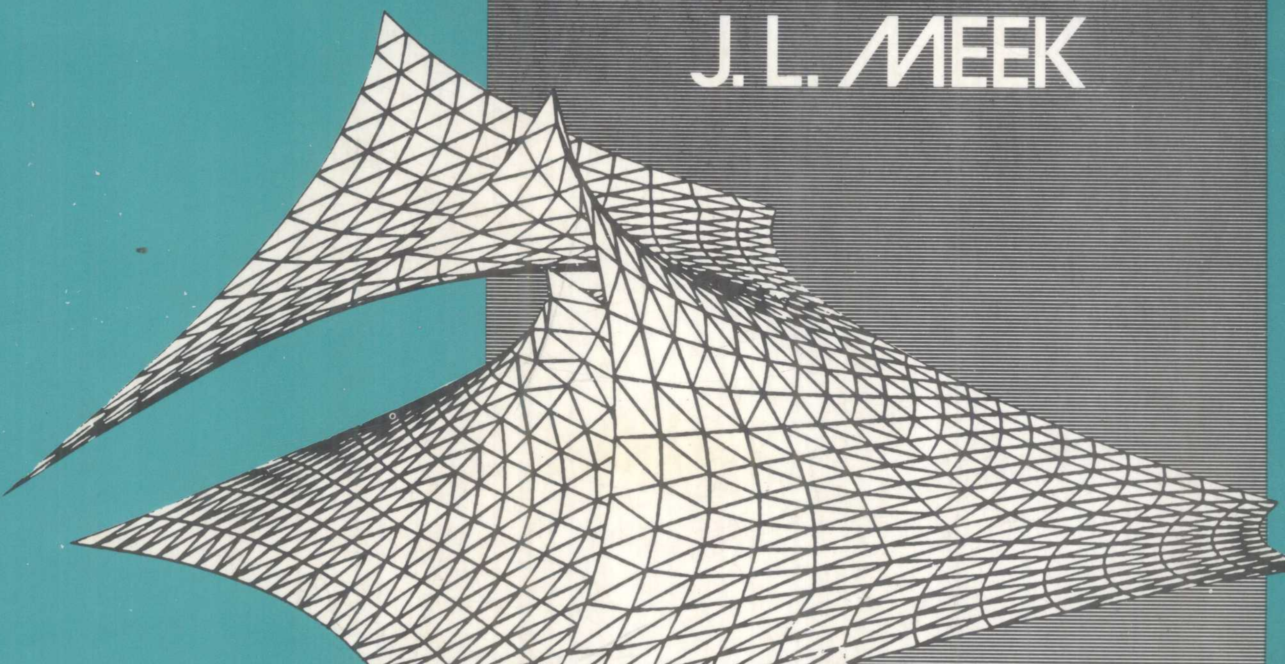
COMPUTER METHODS IN STRUCTURAL ANALYSIS

J. L. MEEK



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Preface

This book grew out of experience gained in three decades of development of matrix structural analysis, firstly as a research tool, then as post graduate lecture material and finally as an integral part of the undergraduate curriculum. The basic building blocks of a unified theory for structural analysis are to be found in matrix and linear algebra. It is with the use of matrix theory that the duality of the concepts of equilibrium and compatibility can be brought together via the principles of virtual displacements and virtual forces and expressed as the contregredient law. For many of these basic ideas the author is indebted to Dr K. Liversley of Cambridge University, Mr J.C. DeC Henderson formerly of Imperial College and the late Mr R.S. Jenkins of Ove Arup and Partners. The spirit of the book is to look firstly at these underlying principles of mechanics, developing them as a coherent and unified theory which can then be used in the approximate solution of the partial differential equations of solid mechanics, as applied to various structural forms such as skeletal frames, plates, shells and the solid continuum.

Because the line element structure (truss, frame and cable) plays an important role in the teaching of elementary structural mechanics, this form has received a reasonable amount of attention (Chapters 4, 5 and 6). The plan for the book then is to start with an introduction to vector, tensor and matrix notations in Chapter 1, together with a discussion of the interpolation theory which forms a basic part of the finite element method. In Chapter 2, the fundamental theorems necessary to apply the principles of virtual displacements and forces to both discrete and continuous structures are fully developed. It is found convenient to use the ubiquitous Gauss divergence theorem for the latter case. Material constitutive laws are discussed and the incremental elasto-plastic constitutive equations developed so that the reader will be able to apply the theory to a variety of material models currently available.

Chapter 3 introduces the finite element method in a number of its forms, and the use of the contregredient law in the development of the finite element stiffness matrices explored for the displacement, hybrid and equilibrium types of element. With nearly 40 000 papers now written on the finite element method this Chapter must by its very briefness be one of highly distilled material.

Chapter 4 deals with the force method of analysis in abbreviated form. Although the stiffness method has gained ascendency in use in the numerous computer software packages commercially available for structural analysis, the principles of statics, deflection calculation etc., are still taught in the first instance via the force method. Thus, there is still rationale for an understanding of the force method and it is still valid to observe that computer software for both the force and the displacement methods can be made to look identical at the user interface. Only for large scale analyses does the displacement method have significant advantages.

In Chapter 5 the displacement method is detailed and in this context attention is focused on the direct stiffness method. It must be realized that the direct stiffness method is only the name given to the process in which the various element stiffness properties are firstly expressed in the common global coordinate system and then assembled via unit matrix transformations which reflect the structure connectivity or topology.

In Chapter 6 some attention has been give to the calculation of elastic critical loads, not only because this is a useful exercise, but also because it now proves to be a rather minor yet elegant extension of the simple linear theory. A treatment has been given in this Chapter of the geometric non-linear analysis, both for line elements and membrane shell structures, because these forms have an important application to the shape finding, analysis and design, of tension structures.

Chapter 7 deals with the finite element analysis of the solid continuum which of course can be divided into the planar situations (plane stress, plane strain and axisymmetry), and the three-dimensional stress state. A feature in this Chapter is the discussion of the natural mode method of Professor J.H. Argyris which succinctly separates the rigid-body motion and straining modes of an element. The St Venant's torsion problem is analysed by the use of the cross section warping displacement function, and the means given for location the shear centre of those cross sections under St Venant's torsion stress.

Chapter 8 introduces a detailed discussion of the small deflection plate bending analysis. Because of its importance in understanding the bending action associated with shell behaviour many element types are given, including the flat facet elements and the degenerated isoparametric and heretosis elements. Comparisons of element computational efficiency and accuracy are given. The DKL (Discrete Kirchoff with Loof nodes) is introduced as a possible contender for the inclusion of plate elements in shell analysis via flat facet elements.

The analysis of shells is given in Chapter 9. With over 4000 papers now published on shell theory it becomes necessary to give a brief review of the literature. This is followed by a description of flat facet a type elements (plane stress plus plate element). Again the DKL element is highlighted for some advantages it appears to possess for moment connections in box type structures as well as in general shell analysis. It is similar to the Morley and semi-Loof type elements in that it has the rotational nodal variables embedded in the element sides rather than at the apex nodes. In looking at curved shell elements one has to make some compromise between computational efficiency and accuracy on the one hand and practical use on the other. It would appear that those elements which require high order derivatives in their nodal variables have limited application. Thus the decision was made to give details of the isoparametric shell element (Ahmad and Irons) and to give a reasonable discussion therein of the problems of membrane and shear locking problems and the means to avoid these troubles.

In studying the book the suggested order is to first read Chapters 1–3, and if the intention then is to pursue finite element analysis, to move to Chapters 5, 7, 8 and 9, in that order. For a lower level of study one may choose parts of Chapters 1 and 2 and then move to Chapter 4 and 5 to obtain a knowledge of matrix structural analysis as applied to line element (skeletal) structures. It is strongly recommended that some of the material of Chapter 6 is included to extend the student's capabilities to the calculation of frame buckling loads.

The author would like to thank the many people who have contributed to the book. Firstly, to Professor R.W. Clough for his introduction to the writer matrix structural analysis at UC (Berkeley) in the spring of 1958 and for the three decades of inspirational research which has come from UC (Berkeley) under the leadership of Professors Popov, Scordelis, Wilson, Powell and many others. To Professor Carlos Fellipa, now at Boulder, CO, USA, whose insight into finite element analysis has always been at the frontiers of knowledge and an inspiration to the writer. One cannot write a preface such as this without referring to the forty years of leadership provided by Professor J.H. Argyris with his marvellously simple natural mode technique, and to the quiet courage of the late Professor Bruce Irons who gave his all to the cause.

My thanks too, to the many students who have contributed through their research efforts: Drs W. Tranberg, H.S. Tan, K.Y. Tan and S. Loganathan, and to W. Lin and H. Goa for assisting with proof reading of the manuscript.

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CONTENTS

CHAPTER 1

Mathematical preliminaries

1.1 INTRODUCTION

In writing this book the intention is to assist the student in the learning process for structural and finite element analysis, in which he must master, not only the numerical techniques available, but also the mathematical skills necessary for the efficient description of the physics of the problem at hand. As such it is a teaching book. The format, then, is first to present a discussion on mathematical preliminaries in which the concepts of vectors, tensors and matrices are introduced. Which notation should be used? Vector, tensor or matrix? The answer is not simple, and the three notations are certainly not mutually exclusive. Following a discussion of the merits of the various notations, the reader is introduced to the ubiquitous Gauss' divergence theorem. This theorem forms the basis of most descriptions of the integral of the rate of change of a variable over a region to its values on the surface of the region. In structural mechanics the relationship has been discovered independently and given such names as principle, of virtual displacements, principle of virtual forces, or simply virtual work. The advantage of the more general formulation by Gauss is in its adaptability to a variety of other physical situations where the concepts of static equilibrium are not available. In structural mechanics it is possible to extend the concept of virtual displacements (forces) to the contragredient principle which succinctly describes the relationship between statically equivalent force systems and their compatible displacements. The contragredient principle can be loosely described as a reflective principle (for the purposes of memory only). That is, if the force systems (P, Q) are connected through the relationship,

$$\{P\} = [B]\{Q\}$$

then the corresponding compatible displacements (p, q) are connected as follows:

$$\{q\} = [B]^T\{p\}$$

Without the ability to apply both Gauss' celebrated theorem and the contragredient principle, the student's capabilities of analysis are seriously restricted. The finite element method can be classified as a means for the approximate solution of the partial differential equations of mathematical physics. It requires the subdivision of the region under consideration into a number of geometrically definable domains. In each domain, the unknown variables (for which the approximation is desired), are expressed in terms of certain values (either generalized coordinates or nodal values), by functions of the coordinate system. A study of this problem and the choice of nodal values rather than generalized coordinates lead to the definition and construction of orthogonal interpolation functions. The chapter concludes with a discussion of coordinate systems and the various element domains (isoparametric and triangular) and the calculation of function derivatives in, and integrals over, the domain. In subsequent chapters, the various physical situations encountered in both line structures and continuum mechanics are introduced and their finite element approximations discussed.

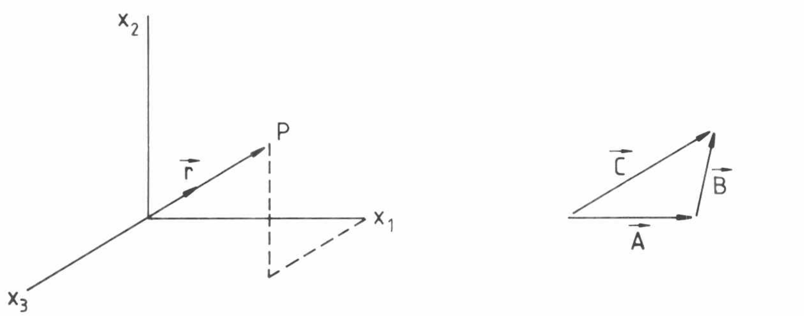
An attempt is made to give a fairly complete description of a wide selection of finite element types. In the first instance, attention is focused on small displacement, linear elasticity. In general the emphasis will be on the displacement element formulation with compatible displacements at the element interfaces. It will be shown that this approach may also be formulated as a Galerkin method of weighted residuals (again a useful concept in non–structural applications). The discussion will, however, by no means be restricted only to displacement models, and sections will be reserved for discussions of the natural mode technique, developed by Argyris [1], and the hybrid stress elements of Pian [2]. It may be pertinent at this stage in the book to draw the reader's attention to the work, *Energy Theorems and Structural Analysis* by Argyris and Kelsey [3], published in 1959. The material in this work belies its title, and the work actually contains a very modern discussion of the matrix methods of structural analysis. Because the calculation of elastic critical loads is merely an extension of the linear theory to include the effects of axial forces when the equilibrium equations are written in the deformed rather than the initial position, it is natural that the book includes a chapter on critical load (or eigenvalue) calculation and the corresponding mode shapes. This is a break in tradition from classical texts in that the computer has made the calculation of the linear eigenvalue problem for the first critical load a simple extension of the first–order theory. Because the effectiveness of the finite element method lies in part in its successful application to non–linear analysis problems of both a material and geometric nature, the text concludes with some of the techniques currently available for these analyses. In particular a discussion is given of net and membrane structures and shape finding techniques and of the non–linear analysis of structures which may present rather mischievous behaviour in their load deflection relationships. The solution to the transient heat flow problem and the related elasto–plastic thermal stresses will be given as an example of the non–linear material behaviour.

1.2 NOTATION: VECTOR – MATRIX – INDEX

1.2.1 Vector notation

The concept of vectors arises from the geometrical representation of forces, displacements and their time derivatives and from the parallelogram law of addition of such quantities. A vector is defined as a quantity in space which has direction as well as magnitude, and it will be denoted by $\vec{F}$, $\vec{R}$, etc. A point P in space is defined by its position vector $\vec{r}$, as in Fig. 1.1(a). In Fig. 1.1(b), the vector addition of $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{C} = \vec{A} + \vec{B} \tag{1.1}$$



(a) Position vector $\vec{r}$ (b) Addition of vectors

Fig. 1.1 Vector components - vector addition.

It is soon found, however, that further operations are necessary for the satisfactory manipulation of vector quantities; the two most commonly used are the dot and cross products. The first produces a scalar and the second a vector.

Dot product

$$\vec{A} \cdot \vec{B} = |A||B|\cos\theta \tag{1.2}$$

Cross product

$$\vec{A} \times \vec{B} = |A||B|\sin\theta\ \hat{n} \tag{1.3}$$

The sense of $\hat{n}$ in Fig. 1.2(b) is given by the right–hand screw rule from $\vec{A}$ to $\vec{B}$. The use of these two operators is illustrated in the examples which follow. Firstly, consider the position vector $\vec{r}$ given as the vector sum of its three components in the directions of the Cartesian coordinates (x_1, x_2, x_3), base unit vectors $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$.

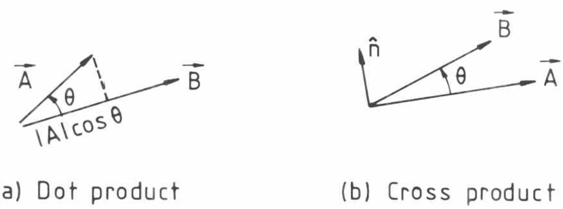


Fig 1.2 Vector operations.

That is,

$$\vec{r} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3 \tag{1.4}$$

The i th component of $\vec{r}$ is given by

$$x_i = \vec{r} \cdot \hat{e}_i \tag{1.5}$$

The magnitude of the vector $\vec{F}$ is given by

$$|F| = (\vec{F} \cdot \vec{F})^{1/2} \tag{1.6}$$

Consider now, the force vector $\vec{F}$ acting through the point P, position vector $\hat{r}$. It is required to calculate the moment vector $\vec{M}_0$ of $\vec{P}$ about O. From Fig. 1.3(a), the magnitude of this moment is given by

$$|M_0| = |F|p \tag{1.7}$$

where p is the perpendicular distance from O to the line of $\vec{F}$. The moment $\vec{M}_0$ is perpendicular to the plane of $\vec{F}$ and $\vec{r}$ and in the sense shown in Fig. 1.3(a). Thus,

$$|\vec{M}_0| = \vec{r} \times \vec{F} = |F||r|\sin\theta\ \hat{n} = |F|p\hat{n} \tag{1.8}$$