Electrical and Mechanical Networks

AN INTRODUCTION TO THEIR ANALYSIS

WILLIS W. HARMAN
Professor of Electrical Engineering
Stanford University

DEAN W. LYTLE
Associate Professor of Electrical Engineering
University of Washington

McGRAW-HILL ELECTRICAL AND
ELECTRONIC ENGINEERING SERIES

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STORER Passive Network Synthesis
STRAUSS Wave Generation and Shaping
TERMAN Electronic and Radio Engineering
TERMAN AND PETTIT Electronic Measurements
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THALER AND PASTEL Analysis and Design of Nonlinear Feedback Control Systems
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TOU Digital and Sampled-data Control Systems
TRUXAL Automatic Feedback Control System Synthesis
VALDES The Physical Theory of Transistors
WEINBERG Network Analysis and Synthesis
WILLIAMS AND YOUNG Electrical Engineering Problems

preface

When the notes on which the first half of this book is based were first used to teach sophomores at Stanford University in 1953, the approach was quite radical; now, as this material reaches publication stage, it may seem "old hat" to some. So rapidly do fashions change in our field. There are those among us who can recall when new texts were proudly advertised as "classroom-tested for fifteen years before publication." Today, however, when publishing contracts are often granted while the embryonic book is still little more than a tentative collection of chapter headings and when the time span between rough notes and published book is sometimes only a matter of months, the distinction of having been used in note form for eight years may seem to some a dubious one for this text to have. It makes the book seem almost by definition out of date. Nevertheless, it does mean that the material presented here, when supplemented by classroom aid and encouragement, has been found to be comprehensible to students at the sophomore-junior level.

The approach to the analysis of linear systems represented by the first five chapters is one which was introduced to the engineering profession by A. E. Kennelly and Vannevar Bush before World War I† and then lost sight of for over twenty years. This is the conceptually simple, but potentially rigorous and powerful, method of describing the linear network by finding the ratio of excitation to response when the excitation is of the form e^{st}. This ratio (impedance in the case of a one-port electrical network in which the voltage is excitation and the current is response) is used to find the complete transient response. The forced term is obtained by writing the excitation as the sum (or integral) of terms of the form e^{st} and dividing each term by the excitation-response ratio with the appropriate value of s inserted. The

† A. E. Kennelly, The Impedances, Angular Velocities and Frequencies of Oscillating-current Circuits, *Proc. IRE*, 4:47–48 (1916); V. Bush, Oscillating-current Circuits by the Method of Generalized Angular Velocities, *Trans. AIEE*, **36**:207–221 (1917).

natural behavior is obtained as a sum of terms of the form $A_i e^{s_i t}$, where the A_i are constants determined by initial conditions and the s_i are the values of s which make the excitation-response function equal to zero.

This approach is developed through a carefully chosen, graded series of examples in which the student is led to consider how the analysis methods he is to learn might have been originally devised. In later chapters, the methods are presented in a more systematic context, but initially the student is repeatedly invited to go through the experience of facing a new problem and inventing the analysis technique which will yield the solution. In Chapter 2, for example, he "discovers" the use of the impedance function for finding the natural behavior of simple two-element networks. In Chapter 3, faced with the new situation of the sinusoidal steady state, he finds that a simple extension allows him to handle this as well. Chapter 4 leads him to complete transient solutions, and Chapter 5 to resonance phenomena and the complex-frequency plane. In this modified case method of presentation, each advance in analytical technique is "invented" in response to a new challenge. (This approach is in contrast to the logically attractive, but psychologically less effective, method of first laying a solid foundation of topology, matrix algebra, and network theorems, after which the weary student is ready to solve an *RC* transient problem.)

Chapter 6 constitutes an interlude on the subject of electric power and related topics. It can be omitted without incurring serious interruption of the main discussion.

Chapter 7 extends the usefulness of the discovered methods through the table of unit-impulse responses (the Laplace transform table). The power of the tools now available is demonstrated by application to several practical examples. (The relationship of this approach to the Fourier-Laplace approach is discussed in Chapter 13, which can be taken up any time after Chapter 7.)

Chapters 8 and 9 deal with systematic methods of writing and solving network equations for more complex situations. Chapter 10 is another interlude primarily on electrical topics this time on mutual inductance.

Chapters 11 and 12 apply the analysis methods previously developed for electrical and mechanical networks to electromechanical energy-conversion and control systems, serving as introductions to more advanced work in these areas. Chapter 14, too, points to the future, with brief discussions of approaches to be used when networks are nonlumped (distributed), non-time-invariant, or nonlinear.

To ponder on the task of listing acknowledgments is to realize how many persons, contemporary and historical, have contributed to the writing of this text by their own writings, by discussions with the

authors, and by words of encouragement. Since it seems futile to attempt to list them, perhaps they will forgive omission of all names except that of Marilyn Marguerite Lytle, whose forbearance during the many hours her husband was at the writing desk and whose typing of hundreds of pages of manuscript went far beyond the wifely duties agreed to in the marriage contract.

Willis W. Harman
Dean W. Lytle

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1

simple electrical and mechanical systems

Engineers are, for the most part, concerned with the design, construction, and operation of complex systems for accomplishing certain physical tasks. Some of these complexes deal primarily with material highway systems for transportation of goods and persons, industrial systems for quantity production of refrigerators and television sets, apparatus for large-scale chemical and metallurgical processing, water-supply and sewage-removal systems, and food-processing plants, to name a few. Others have to do with energy—electric power systems for transforming chemical energy in coal to electrical energy in the home, power plants for transportation equipment of all types, servomechanisms for the accurate control of energy, refrigeration and air-conditioning systems. A third group, including communication systems and computing machines, deal with the transmission and the processing of information or data.

In all phases of this work the need continually arises for answers to such questions as the following:

> How should a particular portion of one of these complexes be designed so as to accomplish its task most effectively and economically?
>
> What is the best method of operation from the standpoints of economy, reliability, and effectiveness?
>
> How will operation be affected by the failure of a certain component, and how can the seriousness of the consequences be minimized?

Very few such questions can be answered directly. Most often the answers must be obtained by choosing a tentative solution or design, testing it by experiment or analysis, devising a new solution on the basis of what has been learned, testing this, and so on. Because testing by mathematical analysis is usually cheaper than testing by experiment, and also because the results are usually in more general form and give more insight into the relative effects of the various elements of the problem, analysis plays an increasingly important role in engineering.

ANALYSIS OF COMPONENT SYSTEMS Because of the complexity of the over-all systems it is ordinarily most practicable to subdivide them into smaller component systems and to deal with these individually.

The essential steps in analyzing these component systems are the following:

1. The physical system to be analyzed is represented by a mathematical model. This always involves approximations and assumptions which limit the application of the results of the analysis. It is most important, therefore, that the approximations involved be clearly stated and kept in mind.

2. Solutions are found for the mathematical equations which represent (although imperfectly) the physical system. This step is mathematics, whereas the first and third are engineering.

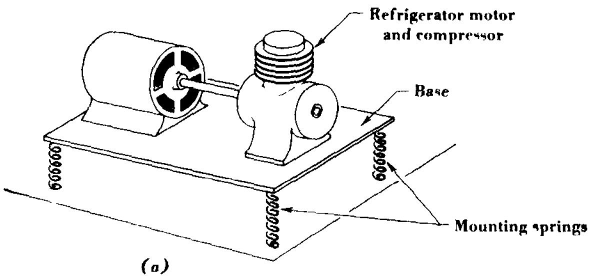
3. The mathematical solutions are interpreted physically. They are examined to evaluate the effects of assumptions and approximations and to make certain that they are physically reasonable. Finally, the information is presented in the most useful form (design formulas, tables, curves, etc.).

We propose, in this book, to study methods of analyzing mechanical and electrical systems. That is to say, we wish to examine physical systems such as those pictured in Fig. 1-1 with the ultimate aim of predicting their behavior when they are subjected to certain forces or conditions.

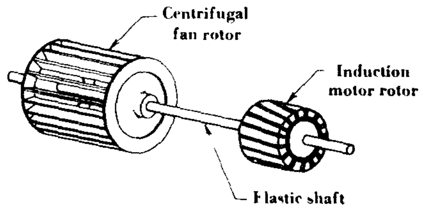
The universe is so constructed that the same mathematical equations often describe a number of different physical situations. In this case the various physical systems are called *analogues*. It is profitable to study such systems side by side, since the methods of analysis which are developed for one will apply equally well to the others. Furthermore, systems in which electrical and mechanical elements are combined are common—the entire field of servomechanisms is of this nature. Thus, there are compelling reasons for developing analysis methods which will be generally applicable.

1-1 MECHANICAL NETWORK ELEMENTS

Let us begin by considering the spring-mounted refrigerator motor and compressor of Fig. 1-1*a*. The first step to take in the study

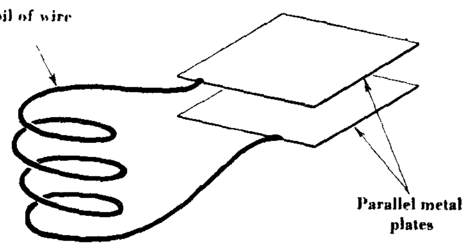


(a)



(b)

Fig. 1-1. Examples of physical systems.



(c)

of this system is to reduce it to an approximate model which is simpler than the actual system and yet adequate for the purpose at hand. For example, if only vertical oscillations are of interest, the combined effect of the four mounting springs may be approximated by a single equivalent spring. The combined mass of the motor, compressor, and base may be considered to be at a point, and the over-all frictional power

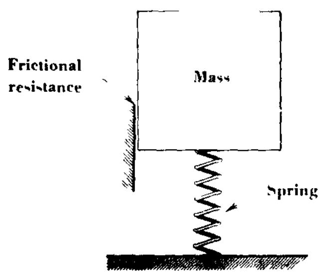


Fig. 1-2. Approximate model for Fig. 1-1a.

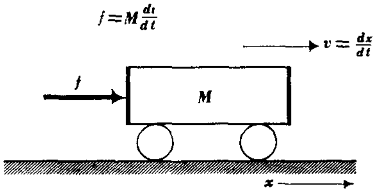


Fig. 1-3. The relation among mass, force, and acceleration.

loss may be concentrated in a small region. This gives the approximate model (mechanical *network*) of Fig. 1-2.†

In studying quantitatively the three mechanical system components which appear in this model, it will be advantageous to start from the concept of energy, since energy has the quality of universality. The principle of conservation of energy states that energy may be converted from one to another of various forms but that it is always the same energy. It may appear as mechanical, electrical, thermal, or chemical energy; it may diffuse as electromagnetic radiation, or it may be wrapped up in a particularly compact form as mass, as in an unexploded atomic bomb; but it is always a quantity which can be accounted for and measured.

MASS The kinetic energy (energy of motion) possessed by a moving body (Fig. 1-3) is found experimentally to be proportional to the square of its velocity v. The constant of proportionality is taken to be $M/2$, where M is the *mass* of the body. Thus, the kinetic energy is written

$$W_k = \tfrac{1}{2}Mv^2 \tag{1-1}$$

From this equation the mass of a body may be defined as a measure of its ability to store kinetic energy.

An alternative definition of mass is deduced by taking the derivative of both sides of Eq. (1-1). The time rate of change of energy is power p; hence

$$\frac{dW_k}{dt} = p = Mv\frac{dv}{dt} \tag{1-2}$$

† *System*, *device*, *model*, *network*, and *circuit* are all used more or less interchangeably in the literature of mechanical and electrical analysis. The unfortunate result is that none of these words retains a precise meaning. In this text, we shall use *network* when we are referring to the mathematical models of mechanical or electrical systems or devices. *System* is such a broad term that no effort will be made to restrict it to either models or actual physical devices. Thus, a term such as *system elements* can occur when either a device or its model is being discussed. *Circuit* will be used interchangeably with *system* in discussing electrical devices.

Another term which can cause confusion is *equivalent*. This word is often used in compounds like *equivalent system*, *equivalent circuit*, and *equivalent model*. In combinations of this sort the term invariably means equivalent in some restricted sense, such as equivalent over a certain range or with regard to some specified property.