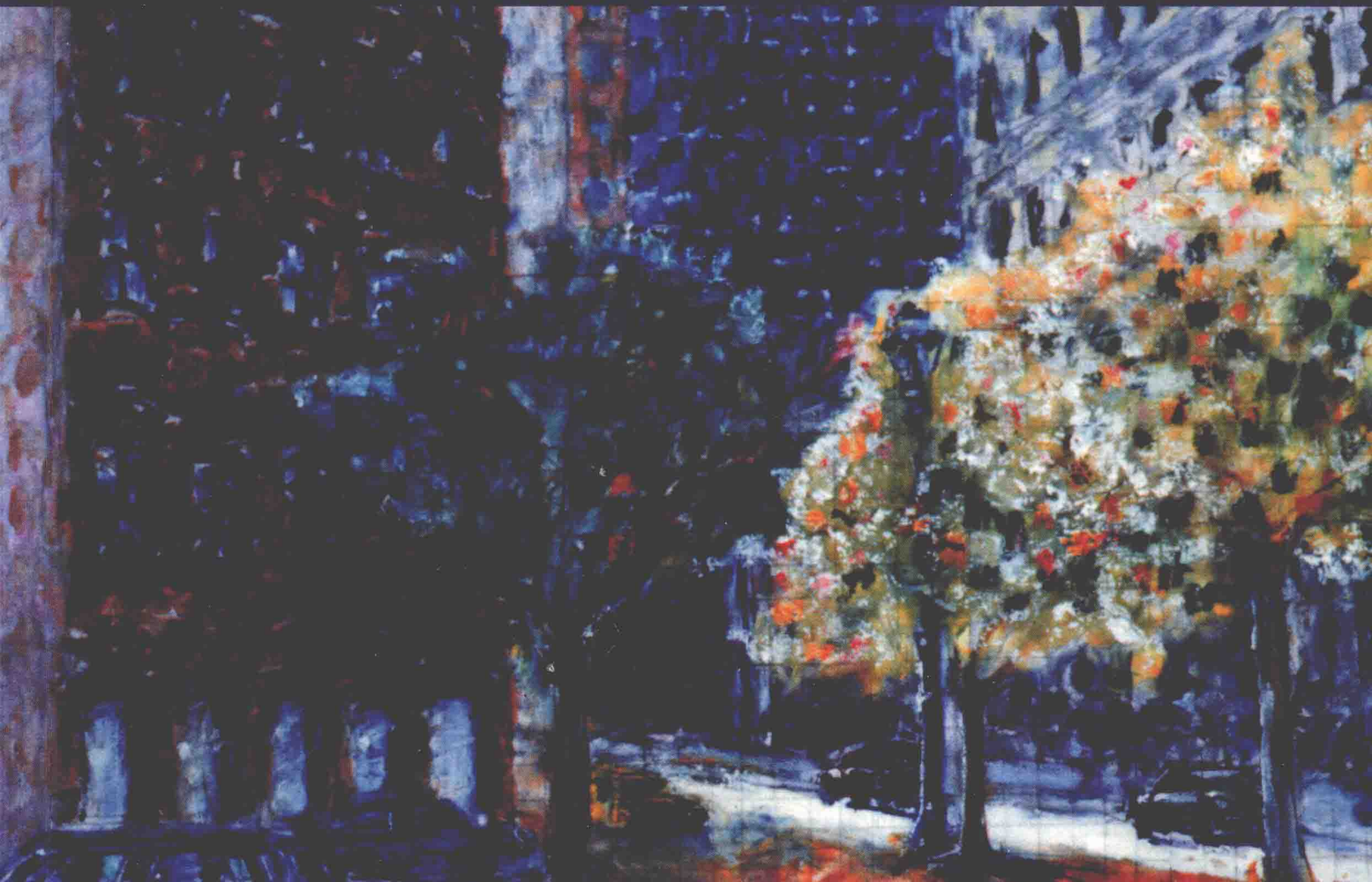
Quantitative Methods for Business

A CONCEPTUAL, EXCEL-BASED APPROACH



MILLIANNE LEHMANN AND PAUL ZEITZ UNIVERSITY OF SAN FRANCISCO / FALL 2004

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Preface for Students

In January 1996, the McLaren Business School of the University of San Francisco asked the mathematics department to overhaul the Quantitative Methods for Business course. We were challenged with the task of creating a mathematics course for business students that was truly relevant. The modern businessperson must be able to understand quantitative information and solve complex problems, but not necessarily with formal mathematical methods such as algebra and calculus. Computers have become so advanced that they can do much of the algebra for us.

The challenge for the 21st-century business student is to learn to co-exist with computers, to use them effectively as problem-solving tools. The key skills now are not delicate algebraic techniques, but instead are methods of formulating sophisticated problems in a way that computers can understand them. The main computer tool that we will use is the spreadsheet program Excel.

We will cover many of the standard mathematics topics, which includes review of algebra, percents, finance, linear equations, optimization and linear programming. But our course will greatly de-emphasize algebra in favor of understanding the concepts and using Excel.

If you think that this means the course is easy, you are mistaken. The beauty of powerful spreadsheet programs like Excel is that they allow us to analyze and solve truly sophisticated problems which would be impossible to tackle with traditional algebraic methods. Thus we will spend much of our time looking at *really hard word problems.* The computer will solve the problems, but you will do the most intellectually challenging part, namely formulating ("setting up") the problems.

This is the preliminary edition of our text. You are the guinea pigs, reading our draft chapters. Please point out typos and give us suggestions for improvement!

A few words about reading this book:

- Please keep pencil, paper and calculator handy. Having a computer nearby is a good idea, as well. Each chapter has an associated computer file (named **chapter1.xls**, etc.). Ask your instructor for the location of these files.
- There are many worked examples (each of which ends with a little filled-in square like this: ■). Read the examples carefully—they are the core of the book.
- We try to use very few formulas, instead focusing on concepts that will help you to think independently. But those formulas and principles which are truly important are placed in a box $\boxed{\text{like this}}$. When you encounter something in a box, mull it over until you have mastered it (and memorized it).

Contents

Chapter 1

Numbers, Equations, and Formulas

1.1 Introduction

Why Math?

A business produces goods and/or services which are then sold, hopefully, at a profit. Goods, services, profit, and debt can all be expressed as numerical quantities. One of the components of any successful business is a careful analysis of these quantities. Here is a very simple and practical example, a study of consumer debt.

Example 1.1.1 *The Credit-Card Problem.* Dana gets a credit card which offers a \$6,000 credit limit and a 24% annual interest rate. This is her first credit card, and she goes wild, accumulating \$6,000 worth of purchases in no time at all (she has "maxed out" the card).

Dana does not have to pay her balance of \$6,000 immediately. Each month, she receives a bill. The credit card company adds the monthly interest to her outstanding balance (calling it a "finance charge") and requires that she at least pay a "minimum payment" which in this case is 3% of the balance, or \$10, whichever is greater. For example, in the first month, her finance charge will be 3% of \$6,000, which is[1]

$$0.03 \times \$6{,}000 = \$180.$$

Dana decides that she will just pay the minimum payment each month. The question is, how long will it take to pay off her debt? And how much money will she pay in doing so? ∎

This problem, which we will solve later in the chapter (see Problems 1.2.20, 1.4.9, 1.5.9–10), is easy to state, yet *cannot be solved using algebraic formulas.* Like most interesting and important problems that we will examine, the solution requires a three-pronged approach, employing logic, simple algebra, and computer assistance. Algebra, while important, is rarely the key. Instead, you will find that

[1] If you don't remember percents, have no fear. This topic is reviewed thoroughly in Section 1.2.

most of your effort goes into the logical analysis of the problem and a careful application of the computer to solve it.

Consequently, while we use algebra and other formal mathematical tools in this book, most of the pages below are devoted not so much to new mathematical ideas, but instead to ways of clearly analyzing problems with math, both by hand as well as with calculators and computers.

What Math?

We hope that this textbook is unlike any other math textbook that you have seen, because it doesn't try to teach you much formal mathematics. Instead, the main goal of this book is to teach you how to attack and solve problems, by resourcefully using common sense, algebra, and, most crucially, the computer. For most problems, even the very hardest ones, the solution is less a matter of technical knowledge than it is just knowing *exactly* when to add, subtract, multiply, or divide numbers.

That's the good news; after all, you already know how to add, subtract, multiply, and divide. The bad news is that this book is essentially a collection of harder and harder story problems, many requiring innovative use of relatively simple tools. To master them, you must get comfortable with numbers, arithmetic, and simple algebra. We assume that you have studied much of this before, but that you may be a little rusty.

To see how much you remember, please take the 30-minute **diagnostic test** in Appendix B on p. 412. The answers are on p. 413, with a table showing you the relevant section of the text for each question. In various sections in this and the next chapter, you will be reminded to "skim or read" sections in the appendix. Use your performance on the diagnostic test to guide you: if you missed one or more questions pertaining to the appendix section, *read the section carefully, and do ALL the exercises*. If you got all the questions right, then, at the very least, skim the section, and try doing some of the exercises. This review process will not take long, and will provide an important foundation for the more challenging work to come.

The text proper begins with thorough review of percents, a subject which you have undoubtedly seen before. Even though percents are a very elementary topic, they are crucial to all aspects of business, and you *must* master them before proceeding. We urge you to read the next section extremely carefully.

The remainder of the chapter is devoted to revisiting elementary arithmetic and algebra using Excel. When you have finished this chapter, you will be well on your way to developing a strong partnership with the computer as more than just a computational tool. These early Excel sections are very important. Especially if you are new to Excel, you should read them slowly and carefully, ideally with a computer in front of you. Almost all of the Excel concepts that you will use in the rest of the book are developed in this first chapter.

Why Excel?

Unlike many business mathematics books, this one takes the point of view that you should use a computer to help you with most of your math homework. We believe that there is nothing wrong with using mechanical aids like computers or calculators. At the time of this writing (2003) the typical computer can perform about as many calculations in a second as a human can do in a year. And software advances have kept pace with hardware: there are now many computer programs that can perform not just arithmetic, but algebra. Essentially, if something can be done by following rigid rules (and most algebraic procedures, such as multiplying polynomials or solving equations, can be done this way), then a computer can do it.

A number of "math processor" software programs have evolved to meet the special needs of engineering, statistics, and architecture. Microsoft Excel is designed for business, and is one of several similar **spreadsheet** programs. Spreadsheet programs organize information in two-dimensional tables. Each table entry (called a **cell**) can be a number or a formula which may refer to other cells.

While Excel may not be the best spreadsheet program, or the least expensive (indeed, there are free spreadsheet programs available from the internet), it has many desirable features.

- It is quite easy to learn. Anyone who has used a word processor and is willing to play around a bit can master the basics of Excel in a few hours.
- It is easy to augment. Indeed, we have created a few custom additions (included with this book) which enhance Excel's graphing and statistical capabilities.
- It is the most popular spreadsheet program, compatible both with Windows and Macintosh operating systems, and used by many businesses, and often included as part of larger "office" applications.
- Because Excel is so popular, mastering it is not a pointless exercise. You will continue to use Excel in other classes, and almost certainly, on the job in the "real world."

Excel is amazingly powerful. It can solve complex equations, deal with huge masses of data, and analyze complex scenarios involving dozens of variables. When properly used, Excel will become an indispensable assistant to you, one that handles virtually all of the heavy lifting and tedium. But there is one thing that Excel cannot do. It cannot think. You need to use your skills and creativity to analyze problems and set them up in such a way that Excel can solve them. This is not an easy task; indeed it is one of the central goals of this book.

1.2 Percents

Please read or skim Section A.1 **before** *starting this section, and please read or skim Section A.2* **during** *your reading of this section.*

Percents are just another notation style for numbers, but a crucially important one, used everywhere and everyday in business. You must become completely fluent with them. For example, you should be able to answer the following question.

13 is 5% of what number?

If you did not get 260, in a few seconds (without a calculator and without using algebra), then you need some practice. Read this section.

Demystifying Percents

Percents are just fractions, with denominators of 100. The word "percent" literally means "per hundred" (from the Latin for "hundred"); for example,

$$6\% = \frac{6}{100} = 0.06.$$

In other words, $x\%$ means *exactly* the same thing as $x/100$, and this means the same thing as "write x in decimal form and then move the decimal point two digits to the left."

What makes percents difficult is not their definition, but their usage. To achieve fluency, you need to become familiar with several common percent numbers as well as some important conventions of language.

First, master the percent equivalencies shown in shown in the percent table found on page 4. Use flash cards if necessary.

Next, a simple language rule:

Percents behave like fractions in that **of** means "times."

For example,

- 10% of 40 is equal to

$$(10\%) \times 40 = 0.10 \times 40 = 4.$$

- 25% of 72 is the same as one-fourth of 72, or

$$\frac{1}{4} \times 72 = \frac{72}{4} = 18.$$

- 1000% of something means multiply it by 10.
- If you are in the top one percent of a graduating class of 1400 students, then 1386 of these students got worse grades than you (because 1% of 1400 is 14 and $1400 - 14 = 1386$).
- 6% of 17 equals $0.06 \times 17 = 1.02$.
- 5% of N means multiply N by $1/20$, and that is the same as dividing N by 20.
- If you are in the 28% tax bracket, then more than one-fourth but less than one-third of your income goes to the government.

As a percent	As a decimal	As a fraction
1%	0.01	$\frac{1}{100}$
2%	0.02	$\frac{2}{100} = \frac{1}{50}$
5%	0.05	$\frac{5}{100} = \frac{1}{20}$
10%	0.1	$\frac{10}{100} = \frac{1}{10}$
20%	0.2	$\frac{20}{100} = \frac{1}{5}$
25%	0.25	$\frac{25}{100} = \frac{1}{4}$
$33\frac{1}{3}\%$	0.333...	$\frac{1}{3}$
50%	0.5	$\frac{50}{100} = \frac{1}{2}$
$66\frac{2}{3}\%$	0.666...	$\frac{2}{3}$
75%	0.75	$\frac{75}{100} = \frac{3}{4}$
100%	1	$\frac{100}{100} = 1$
150%	1.5	$\frac{150}{100} = \frac{3}{2}$
200%	2	$\frac{200}{100} = 2$
300%	3	$\frac{300}{100} = 3$
400%	4	$\frac{400}{100} = 4$
500%	5	$\frac{500}{100} = 5$
1000%	10	$\frac{1000}{100} = 10$

- 19 is 10% of 190 (because 19 is one-tenth of 190).

Now we can easily answer the question posed above, "13 is 5% of what number?" Since 5% is the same as $1/20$, we are asking,

What number, when multiplied by $1/20$, *equals 13?*

But that is the same as

What number, when divided by 20, equals 13?

Consequently, the number we seek is just 13 multiplied by 20, which is 260.

Percent Increase and Decrease

Consider the following problem.

Example 1.2.1 The price of an item is first discounted by 50% and then marked up by 50%. What is the net percentage change of the price of this item?

Solution: Many people mistakenly answer 0%, thinking that the two procedures cancel each other out. To see why this is not correct, **make the problem concrete**—supply an actual starting price for the item, such as \$100, and then actually work everything out.

When the starting price is discounted by 50%, that means it is reduced by 50% of its original price. In other words,

$$\begin{aligned}\text{price after discounting} &= \text{original price} - 0.50 \times \text{original price}\\ &= \$100 - 0.5 \times \$100\\ &= \$100 - \$50\\ &= \$50.\end{aligned}$$

Next, we must mark up the discounted price of \$50; i.e., increase the discounted price by 50% of the discounted price:

$$\begin{aligned}\text{price after markup} &= \text{discounted price} + 0.50 \times \text{discounted price}\\ &= \$50 + 0.5 \times \$50\\ &= \$50 + \$25\\ &= \$75.\end{aligned}$$

Thus the final price is \$25 less than the starting price of \$100. Since \$25 is 25% of \$100, the answer is that the net percentage change is a decrease of 25%. ■

The solution above has some defects. First of all, how do we know if we will get the same answer if we used a different starting price? And also, the solution shows *how*, but doesn't explain *why*. To understand better, we need to use a tiny bit of algebra (for somewhat more detail, see Examples A.2.2 and A.2.3).

Let's first solve a simpler problem, but in a slightly more general form:

Example 1.2.2 What happens when the price of something goes up by 7%?

Solution: Let the starting price be S. Then

$$\text{price increase} = \text{starting price} \times 0.07 = 0.07S,$$

and

$$\begin{aligned}\text{new price (after markup)} &= \text{starting price} + \text{price increase}\\ &= S + 0.07S\\ &= S(1 + .07)\\ &= 1.07S.\end{aligned}$$

■

This reasoning doesn't depend on the value of S, nor on the percent by which we are changing S. So we have a completely general **percent-change formula**:

$$\boxed{\text{new value} = \text{old value} \times (1 + \text{percent change}).} \qquad (1)$$

Please note the following:

- The value of "percent change" must be used in its decimal or fractional form. Thus, in the example above, we used 0.07 in place of 7%; we did not use the value 7!
- The percent change value can be negative, indicating a decrease in value. For example, if we wanted to decrease S by 2%, we would subtract $0.02S$ from S, so the new value would be

  $$S - 0.02S = (1 - 0.02)S = 0.98S.$$

  Again, see Examples A.2.2 and A.2.3 if you had trouble understanding the equation above.

Now we see that our solution to Example 1.2.1 was no fluke. For any starting price, we have

$$\begin{aligned} \text{final price} &= \text{starting price} \times (1 - 0.5) \times (1 + 0.5) \\ &= \text{starting price} \times (0.5)(1.5) \\ &= \text{starting price} \times 0.75, \end{aligned}$$

and since $0.75 = 1 - 0.25$, this represents a net decrease by the fraction 0.25, or 25%.

The percent-change formula is one of the most important formulas in this book. You will use it (by hand and with a computer) thousands of times, so it is a good idea to get very comfortable with it. Here are a few examples.

Example 1.2.3 A quantity grows from 18 to 20. What is the percent change?

Solution: Plug right into the percent-change formula, and use the tiniest bit of algebra. We have

$$20 = 18(1 + \text{percent change}),$$

so

$$1 + \text{percent change} = \frac{20}{18} = 1.111\ldots.$$

Consequently, the percent change is $0.111\ldots$ as a decimal, or $11.11\ldots\%$. How should we round off? That depends on the circumstance. But in general, if you don't have any requirements such as currency which forces precision, an answer should have about as many significant digits as the numbers in the problem. So 11% is probably an adequate answer here. ■

You may be tempted (as are many textbooks) to create a formula for answering questions like the one above. What you need to do is take the percent-change formula and solve for the percent change in terms of the old and new values. The result (see the first two sections of Appendix 3 if you are rusty with your algebra) is

$$\text{percent change} = \frac{\text{new value}}{\text{old value}} - 1.$$

There is nothing wrong with this formula; indeed it is mathematically flawless. However, we do not advocate that you spend much time and effort creating and learning formulas, for the simple reason that too many formulas clutter up your brain and it is very hard to keep track of which one is used where. Our approach is to learn just a few formulas, but to learn those well. Along with the few vital formulas (which we always highlight by putting in a box), you need to master a few examples that use simple numbers. Good examples are easy to remember, and encourage you to use common sense rather than blind faith in poorly-understood formulas.

OK, sermon's over. Here is another practice example, very similar to the previous one, but with a business context.

Example 1.2.4 *Profit*. If you spent \$70,000 on a business which had revenues of \$85,000, what was your profit as a percentage of your investment?

Solution: There are many ways to think about this problem. Here is one way: First, determine what the investment is. This is the \$70,000 that has been spent. The profit, then, is the amount that this investment has grown by. The revenues were \$85,000, so the profit is the difference $\$85,000 - \$70,000 = \$15,000$. Consequently, the profit as a percentage of investment is

$$\frac{15000}{70000} = .214\ldots \approx 21\%.$$

Here is another way, which has the (slight) advantage of not wasting time by subtracting the investment from the revenue. Visualize the "value" of the business as a changing quantity. At first, the value is \$70,000, since that is what has been put into it. But after this investment is transformed into a product which is then sold producing the revenues, these revenues form the new (and higher) value of the business. All that we seek is the percentage change that occurred. Hence

$$85000 = 70000 \times (1 + \text{percent change}),$$

so

$$1 + \text{percent change} = \frac{85000}{70000} = 1.214\ldots.$$

It is easy to just strip off the 1 from this and recover 21% as the answer. ■

Example 1.2.5 The value of stock A rose by 12% in 1997, but fell by 6% in 1998. In contrast, stock B experienced a 6% drop in 1997 and a 12% rise in 1998. Which stock did better?

Solution: The stocks did equally well. Using the percent-change formula twice, we compute that the final (end of 1998) price of stock A is equal to

$$\begin{aligned}\text{starting price} \times (1+0.12) \times (1-0.06) &= \text{starting price} \times (1.12)(0.94)\\ &= \text{starting price} \times 1.0528.\end{aligned}$$