

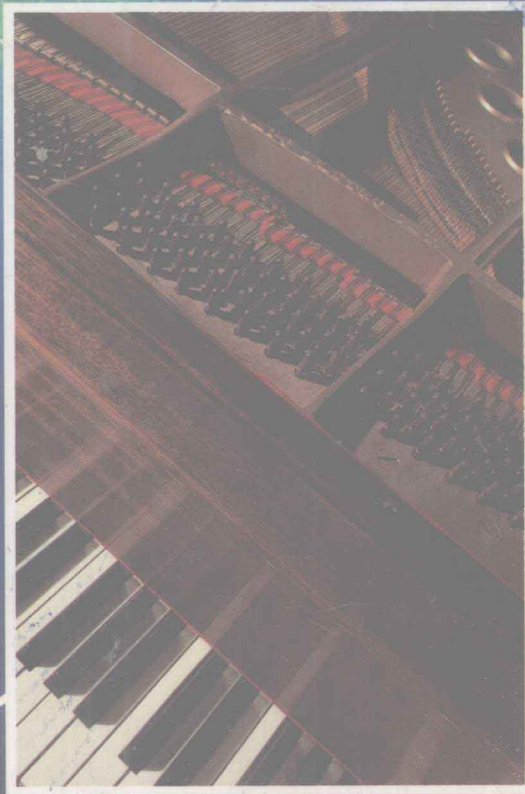
ALGEBRA 1

LARSON, KANOLD, STIFF

$$f(n) = 440 \cdot (\sqrt[12]{2})^n$$

$$\frac{f_2}{f_1} = \frac{6}{5}$$

$$\frac{f_2}{f_1} = \frac{5}{4}$$



24.93

37.64

19.34

10.75

D. C. HEATH

ALGEBRA 1

Roland E. Larson

Timothy D. Kanold

Lee Stiff



D.C. Heath and Company

Lexington, Massachusetts / Toronto, Ontario

About the Cover

The cover shows the relationship of music and mathematics, one of the many real-life applications that you will work with in this text. The background is an enlargement of a Compact Disc (CD), and the lines are actually grooves in the CD. The mathematics shown on the cover represents the algebra of musical harmony. A symphony orchestra “tunes up” before a performance by having an oboist play the note “A-440,” which has a frequency (pitch) of 440 vibrations per second. The frequency of every note on the piano keyboard is a multiple of 440, shown by the formula in the upper right of the cover. The exponent n represents the position of the note, above or below A-440. Two notes harmonize if the ratio of their frequencies, $\frac{f_2}{f_1}$, is an integer or a simple rational number.

As you read the text, see if you can find information that is directly related to the cover.

Acknowledgments

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In Algebra 1, we have included numerous examples and exercises that use real-life data. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

We tried to use the data accurately—to give honest and unbiased portrayals of real-life situations. In the cases where models were fit to data, we used the least-squares method. In all cases, the square of the correlation coefficient, r^2 , was at least 0.95. In most cases, it was 0.99 or greater.

(Acknowledgments continue on page 791)

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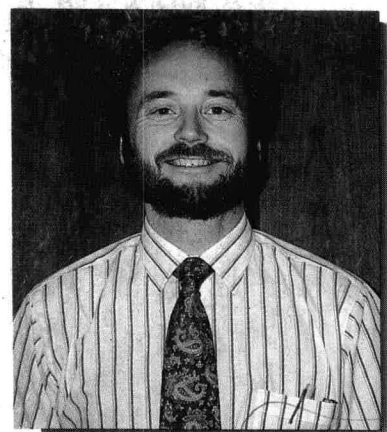
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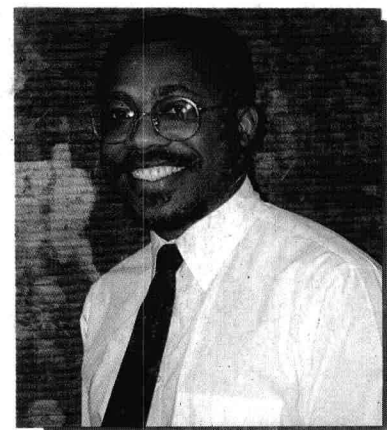
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To the Students

Mathematics evolved over the past few thousand years in many stages. *All* the early stages were centered around the use of mathematics to answer questions about real life. This is very much like the way we wrote *Algebra 1*. We centered the concepts around the real-life use of mathematics.

As mathematics development matured, people began to collect and categorize the different rules, formulas, and properties that had been discovered. This took place independently in many different parts of the world: Africa, Asia, Europe, North America, and South America. The mathematics that we use today is a combination of the work of literally thousands of people.

As you study our algebra book, be sure you understand the value and purpose of the concepts you're learning. Knowing **why** you are learning a concept helps you master it—and understand its relevance to your own life. That's why we begin each lesson explaining what you should learn and why you should learn it.

Remember, math is not a spectator sport—it's a valuable tool you can use in everyday life!



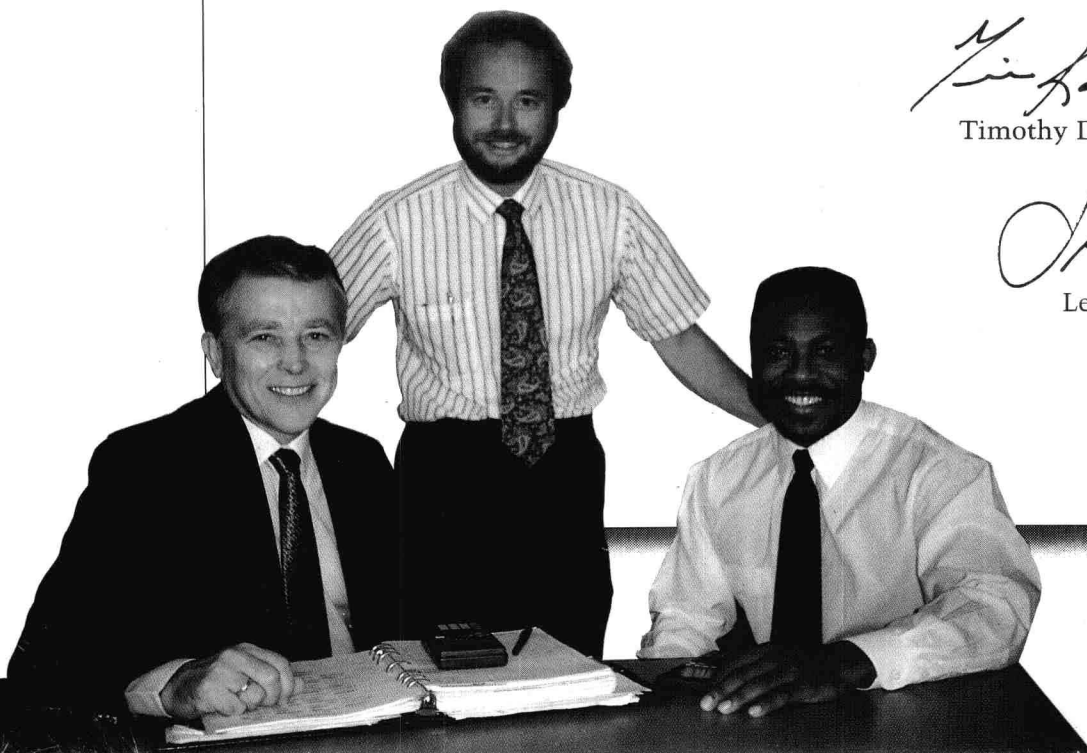
Roland E. Larson



Timothy D. Kanold



Lee Stiff



Real-Life Applications

Look through this list for things that interest you. Then find out how they are used with algebra.

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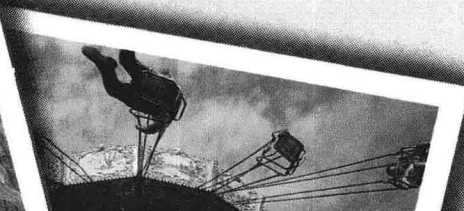
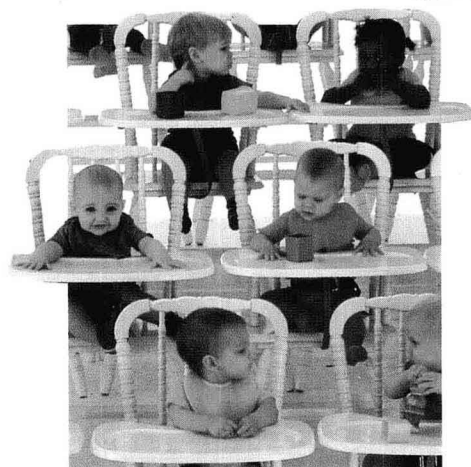


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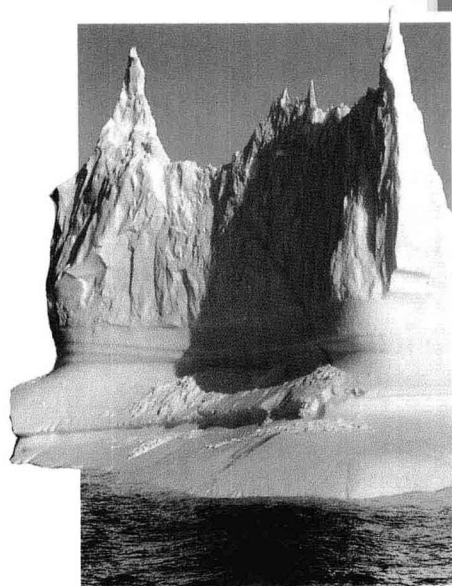
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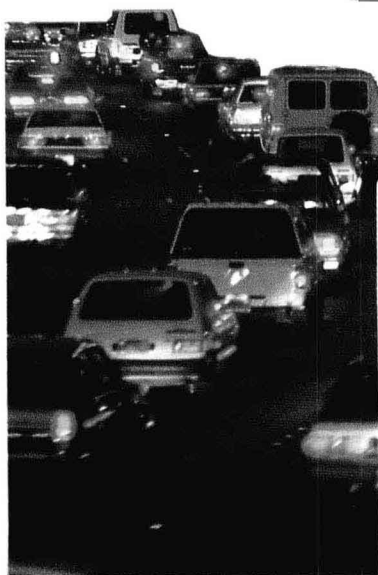
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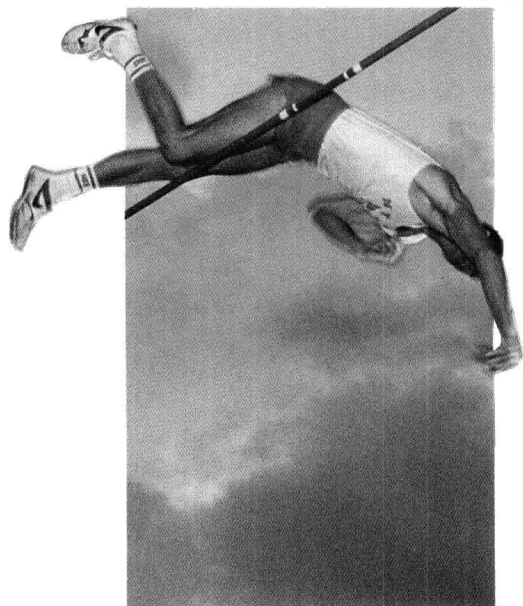
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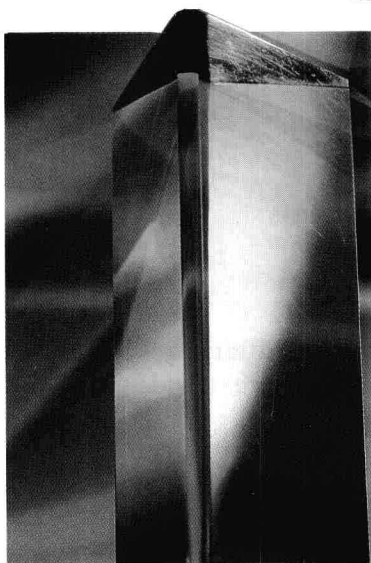
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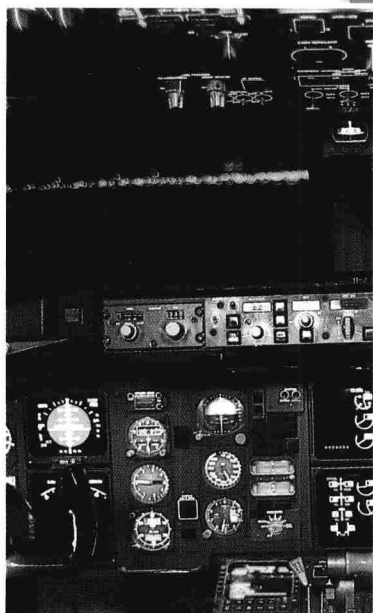
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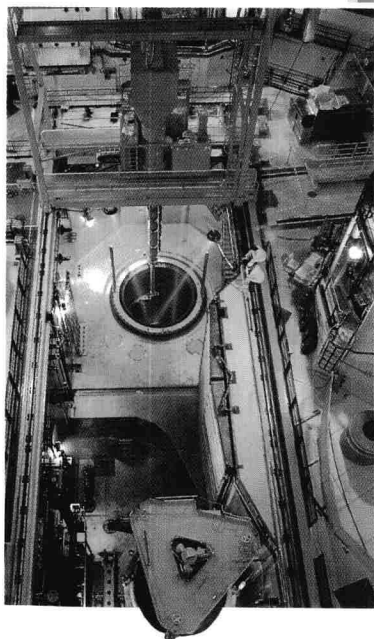
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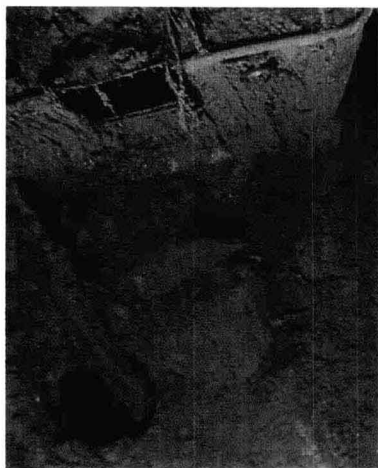
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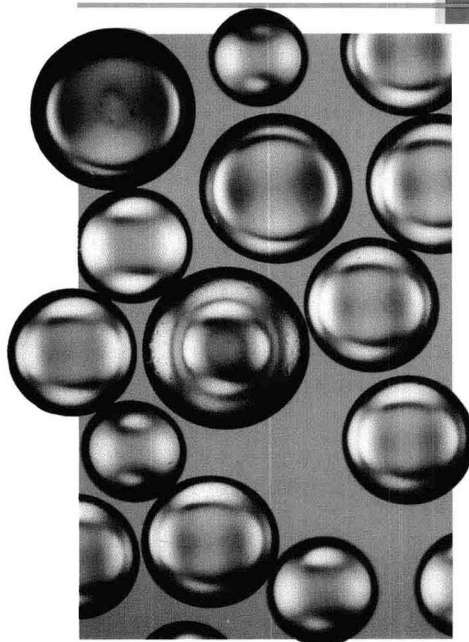
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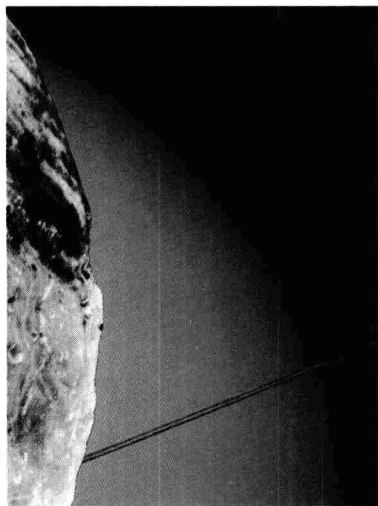
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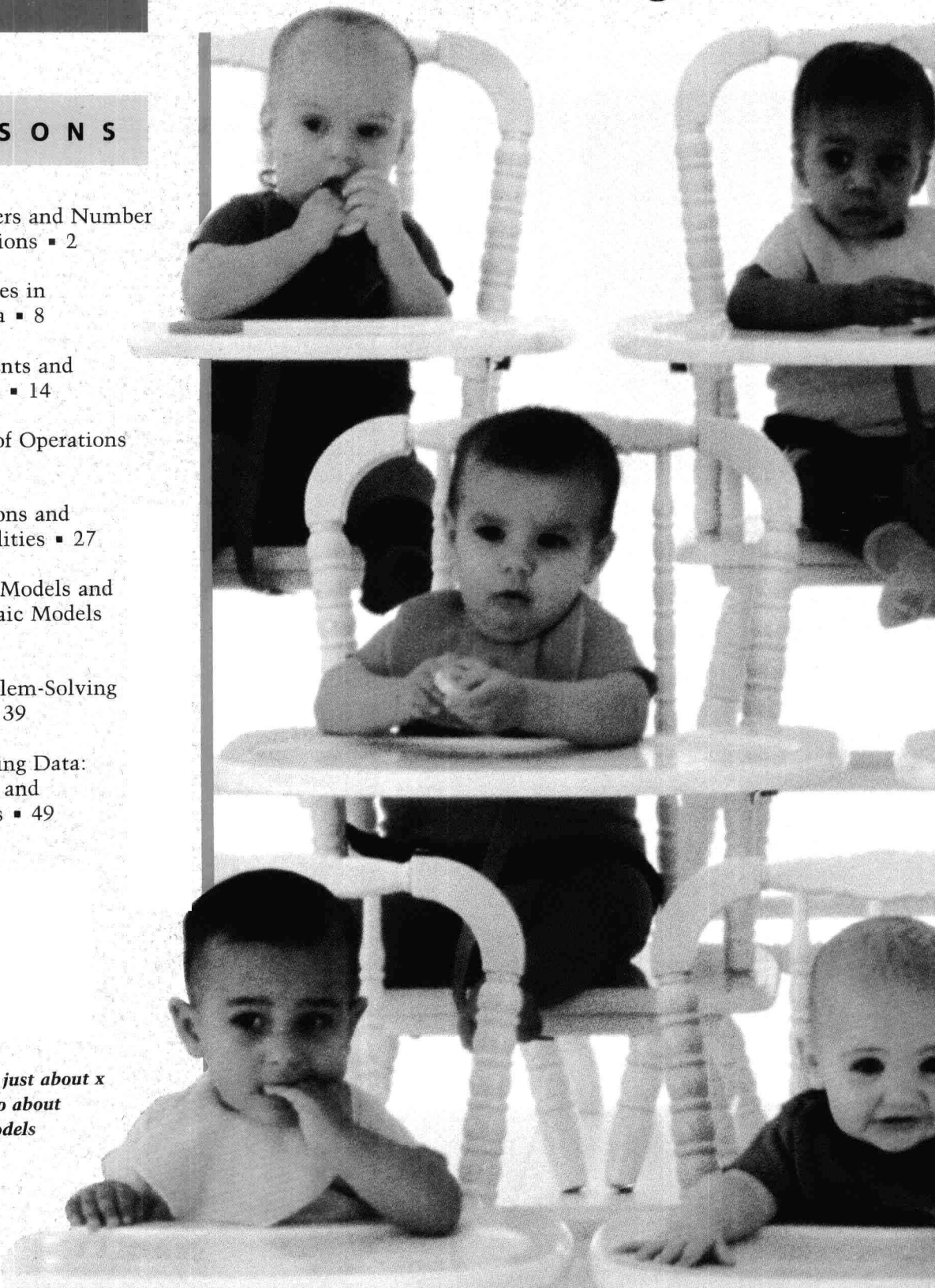
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Connections to Algebra

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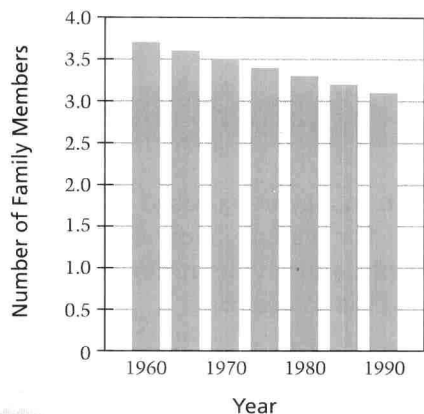
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Algebra is not just about x and y . It is also about toddlers. It models their world.



Real Life Demographics

Average Family Size in U.S.



One of the goals of algebra is to recognize patterns and relationships that occur in real life. *Tables* can help you recognize patterns. For instance, the following table shows that between 1960 and 1990, the average family size in the United States decreased by about one tenth of a family member every five years.

(Source: U.S. Bureau of the Census)

Year	1960	1965	1970	1975	1980	1985	1990
Family Size	3.7	3.6	3.5	3.4	3.3	3.2	3.1

Once a pattern has been found, it can be used to answer questions about the real-life situation. For instance, from the pattern above you could estimate the 1995 average family size to be about 3.0. The *bar graph* gives a picture of the decreasing family size between 1960 and 1990.



1.1

Numbers and Number Operations

What you should learn:

Goal 1 How to represent numbers and number operations

Goal 2 How to use grouping symbols

Why you should learn it:

You can apply what you know about numbers and number operations to help solve problems in algebra.

Goal 1 Numbers and Number Operations

Much of what you will learn in algebra is based on numerical skills that you already have. For instance, you already know how to add, subtract, multiply, and divide numbers, and you will continue to use these skills in algebra.

Different kinds of numbers have different names. For example, the numbers 0, 1, 2, 3, and 4 are **whole numbers**, and the numbers $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{4}$ are **fractions**. What you call a number sometimes depends on the way you write the number. For example, the number $\frac{1}{2}$ can be written in three different forms.

Fraction form: $\frac{1}{2}$

Decimal form: 0.5

Percent form: 50%

Each of these forms represents the same number, which means that you could write $\frac{1}{2} = 0.5$ or $0.5 = 50\%$.

The decimal form of some numbers contains too many digits to write. For instance, no one could write *all* the digits of the decimal for $\frac{3}{7}$:

$$\frac{3}{7} = 0.4285714285 \dots$$

(The three dots mean that the number has more digits than are shown.) You can *approximate* this number to different degrees of accuracy. For instance, rounded to two decimal places, you could write

$$\frac{3}{7} \approx 0.43. \quad \text{Rounded to two decimal places}$$

If you wanted an approximation that is more accurate, you could round to three decimal places and write

$$\frac{3}{7} \approx 0.429. \quad \text{Rounded to three decimal places}$$

The symbol \approx means *is approximately equal to*.

The number of decimal places that you should list when writing a decimal approximation depends on the context of the problem. For instance, with dollar amounts, you probably would not use more than two decimal places.