


PRECISION



MEASUREMENT

PRECISION MEASUREMENT

METHODS AND FORMULAS

BY

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PREFACE

In the higher types of inspection, many parts cannot be measured by actual tool use. Odd curves and angles often prove baffling, especially when the part is necessarily held to close tolerance. Profile gages, forming tools of irregular shape, and fixtures to hold work while being measured, can often be best checked by mathematical calculation. This method usually proves easier and more accurate than attempting to solve the problem by trial and error.

Unfortunately, the average tool inspector has little background for this type of applied mathematics. Often he has a distinct aversion to the subject, dating back to school where problems were worked on the blackboard day after day without practical application. As a result, he often misunderstands the theory and practice of mathematics particularly in relation to precision inspection. In most cases, advanced work requires far more time for calculating the solution than for actual measuring of the work with the tools.

The author has attempted to treat the subject of precision inspection in such a manner as to finally and decisively bridge the existing gap between what is commonly called "school-teacher theory" and the practical down-to-earth problems that confront the inspector in his tool work. The problems presented in this text have been drawn directly from the shop and the accompanying solutions are carried through step by step with the utmost care and simplicity so that no phase of the solution is lost, and yet through the use of a great number of setup drawings and sketches the over-all theory behind the solution is in most cases clearly and unmistakably shown. It should be noted that the solutions to these problems are for the greater part based on setups requiring the simplest of instruments. The uses of balls, discs, or pins in determining the answers to problems are demonstrated. Many jobs cannot be solved by any other medium, and the setups illustrated in this manual can often save days of time and labor by avoiding the building of special jigs and fixtures.

The formulas presented herein are selected as representative of the many problems encountered by an inspector. The student should learn to visualize and apply the basic similarity between the book's problems and those he will encounter in his own shop. Each formula is so written

as to teach the student how to prepare formulas of his own. Formulas should not frighten anyone, since they serve as a means of discarding excess words, leaving a series of facts and figures much easier to keep straight than a hodgepodge of information. This technique is used with success by mathematicians and engineers. No special ability will be necessary other than knowing how to look up information in trigonometric tables.

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PRECISION MEASUREMENT METHODS AND FORMULAS

ELEMENTARY PRINCIPLES: TRIGONOMETRY

FOR many years the simple triangle has been used as a means of calculating distance. Inventors, surveyors, and engineers have applied the principles of trigonometry to their everyday problems. More recently, these principles have been applied to the field of precision inspection, and this book is intended to show the inspector and the toolmaker the practical application of that simple geometric figure, the triangle. Every problem illustrated contains one or more triangles, and their location will become apparent after some experience is gained by using the formulas.

The student will wonder, "Why resort to mathematics?" when apparently all that is necessary is a measuring instrument of some kind. The answer is that we may be forced to apply mathematical solutions in measuring certain dimensions, because the surface to be checked is so inconveniently located that we cannot apply the gage or instrument directly to it. Angled surfaces and curved or hollow parts often cannot be measured with standard tools. Often an angle cannot be precisely measured with a protractor and must be calculated from micrometer measurements. The inspection of threads, accomplished by the use of the three-wire measuring system approved by the United States Bureau of Standards, is based on calculations using trigonometry. The depth, pitch diameter, and other elements are also determined by using this type of mathematics. Gears, a special field in themselves, are not covered in this volume, but their entire structure and shape and the size of tooth are determined by trigonometry. The gear formulas are so involved that the student should not attempt such calculations until he has mastered the underlying principles of trigonometry as set forth in this book.

The sample problems in this book are solved by the use of trigonometry, and this chapter is intended as an elementary introduction to the subject. Sufficient explanation will be given to enable the shop man to understand fully the use of the different trigonometric functions. Advanced trigonometry, which concerns spherical surfaces and compound angles, is not necessary for most inspection problems and hence is not discussed here.

The first step in the study of trigonometry is to learn the names and meaning of the various terms used.*

The important triangle in trigonometry is the right-angled triangle or right triangle. One or more right triangles are found in each of the problems shown in this book. As an aid to the student, they are identified by heavy lines in some of the earlier problems.

Figure 1 shows a right triangle with its three sides named in

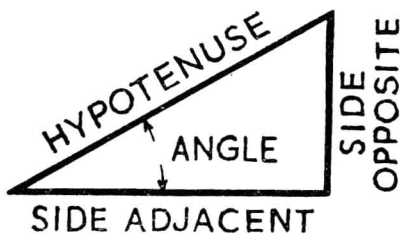


FIG. 1

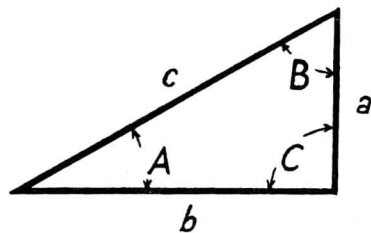


FIG. 2

reference to one angle. The hypotenuse, which is opposite the right angle, is always the longest side. The other two sides can take either designation, depending on which acute angle is being used.

In trigonometry, the angles of a right triangle are usually designated by the capital letters A , B , and C . The sides opposite these angles are designated by the corresponding small letters. Thus in Figure 2, side a is opposite angle A , side b is opposite angle B , and side c is opposite angle C . Accordingly, side a is adjacent to angle B and side b is adjacent to angle A .

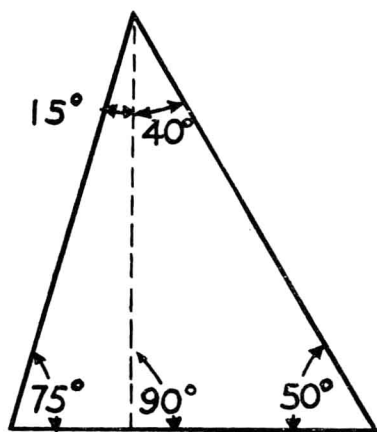


FIG. 3

The sum of interior angles of every triangle, regardless of its size or shape, is equal to 180° . Thus in a right triangle, since one angle equals 90° , the sum of the other two angles is 90° . These angles are therefore less than 90° , and are *acute*. Therefore if one acute angle is known, the other is always 90° minus the known angle.

Many of the problems presented in the text will require the student

* It is suggested that no attempt be made at this point to memorize these terms or functions. Rather, refer back to the sample problems in this chapter when necessary. Constant repetition will cause them eventually to become entrenched in the memory.

to divide large triangles into two or more smaller ones in order to obtain a right triangle. Figure 3 is an example of a triangle divided so as to obtain two right triangles. Note that the perpendicular dotted line produces two square corners or right angles at the bottom. Thus we have two right triangles. Figures 4 and 5 are tests to enable the student to check his understanding of the relationship of the angles.

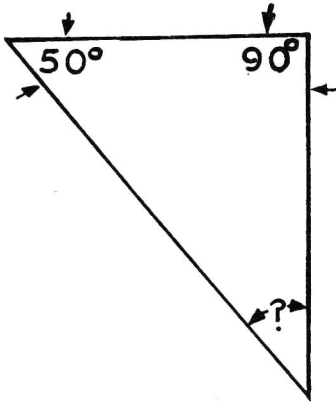


FIG. 4

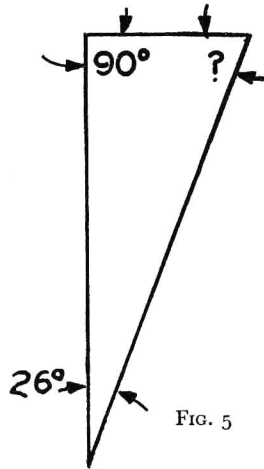


FIG. 5

The basis on which trigonometrical calculation rests is the fact that in any right triangle, the ratio between the lengths of any two of the three sides remains constant regardless of the size of the triangle. If the *shape* changes, the ratio changes. These ratios are called *functions* of the angle since they change as the angle changes. The functions have been calculated for all angles and placed in tables. These tables are known as *tables of trigonometric functions*.

The six trigonometric functions are the sine, cosine, tangent,

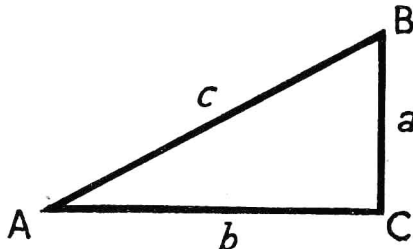


FIG. 6

cotangent, secant, and cosecant. Looking at Figure 6, we see how these functions are obtained in a right triangle.

$$\begin{array}{llll}
 \text{sine} = \frac{\text{side opposite}}{\text{hypotenuse}} & \text{or} & \sin A = \frac{a}{c} \\
 \text{cosine} = \frac{\text{side adjacent}}{\text{hypotenuse}} & \text{or} & \cos A = \frac{b}{c} \\
 \text{tangent} = \frac{\text{side opposite}}{\text{side adjacent}} & \text{or} & \tan A = \frac{a}{b} \\
 \text{cotangent} = \frac{\text{side adjacent}}{\text{side opposite}} & \text{or} & \cot A = \frac{b}{a} \\
 \text{secant} = \frac{\text{hypotenuse}}{\text{side adjacent}} & \text{or} & \sec A = \frac{c}{b} \\
 \text{cosecant} = \frac{\text{hypotenuse}}{\text{side opposite}} & \text{or} & \csc A = \frac{c}{a}
 \end{array}$$

In order to solve trigonometry problems, a table of trigonometric functions of angles must be used. Such a table shows the functions in columns, arranged according to the size of the angle. Here is a simple table of the sines, cosines, tangents, and cotangents of angles from 0° to 10° and from 80° to 90° .

Angle	sin	cos	tan	cot	—
0°	.000	1.000	.000	—	90°
1°	.017	1.000	.017	57.290	89°
2°	.035	.999	.035	28.636	88°
3°	.052	.999	.052	19.081	87°
4°	.070	.998	.070	14.301	86°
5°	.087	.996	.088	11.430	85°
6°	.105	.995	.105	9.514	84°
7°	.122	.993	.123	8.144	83°
8°	.139	.990	.141	7.115	82°
9°	.156	.988	.158	6.314	81°
10°	.174	.985	.176	5.671	80°
—	cos	sin	cot	tan	Angle

To find the sine of 3° , for example, look in the column *under* sin, on the same line where 3° appears in the *left* column; $\sin 3^\circ = .052$, as shown. To find the sine of 87° , however, look in the column *over* sin, on the same line where 87° appears in the right column; $\sin 87^\circ = .999$, as shown. This reading down for angles smaller than 45° and up for angles larger than 45° is usual in tables, but not universal; it is an arrangement to save space. Use this simple table only as you need it to get the general idea; in practical work you will refer to

tables that give the angles in minutes as well as in degrees. They may be arranged somewhat differently. You will have to make yourself familiar with whatever set of tables you use; it will take only a few minutes' attention to do so.

All we need to know about a right triangle, in order to use it for inspection problems, is a set of two pieces of information—either

- (1) the lengths of any two sides, or
- (2) the length of any one side and the size of either acute angle.

If we know either the first or second sets of facts, we can calculate the lengths of all three sides and both acute angles. To show how, we will now use trigonometry to solve a typical shop problem.

Assume that the machine shop has made a wedge of steel (Figure 7) which, according to the blueprint, should have an angle of 26 degrees 33 minutes; your job, as inspector, is to determine whether the wedge

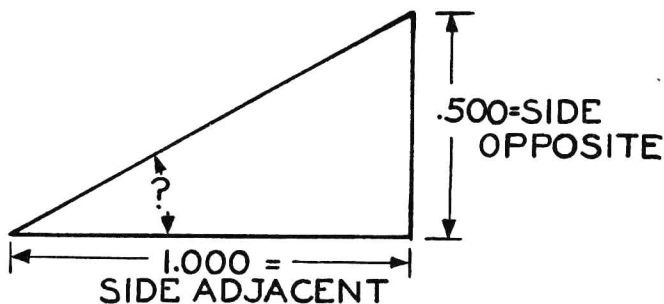


FIG. 7

is in accord with the blueprint specifications. A sine bar would be the best way to check the angle; a bevel protractor also could be used. But if these tools are not available, or not sufficiently precise, you must resort to the application of mathematics. Two things must be known in order to start work on the problem, and in this case they are found by using a micrometer and proceeding in the following steps:

- (1) Measure the side adjacent with a micrometer; the reading is 1.000 inch.
- (2) Measure the side opposite with a micrometer; the reading is $\frac{1}{2}$ inch or .500 inch.
- (3) Divide .500 by 1.000:

$$\begin{array}{r} .500 \\ 1.000 \overline{) .500} \\ \underline{.500} \\ 0 \end{array}$$

What you have done is to calculate the ratio

$$\frac{\text{side adjacent}}{\text{side opposite}}$$

which is the tangent. Your answer .500 is the tangent of the angle you are checking.

- (4) Look up .500 in the tangent column of the trigonometric tables.
- (5) The answer is 26 degrees 33 minutes and the piece is O.K.

There are two kinds of problems that an inspector must solve by trigonometric methods. They are:

- (1) to calculate the length of one side of his right triangle when he knows the size of an acute angle and the length of one other side;
- (2) to calculate the size of an acute angle when he knows the lengths of the two sides or of one side and the hypotenuse.

For other triangle problems he does not need trigonometry: if he knows two of the three sides he can find the third by square-root methods (see page 16); or if he knows one of the acute angles, he finds the other by simple subtraction.

Problems of the first kind mentioned above may occur in six forms. Examples of these six forms follow.

1. The length of a hypotenuse must be calculated. The inspector knows that his right triangle has one acute angle of 30° and that the side adjacent to this angle is 2.000 inches long. He proceeds:

$$\text{hypotenuse} = \text{side adjacent} \div \cosine. \quad (\text{step 1})$$

In Figure 8, this statement means

$$\text{hypotenuse} = 2.000 \div \cos 30^\circ; \quad (\text{step 2})$$

that is,

$$\text{hypotenuse} = 2.000 \div .866; \quad (\text{step 3})$$

therefore the answer is

$$\text{hypotenuse} = 2.309 \text{ inches.} \quad (\text{step 4})$$

If his book of tables lists secants and cosecants, the inspector can solve this problem by an easier method, in which he proceeds:

$$\text{hypotenuse} = \text{side adjacent} \times \secant. \quad (\text{step 1})$$

In Figure 8, this statement means

$$\text{hypotenuse} = 2.000 \times \sec 30^\circ; \quad (\text{step 2})$$

that is,

$$\text{hypotenuse} = 2.000 \times 1.1547; \quad (\text{step 3})$$

therefore the answer is

$$\text{hypotenuse} = 2.309 \text{ inches.} \quad (\text{step 4})$$

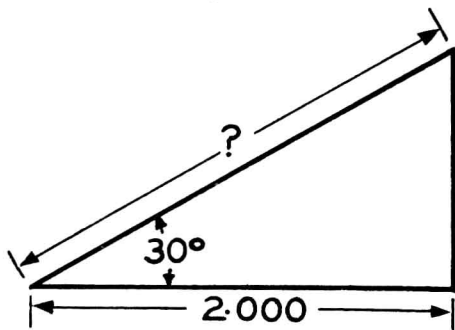


FIG. 8

2. The length of a hypotenuse must be calculated. The inspector knows that his right triangle has one acute angle of 30° and that the side opposite this angle is 1.1547 inches long. He proceeds:

$$\text{hypotenuse} = \text{side opposite} \div \text{sine.} \quad (\text{step 1})$$

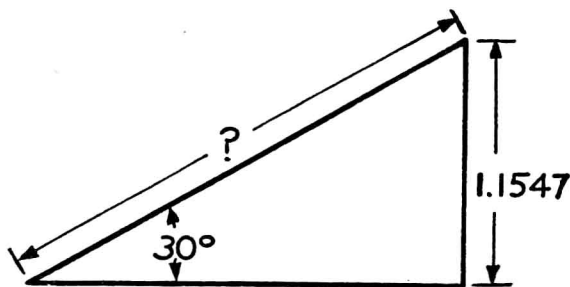


FIG. 9

In Figure 9, this statement means

$$\text{hypotenuse} = 1.1547 \div \sin 30^\circ; \quad (\text{step 2})$$

that is,

$$\text{hypotenuse} = 1.1547 \div .500; \quad (\text{step 3})$$

therefore the answer is

$$\text{hypotenuse} = 2.309 \text{ inches.} \quad (\text{step 4})$$

If his book of tables lists secants and cosecants, the inspector can solve this problem by an easier method, in which he proceeds:

$$\text{hypotenuse} = \text{side opposite} \times \text{secant.} \quad (\text{step 1})$$

In Figure 9, this statement means

$$\text{hypotenuse} = 1.1547 \times \sec 30^\circ; \quad (\text{step 2})$$

that is,

$$\text{hypotenuse} = 1.1547 \times 2.000; \quad (\text{step 3})$$

therefore the answer is

$$\text{hypotenuse} = 2.309 \text{ inches.} \quad (\text{step 4})$$