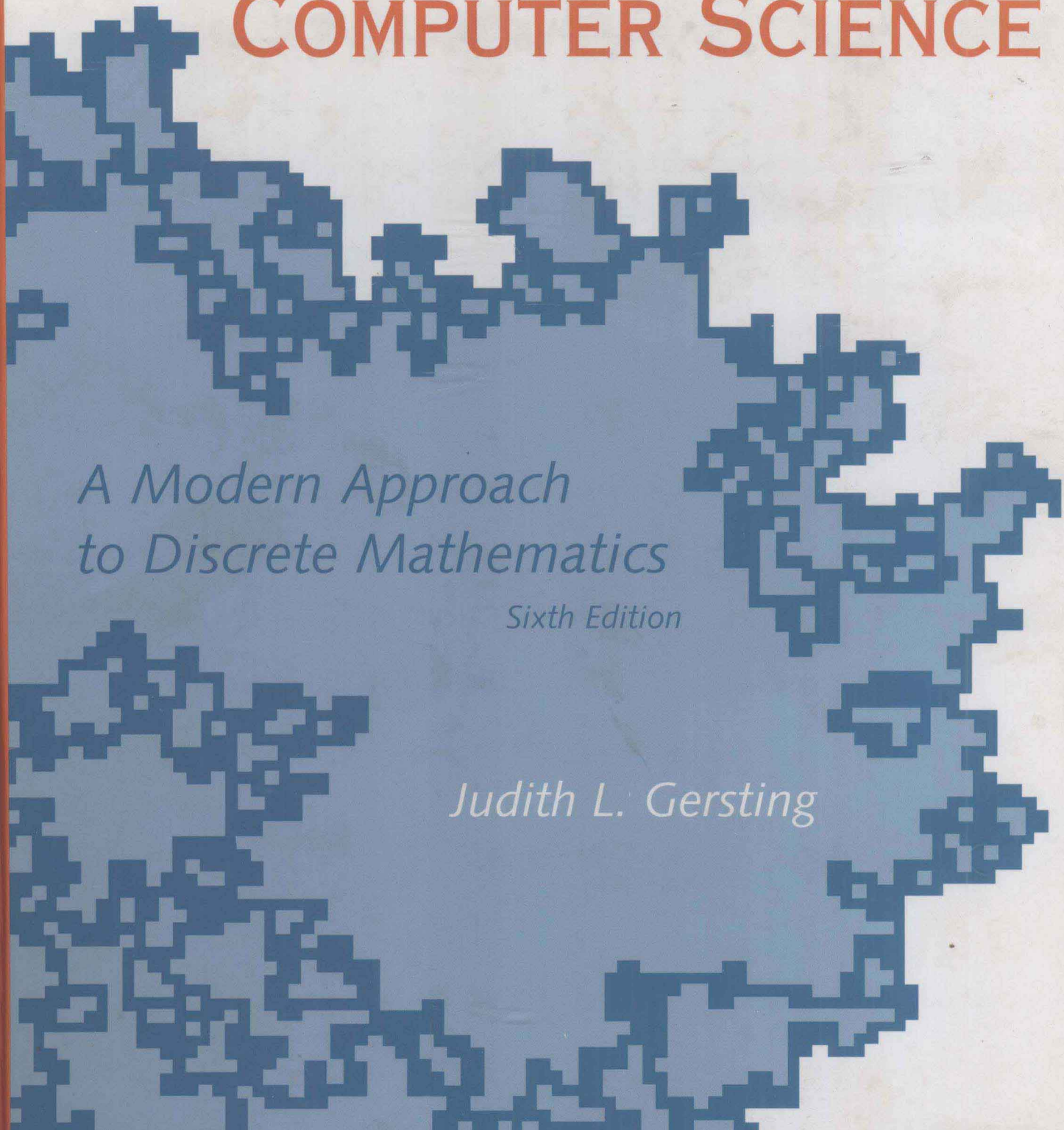
Mathematical Structures for Computer Science



A Modern Approach to Discrete Mathematics

Sixth Edition

Judith L. Gersting

MATHEMATICAL STRUCTURES FOR COMPUTER SCIENCE

A Modern Approach to Discrete Mathematics

SIXTH EDITION

Judith L. Gersting
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Now there are 0101_2 favorite discrete structures:

$(\text{Adam} \wedge \text{Francine}), (\text{Jason} \wedge \text{Cathryn} \rightarrow \text{Samantha})$

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Preface

Discrete structures plays an ever more prominent role in computer science education. Computing Curricula 2001 (CC2001) was a joint undertaking of the Institute for Electrical and Electronic Engineers/Computer Society (IEEE/CS) and the Association for Computing Machinery (ACM). The final report may be found at http://www.acm.org/sigcse/cc2001/. CC2001 identifies a computer science *body of knowledge*, a set of knowledge units "for which there is a broad consensus that the material is essential to an undergraduate degree in computer science." CC2001 describes discrete structures as "foundational material for computer science" and devotes 43 hours of the computer science body of knowledge to discrete structures, more hours than to any other area.

This Sixth Edition of *Mathematical Structures for Computer Science* covers all the topics in the CC2001 discrete structures core and quite a bit more. All topics suggested for a one-semester intensive discrete structures course—and virtually everything suggested for a two-semester version of a discrete structures course—are covered here.

Binding together what otherwise appears to be a collection of disjoint topics are the following themes:

- Importance of logical thinking
- Power of mathematical notation
- Usefulness of abstractions

Of all computer science courses, discrete structures is probably the least appreciated in real time and the most appreciated given more perspective. Two years after the fact, students report, "Every course I had after that used material from discrete structures."

Content Changes in the Sixth Edition

Major changes include the following:

- Reorganization of Chapter 2 to make Recurrence Relations a separate section (Section 2.5), with the addition of material on linear second-order recurrence relations and on divide-and-conquer recurrence relations
- New section on number theory (Section 3.7)
- New section (Section 4.5) on "the mighty mod function," giving a number of applications of the mod function in hashing, cryptography, identification codes, and modular arithmetic designs, among others
- Restoration, at the request of some reviewers, of a section (Section 6.5) on articulation points in graphs that had been deleted from the Fifth Edition.

New Exercises have also been added to each section. It appears that exercises, like chocolate chip cookies, are something you just can't have too many of. Answers to all Practice Problems are given at the back of the book, as are answers to all starred Exercises.

A complete **Solutions Manual** is available to instructors from the publisher.

Web Site

Online Study Guide

A Web site for the book may be found at www.whfreeman.com/gersting, accessed using the password "logic." It functions as an online study guide, offering for each chapter a chapter overview, section overviews, and sample problems with audio commentaries outlining the step-by-step solution process. The sample problems (not contained in the text) are representative of many of the end-of-section Techniques. Each Technique that has a corresponding Web page example is marked with the icon  .

Each sample problem on the Web first states the problem, and then, as the student navigates the pages, the solution unfolds step-by-step much as the student would be expected to write it. A compressed audio file is also part of each Web page after the initial problem statement. The audio file contains a first-person stream-of-consciousness thought process about that step of the solution—why it occurred to the narrator to try it, why it looked promising, what knowledge was being called upon to suggest that it should come next, and so on. Students see perfect and complete worked-out proofs in the textbook and often also see them performed by the instructor. Yet when a student goes home and tries to produce a solution alone, he or she is unsure where to start or how to think about the problem or how to discern any pattern to enable a guess as to what to do next. Consequently the student gives up in frustration. The purpose of the audio narration is to share the "secret picture" that mathematicians use to solve problems.

To access the problems, after you go to www.whfreeman.com/gersting, select a chapter, log in with the password "logic," select a chapter section, then select a sample problem and follow its step-by-step process with the "Next" button. Several sample problems are provided for each section of each chapter.

PowerPoint Slides

PowerPoint slides accompanying each section of the text are available on the Web site at www.whfreeman.com/gersting.

Acknowledgments

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My deepest thanks go to my husband, John, ever my most ardent supporter and dearest friend.

Note to the Student

As you go through this book, you'll encounter many new terms and new ideas. Try reading with pencil and paper at hand and work the Practice Problems as you encounter them. They are intended to reinforce or clarify terminology or a method just introduced; answers are given at the back of the book. Pay attention also to the Reminders that point out common pitfalls or provide helpful hints. Be sure to visit the Web site at www.whfreeman.com/gersting, using the password "logic," for detailed, worked-out solutions to additional example problems tied to the Techniques at the end of each section. The Web site solutions are accompanied by audio files that explain each step.

You may find at first that the thought processes required to solve the exercises in the book are new and difficult. Your biggest attribute for success will be perseverance. Here's what I tell my students: "If you do not see at first how to solve a problem, don't give up. Think about it some more; be sure you understand all the terminology used in the problem, play with some ideas. If no approach presents itself, put it aside and think about it again later. Repeat this process for days on end. When you finally wake up in the middle of the night with an idea, you'll know you are putting in the right amount of effort for this course." Mathematical results don't spring fully formed from the foreheads of mathematical geniuses (well, maybe from mathematical geniuses); but for the rest of us, it takes work, patience, false starts, and **perseverance**.

Enjoy the experience!

Contents in Brief

Contents

Formal Logic

CHAPTER 1

CHAPTER OBJECTIVES

After studying this chapter, you will be able to:

- Use the formal symbols of propositional logic.
- Find the truth value of an expression in propositional logic.
- Construct formal proofs in propositional logic, and use such proofs to determine the validity of English language arguments.
- Use the formal symbols of predicate logic.
- Find the truth value in some interpretation of an expression in predicate logic.
- Use predicate logic to represent English language sentences.
- Construct formal proofs in predicate logic, and use such proofs to determine the validity of English language arguments.
- Understand how the programming language Prolog is built on predicate logic.
- Mathematically prove the correctness of programs that use assignment statements and conditional statements.

You have been selected to serve on jury duty for a criminal case. The attorney for the defense argues as follows:

If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on October 10, it follows that Jason Pritchard did not see the knife. Furthermore, if the knife was there on October 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

QUESTION **Is the attorney's argument sound? How should you vote?**

It's much easier to answer this question if the argument is recast in the notation of formal logic. Formal logic strips away confusing verbiage and allows us to concentrate on the underlying reasoning being applied. In fact, formal logic—the subject of this chapter—provides the foundation for the organized, careful method of thinking that characterizes any reasoned activity—a criminal investigation, a scientific experiment, a sociological study. In addition, formal logic has direct applications in computer science. The last two sections of this chapter explore a programming language based on logic and the use of formal logic to verify the