Special Relativity and Quantum Theory

A Collection of Papers on the Poincaré Group

edited by

M. E. Noz

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dedicated to Professor Eugene Paul Wigner on the 50th Anniversary of His Paper on Unitary Representations of the Inhomogeneous Lorentz Group (completed in 1937 and published in 1939)

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Preface

Special relativity and quantum mechanics are likely to remain the two most important languages in physics for many years to come. The underlying language for both disciplines is group theory. Eugene P. Wigner's 1939 paper on the Unitary Representations of the Inhomogeneous Lorentz Group laid the foundation for unifying the concepts and algorithms of quantum mechanics and special relativity. In view of the strong current interest in the space-time symmetries of elementary particles, it is safe to say that Wigner's 1939 paper was fifty years ahead of its time. This edited volume consists of Wigner's 1939 paper and the major papers on the Lorentz group published since 1939.

This volume is intended for graduate and advanced undergraduate students in physics and mathematics, as well as mature physicists wishing to understand the more fundamental aspects of physics than are available from the fashion-oriented theoretical models which come and go. The original papers contained in this volume are useful as supplementary reading material for students in courses on group theory, relativistic quantum mechanics and quantum field theory, relativistic electrodynamics, general relativity, and elementary particle physics.

This reprint collection is an extension of the textbook by the present editors entitled "**Theory and Applications of the Poincaré Group.**" Since this book is largely based on the articles contained herein, the present volume should be viewed as a continuation of and supplementary reading for the previous work.

We would like to thank Professors J. Bjorken, R. Feynman, R. Hofstadter, J. Kuperzstych, L. Michel, M. Namiki, L.Parker, S. Weinberg, E.P. Wigner, A.S. Wightman, and Drs. P. Hussar, M. Ruiz, F. Rotbart, and B. Yurke for allowing us to reprint their papers. We are grateful to Mrs. M. Dirac and Mrs. S. Yukawa for giving us permission to reprint the articles of Professors P.A.M. Dirac and H. Yukawa respectively.

We wish to thank the Annals of Mathematics for permission to reprint Professor Wigner's historic paper. We thank the American Physical Society, the American Association of Physics Teachers, The Royal Society of London, il Nuovo Cimento and Progress in Theorectical Physics for permission to reprint the articles which appeared in their journals and for which they hold the copyright. The excerpt from *Albert Einstein: Historical and Cultural Perspective: The Centennial Symposium in Jerusalem* is reprinted with permission of Princeton University Press; that from

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Introduction

One of the most fruitful and still promising approaches to unifying quantum mechanics and special relativity has been and still is the covariant formulation of quantum field theory. The role of Wigner's work on the Poincaré group in quantum field theory is nicely summarized in the fourth paragraph of an article by V. Bargmann *et al.* in the commemorative issue of the Reviews of Modern Physics in honor of Wigner's 60th birthday [Rev. Mod. Phys. *34*, 587 (1962)], which concludes with the sentences:

> "Those who had carefully read the preface of Wigner's great 1939 paper on relativistic invariance and had understood the physical ideas in his 1931 book on group theory and atomic spectra were not surprised by the turn of events in quantum field theory in the 1950's. A fair part of what happened was merely a matter of whipping quantum field theory into line with the insights achieved by Wigner in 1939".

It is important to realize that quantum field theory has not been and is not at present the only theoretical machine with which physicists attempt to unify quantum mechanics and special relativity. Indeed, Dirac devoted much of his professional life to this important task, but, throughout the 1950's and 1960's, his form of relativistic quantum mechanics was overshadowed by the success of quantum field theory. However, in the 1970's, when it was necessary to deal with quarks confined permanently inside hadrons, the limitations of the present form of quantum field theory become apparent. Currently, there are two different opinions on the difficulty of using field theory in dealing with bound-state problems or systems of confined quarks. One of these regards the present difficulty merely as a complication in calculation. According to this view, we should continue developing mathematical techniques which will someday enable us to formulate a bound-state problem with satisfactory solutions within the framework of the existing form of quantum field theory. The opposing opinion is that quantum field theory is a model that can handle only scattering problems in which all particles can be brought to free-particle asymptotic states. According to this view we have to make a fresh start for relativistic bound-state problems.

These two opposing views are not mutually exclusive. *Bound-state models developed in these two different approaches should have the same space-time symmetry*. It is quite possible that independent bound-state models, if successful in explaining what we see in the real world, will eventually complement field theory. One of the purposes of this book is to present the fundamental papers upon which a relativistic bound-state model that can explain basic hadronic features observed in high-energy laboratories could be build in accordance with the principles laid out by Wigner in 1939.

Wigner observed in 1939 that Dirac's electron has an SU(2)-like internal space-time symmetry. However, quarks and hadrons were unknown at that time. Dirac's form of relativistic bound-state quantum mechanics, which starts from the representations of the Poincaré group, makes it possible to study the O(3)-like little group for massive particles and leads to hadronic wave functions which can describe fairly accurately the distribution of quarks inside hadrons. Thus a substantial portion of hadronic physics can be incorporated into the O(3)-like little group for massive particles.

Another important development in modern physics is the extensive use of gauge transformations in connection with massless particles and their interactions. Wigner's 1939 paper has the original discussion of space-time symmetries of massless particles. However, it was only recently recognized that gauge-dependent electromagnetic four-potentials form the basis for a finite-dimensional non-unitary representation of the little group of the Poincaré group. This enables us to associate gauge degrees of freedom with the degrees of freedom left unexplained in Wigner's work. Hence it is possible to impose a gauge condition on the electromagnetic four-potential to construct a unitary representation of the photon polarization vectors.

Wigner showed that the internal space-time symmetry group of massless particles is locally isomorphic to the Euclidian group in two-dimensional space. However, Wigner did not explore the content of this isomorphism, because the physics of the translation-like transformations of this little group was unknown in 1939. Neutrinos were known only as "Dirac electrons without mass", although photons were known to have spins either parallel or antiparallel to their respective momenta. We now know the physics of the degrees of freedom left unexplained in Wigner's paper. Much more is also known about neutrinos today that in 1939. For instance, it is firmly established that neutrinos and anti-neutrinos are left and right handed respectively. Therefore, it is possible to discuss internal space-time symmetries of massless particles starting from Wigner's E(2)-like little group. Recently, it was observed that the O(3)-like little group becomes the E(2)-like group in the limit of small mass and/or large momentum.

Indeed, group theory has become the standard language in physics. Until the 1960's, the only group known to the average physicist had been the three-dimensional rotation group. Gell-Mann's work on the quark model encouraged physicists to study the unitary groups, which are compact groups. The Weinberg-Salam model enhanced this trend. The emergence of supersymmetry in the 1970's has brought the space-time group closer to physicists. These groups are non-compact, and it is difficult to prove or appreciate mathematical theorems for them.

The Poincaré group is a non-compact group. Fortunately, the representations of this group useful in physics are not complicated from the mathematical point of view.

The application of the Lorentz group is not restricted to the symmetries of elementary particles. The (2 + 1)-dimensional Lorentz group is isomorphic to the two-dimensional symplectic group, which is the symmetry group of homogeneous linear canonical transformations in classical mechanics. It is also useful for studying coherent and squeezed states in optics. It is likely that the Lorentz group will serve useful purposes in many other branches of modern physics.

This reprint volume contains the fundamental paper by Wigner, and the papers on applications of his paper to physical problems. This book starts with Wigner's review paper on relativistic invariance and quantum phenomena. The reprinted papers are grouped into nine chapters. Each chapter starts with a brief introduction.

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Chapter I

Perspective View of Quantum Space-Time Symmetries

When Einstein formulated his special theory of relativity in 1905, quantum mechanics was not known. Einstein's original version of special relativity deals with point particles without space-time structures and extension. These days, we know that elementary particles can have intrinsic space-time structure manifested by spins. In addition, many of the particles which had been thought to be point particles now have space-time extensions.

The hydrogen atom was known to be a composite particle in which the electron maintains a distance from the proton. Therefore, the hydrogen atom is not a point particle. The proton had been regarded as a point particle until, in 1955, the experiment of Hofstadter and McAllister proved otherwise. These days, the proton is a bound state of more fundamental particles called the quarks. We still do not know whether the quarks have non-zero size, but assume that they are point particles. We assume also that electrons are point particles. However, it is clear that these particles have intrinsic spins. The situation is the same for massless particles. For intrinsic spins, the Wigner's representation of the Poincaré group is the natural scientific language.

As for nonrelativistic extended particles, such as the hydrogen atom, the present form of quantum mechanics with the probability interpretation is quite adequate. If the proton is a bound state of quarks within the framework of quantum mechanics, the description of a rapidly moving proton requires a Lorentz transformation of localized probability distribution. In addition, this description should find its place in Wigner's representation theory of the Poincaré group.

This Chapter consists of one article by Wigner on relativistic invariance of quantum phenomena, and one article by Dirac. As he said in his 1979 paper, Dirac was concerned with the problem of fitting quantum mechanics in with relativity, right from the beginning of quantum mechanics. Dirac suggests that the ideal mechanics should be both relativistic and deterministic. It would be too ambitious to work with both the relativistic and deterministic problem at the same time. Perhaps the easier way is to deal with one aspect at a time. Then there are two routes to the ideal mechanics, as are illustrated in Figure 1. The current literature indicates that it would be easier to make quantum mechanics relativistic than deterministic. In this book, we propose to study the easier problem first.

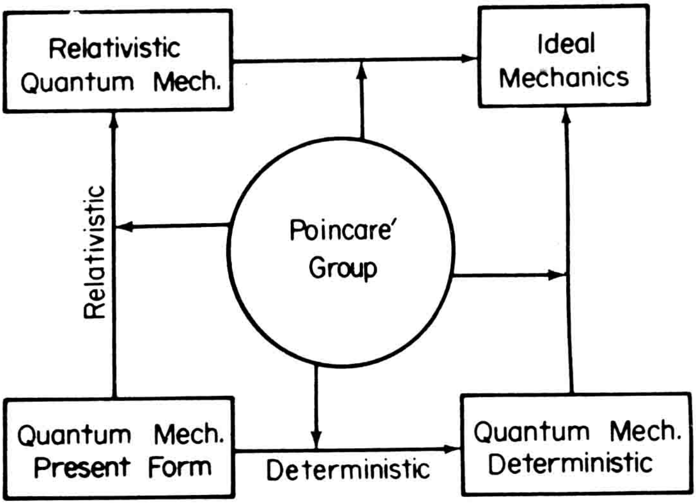

FIG. 1. Two different routes to the ideal mechanics. Covariance and determinism are the two main problems. In approaching these problems, there are two different routes. In either case, the Poincaré group is likely to be the main scientific language.

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Relativistic Invariance and Quantum Phenomena*

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INTRODUCTION

THE principal theme of this discourse is the great difference between the relation of special relativity and quantum theory on the one hand, and general relativity and quantum theory on the other. Most of the conclusions which will be reported on in connection with the general theory have been arrived at in collaboration with Dr. H. Salecker,[1] who has spent a year in Princeton to investigate this question.

The difference between the two relations is, briefly, that while there are no conceptual problems to separate the theory of special relativity from quantum theory, there is hardly any common ground between the general theory of relativity and quantum mechanics. The statement, that there are no conceptual conflicts between quantum mechanics and the special theory, should not mean that the mathematical formulations of the two theories naturally mesh. This is not the case, and it required the very ingenious work of Tomonaga, Schwinger, Feynman, and Dyson[2] to adjust quantum mechanics to the postulates of the special theory and this was so far successful only on the working level. What is meant is, rather, that the concepts which are used in quantum mechanics, measurements of positions, momenta, and the like, are the same concepts in terms of which the special relativistic postulate is formulated. Hence, it is at least possible to formulate the requirement of special relativistic invariance for quantum theories and to ascertain whether these requirements are met. The fact that the answer is more nearly *no* than *yes*, that quantum mechanics has not yet been fully adjusted to the postulates of the special theory, is perhaps irritating. It does not alter the fact that the question of the consistency of the two theories can at least be formulated, that the question of the special relativistic invariance of quantum mechanics by now has more nearly the aspect of a puzzle than that of a problem.

This is not so with the general theory of relativity. The basic premise of this theory is that coordinates are only auxiliary quantities which can be given arbitrary values for every event. Hence, the measurement of position, that is, of the space coordinates, is certainly not a significant measurement if the postulates of the general theory are adopted: the coordinates can be given any value one wants. The same holds for momenta. Most of us have struggled with the problem of how, under these premises, the general theory of relativity can make meaningful statements and predictions at all. Evidently, the usual statements about future positions of particles, as specified by their coordinates, are not meaningful statements in general relativity. This is a point which cannot be emphasized strongly enough and is the basis of a much deeper dilemma than the more technical question of the Lorentz invariance of the quantum field equations. It pervades all the general theory, and to some degree we mislead both our students and ourselves when we calculate, for instance, the mercury perihelion motion without explaining how our coordinate system is fixed in space, what defines it in such a way that it cannot be rotated, by a few seconds a year, to follow the perihelion's apparent motion. Surely the x axis of our coordinate system could be defined in such a way that it pass through all successive perihelions. There must be some assumption on the nature of the coordinate system which keeps it from following the perihelion. This is not difficult to exhibit in the case of the motion of the perihelion, and it would be useful to exhibit it. Neither is this, in general, an academic point, even

* Address of retiring president of the American Physical Society, January 31, 1957.
[1] This will be reported jointly with H. Salecker in more detail in another journal.
[2] See, e.g., J. M. Jauch and F. Rohrlich, *The Theory of Protons and Electrons* (Addison-Wesley Press, Cambridge, Massachusetts, 1955).

Reprinted from *Rev. Mod. Phys.* 29, 255 (1957).

though it may be academic in the case of the mercury perihelion. A difference in the tacit assumptions which fix the coordinate system is increasingly recognized to be at the bottom of many conflicting results arrived at in calculations based on the general theory of relativity. Expressing our results in terms of the values of coordinates became a habit with us to such a degree that we adhere to this habit also in general relativity where values of coordinates are not *per se* meaningful. In order to make them meaningful, the mollusk-like coordinate system must be somehow anchored to space-time events and this anchoring is often done with little explicitness. If we want to put general relativity on speaking terms with quantum mechanics, our first task has to be to bring the statements of the general theory of relativity into such form that they conform with the basic principles of the general relativity theory itself. It will be shown below how this may be attempted.

RELATIVISTIC QUANTUM THEORY OF ELEMENTARY SYSTEMS

The relation between special theory and quantum mechanics is most simple for single particles. The equations and properties of these, in the absence of interactions, can be deduced already from relativistic invariance. Two cases have to be distinguished: the particle either can, or cannot, be transformed to rest. If it can, it will behave, in that coordinate system, as any other particle, such as an atom. It will have an intrinsic angular momentum called J in the case of atoms and spin S in the case of elementary particles. This leads to the various possibilities with which we are familiar from spectroscopy, that is spins 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, $\cdots$ each corresponding to a type of particle. If the particle cannot be transformed to rest, its velocity must always be equal to the velocity of light. Every other velocity can be transformed to rest. The rest-mass of these particles is zero because a nonzero rest-mass would entail an infinite energy if moving with light velocity.

Particles with zero rest-mass have only two directions of polarization, no matter how large their spin is. This contrasts with the $2S+1$ directions of polarization for particles with nonzero rest-mass and spin S. Electromagnetic radiation, that is, light, is the most familiar example for this phenomenon. The "spin" of light is 1, but it has only two directions of polarization, instead of $2S+1=3$. The number of polarizations seems to jump discontinuously to two when the rest-mass decreases and reaches the value 0. Bass and Schrödinger[3] followed this out in detail for electromagnetic radiation, that is, for $S=1$. It is good to realize, however, that this decrease in the number of possible polarizations is purely a property of the Lorentz transformation and holds for any value of the spin.

There is nothing fundamentally new that can be said about the number of polarizations of a particle and the principal purpose of the following paragraphs is to illuminate it from a different point of view.[4] Instead of the question: "Why do particles with zero rest-mass have only two directions of polarization?" the slightly different question, "Why do particles with a finite rest-mass have more than two directions of polarization?" is proposed.

The intrinsic angular momentum of a particle with zero rest-mass is parallel to its direction of motion, that is, parallel to its velocity. Thus, if we connect any internal motion with the spin, this is perpendicular to the velocity. In case of light, we speak of transverse polarization. Furthermore, and this is the salient point, the statement that the spin is parallel to the velocity is a relativistically invariant statement: it holds as well if the particle is viewed from a moving coordinate system. If the problem of polarization is regarded from this point of view, it results in the question, "Why can't the angular momentum of a particle with finite rest-mass be parallel to its velocity?" or "Why can't a plane wave represent transverse polarization unless it propagates with light velocity?" The answer is that the angular momentum *can* very well be parallel to the direction of motion and the wave *can* have transverse polarization, but these are not Lorentz invariant statements. In other words, even if velocity and spin are parallel in one coordinate system, they do not appear to be parallel in other coordinate systems. This is most evident if, in this other coordinate system, the particle is at rest: in this coordinate system the

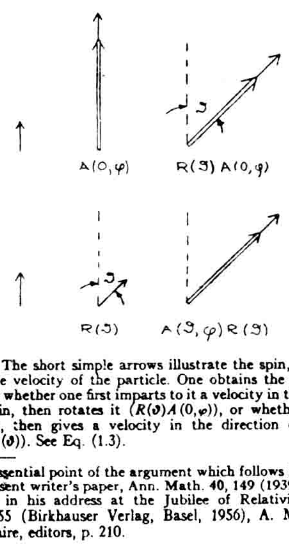

FIG. 1. The short simple arrows illustrate the spin, the double arrows the velocity of the particle. One obtains the same state, no matter whether one first imparts to it a velocity in the direction of the spin, then rotates it ($R(\vartheta)A(0,\varphi)$), or whether one first rotates it, then gives a velocity in the direction of the spin ($A(\vartheta,\varphi)R(\vartheta)$). See Eq. (1.3).

[3] L. Bass and E. Schrödinger, Proc. Roy. Soc. (London) **A232**, 1 (1955).

[4] The essential point of the argument which follows is contained in the present writer's paper, Ann. Math. **40**, 149 (1939) and more explicitly in his address at the Jubilee of Relativity Theory, Bern, 1955 (Birkhauser Verlag, Basel, 1956), A. Mercier and M. Kervaire, editors, p. 210.

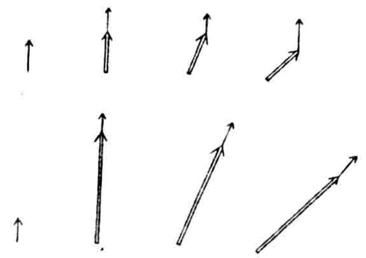

FIG. 2. The particle is first given a small velocity in the direction of its spin, then increasing velocities in a prependicular direction (upper part of the figure). The direction of the spin remains essentially unchanged; it includes an increasingly large angle with the velocity as the velocity in the perpendicular direction increases. If the velocity imparted to the particle is large (lower part of the figure), the direction of the spin seems to follow the direction of the velocity. See Eqs. (1.8) and (1.7).

angular momentum should be parallel to nothing. However, every particle, unless it moves with light velocity, can be viewed from a coordinate system in which it is at rest. In this coordinate system its angular momentum is surely not parallel to its velocity. Hence, the statement that spin and velocity are parallel cannot be universally valid for the particle with finite rest-mass and such a particle must have other states of polarization also.

It may be worthwhile to illustrate this point somewhat more in detail. Let us consider a particle at rest with a given direction of polarization, say the direction of the z axis. Let us consider this particle now from a coordinate system which is moving in the $-z$ direction. The particle will then appear to have a velocity in the z direction and its polarization will be parallel to its velocity (Fig. 1). It will now be shown that this last statement is nearly invariant if the velocity is high. It is evident that the statement is entirely invariant with respect to rotations and with respect to a further increase of the velocity in the z direction. This is illustrated at the bottom of the figure. The coordinate system is first turned to the left and then given a velocity in the direction opposite to the old z axis. The state of the system appears to be exactly the same as if the coordinate system had been first given a velocity in the $-z$ direction and then turned, which is the operation illustrated at the top of the figure. The state of the system appears to be the same not for any physical reason but because the two coordinate systems are identical and they view the same particle (see Appendix I).

Let us now take our particle with a high velocity in the z direction and view it from a coordinate system which moves in the $-y$ direction. The particle now will appear to have a momentum also in the y direction, its velocity will have a direction between the y and z axes (Fig. 2). Its spin, however, will not be in the direction of its motion any more. In the nonrelativistic case, that is, if all velocities are small as compared with the velocity of light, the spin will still be parallel to z and it will, therefore, enclose an angle with the particle's direction of motion. This shows that the statement that the spin is parallel to the direction of motion is not invariant in the nonrelativisitic region. However, if the original velocity of the particle is close to the light velocity, the Lorentz contraction works out in such a way that the angle between spin and velocity is given by

$$\tan\text{ (angle between spin and velocity)} = (1-v^2/c^2)^{\frac{1}{2}} \sin\vartheta, \quad (1)$$

where ϑ is the angle between the velocity v in the moving coordinate system and the velocity in the coordinate system at rest. This last situation is illustrated at the bottom of the figure. If the velocity of the particle is small as compared with the velocity of light, the direction of the spin remains fixed and is the same in the moving coordinate system as in the coordinate system at rest. On the other hand, if the particle's velocity is close to light velocity, the velocity carries the spin with itself and the angle between direction of motion and spin direction becomes very small in the moving coordinate system. Finally, if the particle has light velocity, the statement "spin and velocity are parallel" remains true in every coordinate system. Again, this is not a consequence of any physical property of the spin, but is a consequence of the properties of Lorentz transformations: it is a kind of Lorentz contraction. It is the reason for the different behavior of particles with finite, and particles with zero, rest-mass, as far as the number of states of polarization is concerned. (Details of the calculation are in Appendix I.)

The preceding consideration proves more than was intended: it shows that the statement "spin and velocity are parallel for zero mass particles" is invariant and that, for relativistic reasons, one needs only *one* state of polarization, rather than *two*. This is true as far as proper Lorentz transformations are concerned. The second state of polarization, in which spin and velocity are antiparallel, is a result of the reflection symmetry. Again, this can be illustrated on the example of light: right circularly polarized light appears as right circularly polarized light in all Lorentz frames of reference which can be continuously transformed into each other. Only if one looks at the right circularly polarized light in a mirror does it appear as left circularly polarized light. The postulate of reflection symmetry allows us to infer the existence of left circularly polarized light from the existence of right circularly polarized light—if there were no such reflection symmetry in the real world, the existence of *two* modes of polarization of light, with virtually identical properties, would appear to be a miracle. The situation is entirely different for particles with