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Peter J. Olver
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Computer Algebra and Geometric Algebra with Applications

6th International Workshop, IWMM 2004
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Preface

Mathematics Mechanization consists of theory, software and application of computerized mathematical activities such as computing, reasoning and discovering. Its unique feature can be succinctly described as AAA (Algebraization, Algorithmization, Application). The name "*Mathematics Mechanization*" has its origin in the work of Hao Wang (1960s), one of the pioneers in using computers to do research in mathematics, particularly in automated theorem proving. Since the 1970s, this research direction has been actively pursued and extensively developed by Prof. Wen-tsun Wu and his followers. It differs from the closely related disciplines like Computer Mathematics, Symbolic Computation and Automated Reasoning in that its goal is to make algorithmic studies and applications of mathematics the major trend of mathematics development in the information age.

The International Workshop on Mathematics Mechanization (IWMM) was initiated by Prof. Wu in 1992, and has ever since been held by the Key Laboratory of Mathematics Mechanization (KLMM) of the Chinese Academy of Sciences. There have been seven workshops of the series up to now. At each workshop, several experts are invited to deliver plenary lectures on cutting-edge methods and algorithms of the selected theme. The workshop is also a forum for people working on related subjects to meet, collaborate and exchange ideas.

There were two major themes for the IWMM workshop in 2004. The first was "Constructive and Invariant Methods in Algebraic and Differential Equations," or, in short, "Computer Algebra with Applications." The second was "Geometric Invariance and Applications in Engineering"(GIAE), or, in short, "Geometric Algebra with Applications." The two themes are closely related to each other. On the one hand, essentially due to the efforts of D. Hestenes and his followers, recent years have witnessed a dramatic resurgence of the venerable subject Geometric Algebra (which dates back to the 1870s), with dramatic new content and applications, ranging from mathematics and physics, to geometric reasoning, neural networks, robotics, computer vision and graphics. On the other hand, the rise of computer algebra systems and algorithms has brought previously infeasible computations, in particular those in geometric algebra and geometric invariance, within our grasp. As a result, the two intertwined subjects hold a particular fascination, not only for students and practitioners, but also for mathematicians, physicists and computer scientists working on effective geometric computing.

Since it is very difficult to put the two major themes into a single conference without parallel sessions, the organizers decided to split this year's *IWMM* workshop into two conferences, one for each theme. The first workshop was held in a beautiful quiet riverside town near Shanghai, called ZhuJiaJiao, from May 19 to 21. The second workshop was held in a glorious conference hall of the Xi'an

Hotel, Xi'an, from May 24 to 28. Altogether 169 scholars from China, USA, UK, Germany, Italy, Japan, Spain, Canada, Mexico and Singapore were attracted to the conferences and presented 65 talks. The following invited speakers presented the plenary talks:

Wen-tsun Wu (China)	Gerald Sommer (Germany)
Peter J. Olver (USA)	Alyn Rockwood (USA)
Anthony Lasenby (UK)	Joan Lasenby (UK)
Quan Long (Hong Kong, China)	Jingzhong Zhang (China)
Neil White (USA)	Timothy Havel (USA)
Jose Cano (Spain)	Greg Reid (Canada)
Andrea Brini (Italy)	Dongming Wang (China & France)
Xiaoshan Gao (China)	William Chen (China)
Hongqing Zhang (China)	Ke Wu (China)

The two conferences were very successful, and the participants agreed on the desirability of publication of the postproceedings by a prestigious international publishing house, these proceedings to include the selected papers of original and unpublished content. This is the background to the current volume.

Each paper included in the volume was strictly refereed. The authors and editors thank all the anonymous referees for their hard work. The copyediting of the electronic manuscript was done by Ms. Ronghua Xu of KLMM. The editors express their sincere appreciation for her dedication.

It should be emphasized that this is the first volume to feature the combination and interaction of the two closely related themes of Computer Algebra and Geometric Algebra. It is the belief of the editors that the volume will prove to be valuable for those interested in understanding the state of the art and for further combining and developing these two powerful tools in geometric computing and mathematics mechanization.

Beijing, Minneapolis, Kiel
March 2005

Hongbo Li
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Table of Contents

Part I : Computer Algebra and Applications

On Wintner's Conjecture About Central Configurations

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Abstract. According to Wintner, the study of central configurations in celestial mechanics may be reduced to an extremality problem. Wintner's Conjecture amounts to saying that the corresponding extremal zeroes for each fixed number n of different masses is finite. By the author's Finite Kernel Theorem it follows that the corresponding number of extremal values is finite for each fixed n. Thus, Wintner's Conjecture will be true or false according to whether there will be only a finite number of extremal zeroes or not. This gives thus a new way of attacking Wintner's Conjecture.

Keywords: Celestial Mechanics, Central Configurations, Wintner's Conjecture, Extremal Values, Extremal Zeroes, Finite Kernel Theorem.

The determination of central configurations in astromechanics is a fascinating problem since the time of Euler and Lagrange. In fact, Euler had shown that for three arbitrarily given distinct masses, there exist exactly three distinct collinear central configurations, and Lagrange had shown that the equilateral triangle and only the equilateral triangle is a non-collinear central configuration for three arbitrary masses. In the last century Moulton had proved that for arbitrary n distinct masses the number of collinear central configurations is $n!$ or $n!/2$, cf. [1]. In more recent time, A. Wintner had shown that, for four masses the regular tetrahedron and only the regular tetrahedron is a non-flat central configuration. Wintner had even announced a conjecture to the effect that for arbitrarily fixed number n of masses, the number of possible central configurations is always **finite**. This conjecture had resisted all attacks by quite a number of specialists, including e.g. Smale and his followers by some topological method created *ad hoc*, cf. [5, 6] and [2].

The actual determination of central configurations for a fixed n is not simple. Thus, in the literature there are only sporadic results of not great significance. In the nineties of the last century the present author had formulated a method of the determination of central configurations in reducing it to the solving of a system of polynomial equations under restrictions also in the form of polynomial equations. It is applied to give an alternative proof of theorems of Euler and Lagrange that the central configurations found by them are the only possible

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ones for $n = 3$, cf. [9]. In recent years H. Shi and others had applied this method to find various central configurations of special types for $n \geq 4$, cf. their papers, [3, 4, 11].

What is of great significance is this: Wintner in his classic on celestial mechanics, viz. [8], had shown that the central configurations are in correspondence with the extremal zeroes of some extremalization problem of rational function type and hence also of polynomial type. In applying the *Finite Kernel Theorem*, (cf. [10], Chap. 5, §5) on such extremality problems, we know that the extremal values of such problems are necessarily *finite* in number. If it can be shown that for each extremal value there can only be associated a *finite* number of extremal zeroes, then the total number of extremal zeroes of the problem will be *finite*, which is just the Wintner Conjecture in question. In any way the above gives an alternative method to attack the interesting and difficult conjecture of Wintner.

To begin with, let us first recall the definition of *Central Configuration*. Thus, let t be the time which is supposed to be fixed, and $m_i, (i = 1, \cdots, n)$ be n masses in question. Let us take a barycentric coordinate system with the center of mass of the system $\{m_1, \cdots, m_n\}$ at the origin (see §322 of [8], similar for references below). With respect to such a barycentric coordinate system let $\xi_i = (x_i, y_i, z_i)$ be the barycentric position of $m_i, (i = 1, \cdots, n)$. Set

$$\rho_{jk} = |\xi_j - \xi_k| = [(x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2]^{\frac{1}{2}}. \tag{1}$$

Then the potential energy of the system is given by

$$U = \frac{\Sigma_{1 \leq j < k \leq n} m_j m_k}{\rho_{jk}}. \tag{2}$$

The Newtonian force acting on the mass m_i is then given by

$$U_{\xi_i} = (U_{x_i}, U_{y_i}, U_{z_i}), \tag{3}$$

in which $U_{x_i} = \frac{\partial U}{\partial x_i}$, etc.

By Wintner's definition, the system $\{m_i, \xi_i | i = 1, \cdots, n\}$ is said to form a **Central Configuration** if the force of gravitation acting on each m_i, ξ_i is proportional to both the mass m_i and the barycentric position vector ξ_i, i.e.,

$$U_{\xi_i} = \sigma m_i \xi_i, \quad \text{for} \quad i = 1, \cdots, n, \tag{4}$$

where σ is some scalar independent of i.

It is proved by Wintner that

$$\sigma = -\frac{U}{J}, \tag{5}$$

in which (§322 bis)

$$J = \frac{1}{\mu} \Sigma_{1 \leq j < k \leq n} m_j m_k \rho_{jk}^2, \qquad \mu = \Sigma_{i=1,\cdots,n} m_i. \tag{6}$$

Wintner proved further (§355) that (4), (5), (6), are equivalent to the following equation:

$$JU_{\xi_i} = -\frac{1}{2}UJ_{\xi_i}, \quad i.e.\ (JU^2)_{\xi_i} = 0, \quad \text{for } i = 1, \cdots, n. \tag{7}$$

He further proved (§355, 357) that the system is a central configuration if and only if

$$(JU^2)_{\rho_{ik}} + \Sigma_s \chi_s (R_s)_{\rho_{ik}} = 0, \quad \text{for } 1 \leq i < k \leq n, \tag{8}$$

in which the χ_s's are the Lagrangian multipliers and

$$R_s(\rho_{12}, \cdots, \rho_{n-1,n}) = 0, \quad \text{for } s = 1, \cdots, p \tag{9}$$

are the necessary geometrical relations among the distances ρ_{ik} for $n \geq 4$ in planar case and $n \geq 5$ in spatial case.

In other words, *the system $\{m_i, \xi\}$ forms a central configuration if and only if*

$$JU^2 = Extremum \tag{10}$$

under the restrictions (9).

The Wintner's conjecture is thus reduced to the *extremum* problem (10) under the restricted conditions (9) which are equations in rational functions of various variables involved. We hope that our Finite Kernel Theorem of the present author concerning extremum problems of such type may be useful to arrive at a final proof of **Wintner Conjecture**, as indicated in the beginning of the present paper.

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Polynomial General Solutions for First Order Autonomous ODEs*

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Abstract. For a first order autonomous ODE, we give a polynomial time algorithm to decide whether it has a polynomial general solution and to compute one if it exists. Experiments show that this algorithm is quite effective in solving ODEs with high degrees and a large number of terms.

1 Introduction

To find elementary function solutions for differential equations could be traced back to the work of Liouville. As a consequence, such solutions of differential equations are called *Liouvillian solutions*. In [16], Risch gave an algorithm for finding Liouvillian solutions for the simplest differential equation $y' = f(x)$, that is, to find elementary function solutions to the integration $\int f(x)\mathrm{d}x$. Kovacic presented a method for solving second order linear homogeneous differential equations [13]. Singer established the general framework to find Liouvillian solutions of general homogeneous linear differential equations [18]. Many interesting results on finding Liouvillian solutions of linear ODEs are given in [1, 2, 3, 6, 11, 20, 19]. In [14], Li and Schwarz gave the first method to find rational solutions for a class of partial differential equations.

All these results are limited to linear cases. There seems no general methods to find Liouvillian solutions of nonlinear differential equations. With respect to ODEs of the form $y' = R(x, y)$ where $R(x, y)$ is a rational function, Poincaré made important contributions [15]. More recently, Carnicer also made important progresses in solving the Poincaré problem [5], which is equivalent to finding the degree bound for the algebraic solutions of $y' = R(x, y)$. For ODEs of this form, other work includes: Cano proposed an algorithm to find polynomial solutions [4]; Singer studied the Liouvillian first integrals [18]. On the other hand, Hubert gave a method to compute a basis of the general solutions of first order ODEs and applied it to study the local behavior of the solutions[10]. Bronstein gave an effective method to compute rational solutions of Ricatti equations [2]. In [9], we propose an algorithm to find rational solutions for first order autonomous ODEs. But this algorithm has exponential complexity.

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In this paper, we will give a polynomial time algorithm to find polynomial solutions of first order autonomous ODEs. Instead of finding arbitrary polynomial solutions, we will find the general solutions for ODEs of polynomial type. For example, the general solution for $(\frac{\mathrm{d}y}{\mathrm{d}x})^2 - 4y = 0$ is: $y = (x+c)^2$, where c is an arbitrary constant. Three main results are given in this paper. First, we give a sufficient and necessary condition for an ODE to have polynomial general solutions. Second, we give a detailed analysis of the structure of the first order autonomous ODEs which have polynomial general solutions. This leads to an almost *explicit formula* for the polynomial solutions of the first order autonomous ODE. Third, by introducing a novel method of substituting a polynomial solution into a first order ODE, we get a polynomial time algorithm to find polynomial general solutions of first order autonomous ODEs. Our experiments show that this algorithm is quite effective in solving ODEs with high degree and a large number of terms.

The paper is organized as follows. In section 2, a criterion for an ODE to have polynomial general solutions is given. In section 3, we give the degree bound of polynomial solutions of first order autonomous ODEs. In section 4, we analyze the structure of the first order autonomous ODEs which have polynomial solutions. In section 5, we present a polynomial time algorithm to find polynomial general solutions of first order autonomous ODEs. In section 6, we present the conclusion.

2 Polynomial General Solution to ODEs

Let $\mathbf{K} = \mathbf{Q}(x)$ be the differential field of rational functions in x with differential operator $\frac{\mathrm{d}}{\mathrm{d}x}$ and y an indeterminate over $\mathbf{K}$. We denote by y_i the i-th derivative of y. We use $\mathbf{K}\{y\}$ to denote the ring of differential polynomials over the differential field $\mathbf{K}$, which consists of the polynomials in the y_i with coefficients in $\mathbf{K}$. All differential polynomials in this paper are in $\mathbf{K}\{y\}$, if there is no other statement. Let Σ be a system of differential polynomials in $\mathbf{K}\{y\}$. A *zero* of Σ is an element in a universal extension field of $\mathbf{K}$ [17], which vanishes every differential polynomial in Σ. The totality of the zeros in $\mathbf{K}$ is denoted by $\mathrm{Zero}(\Sigma)$. In this paper, we will use $\mathcal{C}$ to denote the constant field of the universal extension of $\mathbf{K}$.

Let $P \in \mathbf{K}\{y\}/\mathbf{K}$. We denote $\mathrm{ord}(P)$ the highest derivative of y in P, called the *order* of P. Let $o = \mathrm{ord}(P) > 0$ be the order of P. We may write P as follows

$$P = a_d y_o^d + a_{d-1} y_o^{d-1} + \ldots + a_0$$

where a_i are polynomials in $y_1, \ldots, y_{o-1}$ for $i = 0, \ldots, d$ and $a_d \neq 0$. a_d is called the *initial* of P and $S = \frac{\partial P}{\partial y_o}$ is called the *separant* of P. The k-th derivative of P is denoted by $P^{(k)}$. Let S be the separant of P, $o = \mathrm{ord}(P)$ and $k > 0$. Then we have

$$P^{(k)} = S y_{o+k} - R_k \tag{1}$$

where R_k is of lower order than $o + k$.