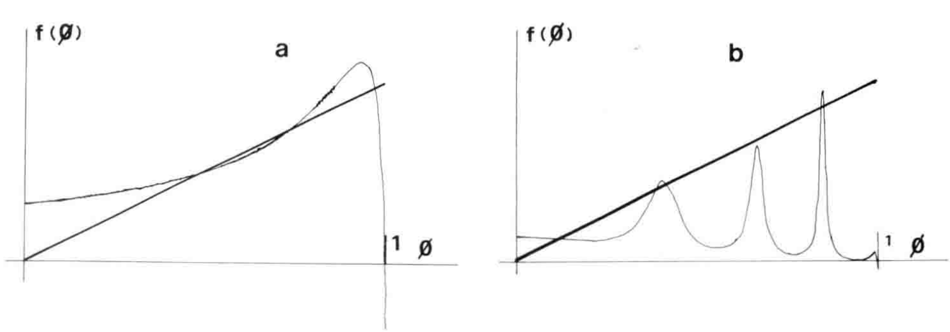



Fig. 2. Response function $f(\phi)$ for a 2-level system in an optical cavity - equation (7) - for R=0.9, $\alpha L=3$ and $\Delta=0$ (left); $\Delta=15$ (right). Intersecting line indicates bistability.

## IV. SWITCHING DYNAMICS

The cycle time of the OB system described by (1,2) will be of order $\Gamma^{-1}$, i.e. controlled by the thermal time constant. With good design this can be submicrosecond, but faster cycling is generally sought via electronic excitations in, especially, semiconductors. One can infer from (2), however, that $I_0/\Gamma \sim$ constant, i.e. that bandwidth and power are roughly proportional, which is a good rule-of-thumb. This rule works also for (7), since $I_s \sim \tau^{-1}$ (ref. 5).

It follows from our stability analysis of (2) that fluctuations around the steady state $\phi_s$ damp at a rate

$$\Gamma_e = \Gamma - I_0 f'(\phi_s).$$

$\Gamma_e$ changes sign at a switch point, where the line is tangent to $f(\phi)$ in Fig. 1, and is thus small in its neighbourhood, which leads to the phenomena of critical slowing-down and critical fluctuations well known in phase transition theory. At the switch point the perturbation analysis has to be taken to second order, leading to a prediction that when the intensity is stepped from below the threshold value $I_{th}$ to above that value, the switching times scale as $(I_0-I_{th})^{-\frac{1}{2}}$, which has been confirmed in a number of systems[1-3,25].

In the "transphasor" regime[26], where the characteristic is not quite bistable, the gain spectrum for a small modulation of $I_0$ at frequency $\omega$ can be calculated as

$$|\text{gain}| \sim \Gamma/(\Gamma_e^2 + \omega^2)^{\frac{1}{2}}$$

which shows that high gain is available close to a switch point, but only at the cost of a bandwidth narrowing in proportion. Nardone and Mandel[27] calculate nonlinear corrections to this gain which remove the singularity as $\Gamma_e \to 0$.

Critical slowing down is manifested when the input is stepped to a new, constant value. An interesting alternative is to consider an "address pulse", i.e. a temporary step in $I_0$ designed to switch the OB device from its lower to its upper state. The first order character of (2) means that whether or not the device switches depends entirely on the value of $\phi$ at the end of the address pulse. If the three intersections with the line describing the "hold" level $I_0$ are $\phi_\ell, \phi_m, \phi_u$, then if at the end of the address pulse $\phi > \phi_m$, the device will switch i.e. $\phi \to \phi_u$. Conversely $\phi < \phi_m$ leads to a decay back to $\phi_\ell$. For $\phi_\ell \sim \phi_m$, $\dot{\phi}$ is small, and the system "dwells" a long time close to $\phi_m$. Mandel has analysed this phenomenon, and terms it "slowing-down". He shows that the dwell-time scales logarithmically, in contrast to critical slowing-down. The effect has been demonstrated experimentally[28]. This shows, incidentally, that the unstable branch is physically observable, and can be traced out by analysis of slowing-down for different $I_0$ and address-pulse area.

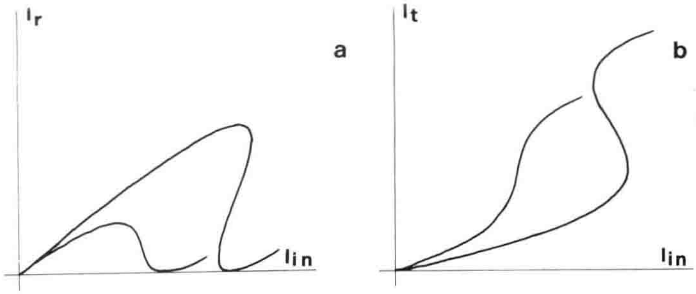



Fig. 3. Reflected (left) and transmitted (right) intensity versus input intensity for a Kerr cavity, for R-0.5, $\alpha L=0$ and $\theta=-1.6$ (bistable); $\theta=-1.1$ (monostable).

We have so far assumed that only a single nonlinear effect is active. In practice, there may be several: in particular in semiconductors electronic OB will inevitably be accompanied by thermal effects, which can also influence the optical properties of the device. Suppose then that two mechanisms are operative, one fast (electronic) and the other slow (thermal), and that they have opposing effects. The dynamics can then be approximated by still using (2), but regarding the slow effect as a "bias" of the response function $f(\phi)$. Since e.g. the thermal load will be different for the states $\phi_\ell$, $\phi_u$, these will have different bias, say $\theta_\ell$, $\theta_u$ corresponding to translation of $f(\phi)$ in Fig. 1.

Consider now a fast "electronic" switch-up. Initially the bias will still be close to $\theta_\ell$, but the device will begin to heat up, and $\theta$ will drift towards $\theta_u$. This means that $f(\phi)$ will move to the right in Fig. 1. If $|\theta_u - \theta_\ell|$ is large enough, OB will be lost, and the device will switch back down (fast). It will then cool down, $f(\phi)$ will move left, and the device will eventually switch up once more. This sequence can repeat indefinitively as a self-sustaining oscillation.

This effect was predicted by McCall[12], and has been observed in a number of OB systems - see Chapter 6 of Gibbs' book[2] for a review.

The presence of slower, possibly opposed, contributions to nonlinearity is an important factor in the design of OB systems. As we have noted, the strength of a given nonlinearity increases as its response time is lengthened, and vice versa. Since the response times of both electronic and thermal excitations are sensitive to surface effects, the balance between these contributions to OB may be radically altered by system geometry. For example, "pixellation" of material to obtain high packing densities of OB devices (see below) will normally affect the response times of both electronic and thermal excitations, and careful design will be necessary to avoid instability in such systems.

This last consideration leads naturally to the topic of transverse effects, which determine the minimum size of OB devices, and together with the response time control the holding powers and switching energies which will determine the viability of OB systems.

## V. TRANSVERSE EFFECTS IN OPTICAL BISTABILITY

Transverse coupling, both within a single bistable element and between adjacent elements, has a major influence on the nature and quality of optical bistability in practical systems. Diffraction is the obvious coupling mechanism, but in many systems the excitation is mobile enough for transverse diffusion to be significant. This is the case for semiconductors such as InSb[26,29], where the excitation is an electron-hole plasma and, indeed, for thermal devices such as interference filters[9].

Basically, these transverse effects require the addition of a diffraction term to (1) and a diffusion term to (2) or their generalisation. For illustration, we concentrate on Kerr nonlinearity and Fabry-Perot cavities.

In this case, the propagation equations for the forward and backward (scaled) field amplitudes $F(\underline{r})$, $B(\underline{r})$ and excitation density $h(\underline{r})$, in an etalon of thickness L, may be expressed as[30]

$$\frac{\partial F}{\partial z} = -\left[\frac{\alpha L}{2} + ih(\underline{r}) + \frac{1}{2} i\mathfrak{z}\nabla_t^2\right]F \tag{8}$$

$$-\frac{\partial B}{\partial z} = \left[-\frac{\alpha L}{2} + ih(\underline{r}) + \frac{1}{2} i\mathfrak{z}\nabla_t^2\right]B \tag{9}$$

$$(\ell_D^2\nabla_t^2 - 1)h = -4\mathfrak{z}\,\mathrm{sgn}(n_2)(|F|^2 + |B|^2) \tag{10}$$

Here $\alpha$ is the linear absorption coefficient and $\mathfrak{z} = L/kw^2$, where we assume a gaussian input beam of width w, and scale transverse distances to w. $\nabla_t^2$ is the transverse Laplacian, and $l_D$ the diffusion length for the medium excitation. These equations, together with Fabry-Perot boundary conditions on F and B, and transverse and surface boundary conditions for h, are the basis of our model for steady-state OB.

A brief discussion of possible approaches to solution of these partial differential equations is appropriate: even in the steady state they require substantial computer effort, especially in two transverse dimensions. For the moment, consider $\ell_D=0$, i.e. the "diffraction-only" case, in which (8) and (9) are third order in $|F|$ and $|B|$.

Perhaps the simplest approach is mode-expansion, thereby reducing the system to a number of ode's equivalent to the number of modes analysed. Because, however, third order linearity couples three modes to a fourth, the computation grows as $N^4$ (see reference 31), and is thus only useful if one mode is dominant, in which case the technique can work well[32] and may even yield analytic results[33].

Even more straightforward is the finite-difference technique, which is flexible and direct, and has proved useful for problems with cylindrical symmetry[34].