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Albert C. J. Luo

Discontinuous Dynamical Systems on Time-varying Domains

动态域上的不连续动力学系统

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With 96 figures, 4 of them in color

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Earthquake and Heartbreak

that shocks one's passion and spirit,
and recalls everyone to a sense of duty.

Preface

This book is about discontinuous dynamical systems on time-varying domains. I had not planed to write this book originally. As a scientist working on dynamics and vibration, the 5.12 earthquake of Wenchuan (Sichuan province, China) shocked my heart and made me feel guilty because my research cannot make any direct contributions to help them. Therefore, I would like to write two words "Zhenhan" in Chinese callig-raphy on the dedication page to express my passion. The meaning of "Zhenhan" is "Earthquake and Heartbreak" that shocks one's passion and spirit, and recalls everyone to a sense of duty. Herein, I would like to accumulate recent research developments of discontinuous dynamical systems on time-varying domains. One likes to use continuous models for discontinuous dynamical systems. However, sometimes such con-tinuous modeling cannot provide adequate descriptions of discontinuous dynamical systems. Recently, researchers have gradually realized that discontinuous modeling may provide an adequate and acceptable predi-cation of engineering systems. Currently, most research still focuses on discontinuous dynamical systems on time-invariant domains. To better describe practical problems, some research on discontinuous systems on time-varying domains is scattered here and there but without a systemati-cal theory. The purpose of this book is to systematically present a theory of discontinuous dynamical systems on time-varying domains for univer-sity students and researchers.

This book mainly focuses on the switchability of discontinuous dy-namical systems on time-varying domains. Based on such concepts, prin-ciples of dynamical system interactions without any connections are pre-sented. This book consists of seven chapters. Chapter 1 discusses two examples to show where discontinuous dynamical systems exist. Chapter 2 presents a basic theory for the switchability of a flow to the separation boundary in discontinuous dynamical systems, and switching bifurca-tions are also addressed. In Chapter 3, transversality and sliding phenomena for a controlled dynamical system to an inclined line boundary of control logic are presented to show how to apply such a new theory. In Chapter 4, dynamics of a frictional oscillator on a traveling belt with time-varying speeds is presented, which is a simple example of discon-tinuous dynamical systems on the time-varying domains. Chapter 5 pre-sents the dynamics mechanism of impacting chatter and stick phenomena in two dynamical systems with impact laws. In Chapter 6, dynamical behaviors of two systems connected with friction are presented. In Chap-ters 3 6, the similar writing styles and formats are adopted to show how to apply the new theory to different problems. In Chapter 7, a generalized theory for the interaction principle of dynamical systems is developed from discontinuous dynamical systems on time-varying domains. I hope the information presented in this book may stimulate more research in the area of discontinuous dynamical systems.

Finally, I would like to appreciate my students (Brandon M. Rapp, Brandon C. Gegg, Dennis O'Connor and Sagun Thapa) for applying the new concepts to mechanical systems and completing all numerical com-putations. This book is dedicated to the people who died during the earthquake of Wenchuan (Sichuan province, China) on May 12, 2008. Finally, I would like to thank my wife (Sherry X. Huang) and my chil-dren (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their tolerance, patience, understanding and support.

Albert C. J. Luo
Edwardsville, Illinois
August, 2008

Contents

Chapter 1
Introduction

In engineering, discontinuous dynamical systems exist everywhere. One usually uses continuous models to describe discontinuous dynamical systems. However such continuous models cannot provide suitable predictions of discontinuous dynamical systems. To better understand discontinuous systems, one should realize that discontinuous models will provide an adequate and real predication of engineering systems. Thus, one considers a global discontinuous system consisting of many continuous sub-systems in different domains. For each continuous subsystem, it possesses dynamical properties different from the adjacent continuous subsystems. Because of such difference between two adjacent subsystems, the switch ability and/or transport laws on their boundaries should be addressed. The investigation on such discontinuous systems mainly focused on the time-independent boundary between two dynamical systems. In fact, the boundary relative to time is more popular. In this book, discontinuous dynamical systems on time-varying domains will be of great interest. A brief survey will be given through two practical examples. Finally, the book layout will be presented, and the summarization of all chapters of the main body of this book will be given.

1.1 Discontinuous systems

In mechanical engineering, there are two common and important contacts between two dynamical systems, i.e., impact and friction. For example, gear transmission systems possess both impact and friction. Such gear transmission systems are used to transmit power between parallel shafts or to change direction. During the power transmission, a pair of two gears forms a resultant dynamical system. Each gear has its own dynamical system connected with shafts and bearings. Because two subsystems are not connected directly, the power transmission is completed through the impact and friction. Because both of sub-systems are independent each other except for impacting and sliding together, such two dynamical systems have a common time-varying boundary for impacting, which causes domains for the two dynamical systems to be time-varying.

In the early investigation, a piecewise stiffness model was used to investigate dynamics of gear transmission systems. Although such a dynamical system is discontinuous, the corresponding domains for vector fields of the dynamical system are time-independent. For instance, den Hartog and Mikina (1932) used a piecewise linear system without damping to model gear transmission systems, and the symmetric periodic motion in such a system was investigated. For low-speed gear systems, such a linear model gave a reasonable prediction of gear-tooth vibrations. With increasing rotation speed in gear transmission systems, vibrations and noise become serious. Ozguven and Houser (1988) gave a survey on the mathematical models of gear transmission systems. The piecewise linear model and the impact model were two of the main mechanical models to investigate the origin of vibration and noise in gear transmission systems. Natsiavas (1998) investigated a piecewise linear system with a symmetric tri-linear spring, and the stability and bifurcation of periodic motions in such a system were investigated through the variation of initial conditions. Based on a piecewise linear model, the dynamics of gear transmission systems was investigated by Comparin and Singh (1989), and Theodossiades and Natsiavas (2000). Pfeiffer (1984) presented an impact model of gear transmissions, and the theoretical and experimental investigations on regular and chaotic motions in the gear box were later carried out by Karagiannis and Pfeiffer (1991).

To model vibrations in gear transmission systems, Luo and Chen (2005) gave an analytical prediction of the simplest periodic motion through a piecewise linear impacting system. In addition, the corresponding grazing of periodic motions was observed, and chaotic motions were simulated numerically through such a piecewise linear system. From the local singularity theory in Luo (2005), the grazing mechanism of the strange fragmentation of the piecewise linear system was discussed by Luo and Chen (2006). Luo and Chen (2007) used the mapping structure technique to analytically predict arbitrary periodic motions of such a piecewise linear system. In that piecewise linear model, it was assumed that impact locations were fixed, and the perfectly plastic impact was considered. The separation of the two gears occurred at the same location as the gears impact. Compared with the existing models, this model can give a better prediction of periodic motions in gear transmission systems, but the relative assumptions may not be realistic to practical transmission systems because all the aforementioned investigations are based on a time-independent boundary or a given motion boundary. To consider the dynamical systems with the time-varying boundary, Luo and O'Connor (2007a,b) proposed a mechanical model to determine the mechanism of the impacting chatter and stick in gear transmission systems. The corresponding analytical conditions for such impacting chatter and stick were developed.

In mechanical engineering, the friction contact between surfaces of two systems is important for motion transmissions (e.g., clutch systems, brake systems, etc.). In addition, the two systems are independent except for the friction contact. Such problem will have time-varying boundary and domains. For such a friction problem, it should return back to the 1930's. den Hartog (1931) investigated the periodic motion of a forced, damped, linear oscillator contacting a surface with friction. Levitan (1961) investigated the existence of periodic motions in a friction oscillator with a periodically driven base. Filippov (1964) investigated the motion existence of a Coulomb friction oscillator, and presented a differential equation theory with discontinuous right-hand sides. The differential inclusion was introduced via the set-valued analysis for the sliding motion along the discontinuous boundary. The investigations of discontinuous differential equations were summarized by Filippov (1988). However, the Filippov's theory mainly focused on the existence and uniqueness of solutions for non-smooth dynamical systems. Such a differential equation theory with discontinuity is difficult to apply to practical problems. Luo (2005a) developed a general theory to handle the local singularity of discontinuous dynamical systems. To determine the sliding and source motions in discontinuous dynamical systems, the imaginary, sink and source flows were introduced by Luo (2005b). The detailed discussions can be referred to Luo (2006).

On the other hand, Hundal (1979) used a periodic, continuous function to investigate the frequency-amplitude response of a friction oscillator. Shaw (1986) investigated non-stick, periodic motions of a friction oscillator through Poincaré mapping. Feeny (1992) analytically investigated the non-smooth of the Coulomb friction oscillator. To verify the analytic results, Feeny and Moon (1994) investigated chaotic dynamics of a dry-friction oscillator experimentally and numerically. Feeny (1996) gave a systematic discussion of the nonlinear dynamical mechanism of stick-slip motion of friction oscillators. Hinrichs, Oestreich and Popp (1997) investigated the nonlinear phenomena in an impact and friction oscillator under external excitations (also see, Hinrichs, Oestreich and Popp (1998)). Natsiavas (1998) developed an algorithm to numerically determine the periodic motion and the corresponding stability of piecewise linear oscillators with viscous and dry friction damping (also see, Natsiavas and Verros (1999)). Ko, Taponat and Pfaifer (2001) investigated the friction-induced vibrations with and without external excitations. Andreaus and Casini (2002) gave a closed form solution of a Coulomb friction-impact model without external excitations. Thomsen and Fidlin (2003) gave an approximate estimation of response amplitude for the stick-slip motion in a nonlinear friction oscillator. Kim and Perkins (2003) investigated stick-slip motions in a friction oscillator via the harmonic balance or Galerkin method. Li and Feng (2004) investigated the bifurcation and chaos in a friction-induced oscillator with a nonlinear friction model. Pilipchuk and Tan (2004) investigated the dynamical behaviors of a 2DOF mass-damper-spring system contacting on a decelerating rigid strip with friction. Awrejcewicz and Pyryev (2004) gave an investigation on frictional periodic processes by accelerating or braking a shaft-pad system. In 2007, Hetzler, Schwarzer and Seemann (2007) considered a nonlinear friction model to analytically investigate the Hopf-bifurcation in a sliding friction oscillator with applications to the low frequency disk brake noise.

In the aforementioned investigations, the conditions for motion switchability to the discontinuous boundary were not considered enough. Luo and Gegg (2005a) used the local singularity theory of Luo (2005a, 2006) to develop the force criteria for motion switchability on the velocity boundary in a harmonically driven linear oscillator with dry-friction (also see, Luo and Gegg (2005b)). Through such an investigation, the traditional eigenvalue analysis may not be useful for motion switching at the discontinuous boundary. Lu (2007) mathematically proved the existence of such a periodic motion in a friction oscillator. Luo and Gegg(2006a,b) investigated the dynamics of a friction-induced oscillator contacting on time-varying belts with friction. Recently, to model the disk brake system, many researchers still considered the mechanical model as in Hetzler, Schwarzer and Seemann (2007). Luo and Thapa (2007) proposed a new method to model the brake system which consists of two oscillators, and the two oscillators are connected through a contacting surface with friction. Based on this model, the nonlinear dynamical behaviors of a brake system under a periodical excitation were investigated.

The other developments on non-smooth dynamical systems in recent decades will be addressed as well. Feigin (1970) investigated the C-bifurcation in piecewise-continuous systems via the Floquet theory of mappings, and the motion complexity was classified by the eigenvalues of mappings, which can be referred to recent publications (e.g., Feigin 1995; di Bernnado et al 1999). The C-bifurcation is also termed the grazing bifurcation by many researchers. Nordmark (1991) used "grazing" terminology to describe the grazing phenomena in a simple impact oscillator. No strict mathematical description was given, but the grazing condition (i.e., the velocity $dx/dt = 0$ for displacement x) in such an impact oscillator was obtained. From Luo (2005a, 2006), such a grazing condition is a necessary condition only. The grazing is the tangency between an n-D flow of dynamical systems and the discontinuous boundary surface. From differential geometry points of view, Luo (2005a) gave the strict mathematic definition of the "grazing", and the necessary and sufficient conditions of the general discontinuous boundary were presented (also see, Luo 2006). Nordmark's result is a special case. Nusse and Yorke (1992) used the simple discrete mapping from Nordmark's impact oscillator and showed the bifurcation phenomena numerically. Based on the numerical observation, the sudden change bifurcation in the numerical simulation is called the *border-collision bifurcation*. So, the similar discrete mappings in discontinuous dynamical system were further developed. Especially, Nordmark and Dankowicz (1999) developed a discontinuous mapping from a general way to investigate the grazing bifurcation, and the discontinuous mapping is based on the Taylor series expansion in the neighborhood of the discontinuous boundary. Following the same idea, di Bernardo et al (2001a,b; 2002) developed a normal form to describe the grazing bifurcation. In addition, di Bernardo et al (2001c) used the normal form to obtain the discontinuous mapping and numerically observed such a border-collision bifurcation through a discontinuous mapping. From the discontinuous mapping and its normal form, the aforementioned bifurcation theory structure was developed for the so-called, codimension 1 dynamical system.

The discontinuous mapping and normal forms on the discontinuous boundary were developed from the Taylor series expansion in the neighborhood of the discontinuous boundary. However, the normal form requires the vector field with the C^r-continuity and the corresponding convergence, where the order r is the highest order of the total power numbers in each term of normal form. For piecewise linear and nonlinear systems, the C^1-continuity of the vector field cannot provide an enough mathematical base to develop the normal form. The normal form also cannot be used to investigate global periodic motions in the discontinuous system. Leine et al (2000) used the Filippov theory to investigate bifurcations in nonlinear discontinuous systems. However, the discontinuous mapping techniques were employed to determine the bifurcation via the Floquet multiplier. The more discussion about the traditional analysis of bifurcation in non-smooth dynamical systems can be referred to Zhusubaliyev and Mosekilde (2003). Based on the recent research, the Floquet multiplier also may not be adequate for periodic motions involved with the grazing and sliding motions in non-smooth dynamical systems. Therefore, Luo (2005a) proposed a general theory for the local singularity of non-smooth dynamical systems on connectable domains (also see, Luo (2006)). To resolve the difficulty, Luo (2007b) developed a general theory for the switching possibilities of flow on the boundary from the passable to non-passable one, and so on. In this book, from recent developments in Luo (2007a; 2008a,b), a generalized theory for discontinuous systems on time-varying domains will be presented.

1.2 Book layout

To help readers easily read this book, the main contents in this book are summarized as follows.

In Chapter 2, from Luo (2008a,b), a switchability theory for a flow to the boundary in discontinuous dynamical systems will be presented. The G-functions for discontinuous dynamical systems will be introduced to investigate singularity in discontinuous dynamical systems. Based on the new G-function, the full and half sink and source, non-passable flows to the separation boundary in discontinuous dynamical systems will be discussed, and the switchability of a flow from a domain to an adjacent one will be addressed. Finally, the switching bifurcations between the passable and non-passable flows will be presented.

In Chapter 3, the switchability of a flow from one domain into another in discontinuous dynamical systems will be presented through a periodically forced, discontinuous dynamical system with a time-invariant boundary. The normal vector-field for flow switching on the separation boundary will be introduced and the passability condition of a flow to the separation boundary will be given through such normal vector fields. The sliding and grazing conditions to the separation boundary will be presented as well. This investigation may help one better understand the sliding mode control.

In Chapter 4, an oscillator moving on the periodically traveling belt with dry friction is investigated as a dynamical system with a time-varying boundary. The conditions of stick and non-stick motions in such an frictional oscillator will be developed, and the periodic motions in the oscillator will be investigated as well. The grazing and stick (or sliding) bifurcations will be investigated. The significance of this investigation is to show how to control motions in such friction-induced oscillators in industry.

In Chapter 5, impact and stick motions of two oscillators at the time-varying boundary and domains will be presented under an impact law. The dynamics mechanism of the impacting chatter with stick at the moving boundary will be investigated from the local singularity theory of discontinuous dynamical systems. The analytical conditions for the onset and vanishing of stick motions will be obtained, and the condition for maintaining stick motion is achieved as well. This chapter will show how two dynamical systems interact.

In Chapter 6, two dynamical systems connected with friction will be presented, and the motion switchability on the moving discontinuous boundary will be discussed through the theory of discontinuous dynamical systems. The onset and vanishing of motions will be discussed through the bifurcation and grazing analyses. It will be observed that two different systems can stick together forever.

In Chapter 7, a theory for two dynamical systems with a general interaction will be presented. Such an interaction occurs at a time-varying boundary. From the discontinuous theory in Chapter 2, a general methodology for two system interactions will be presented in order to determine the complex motion of the two systems, which is caused by the interaction between two systems.

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