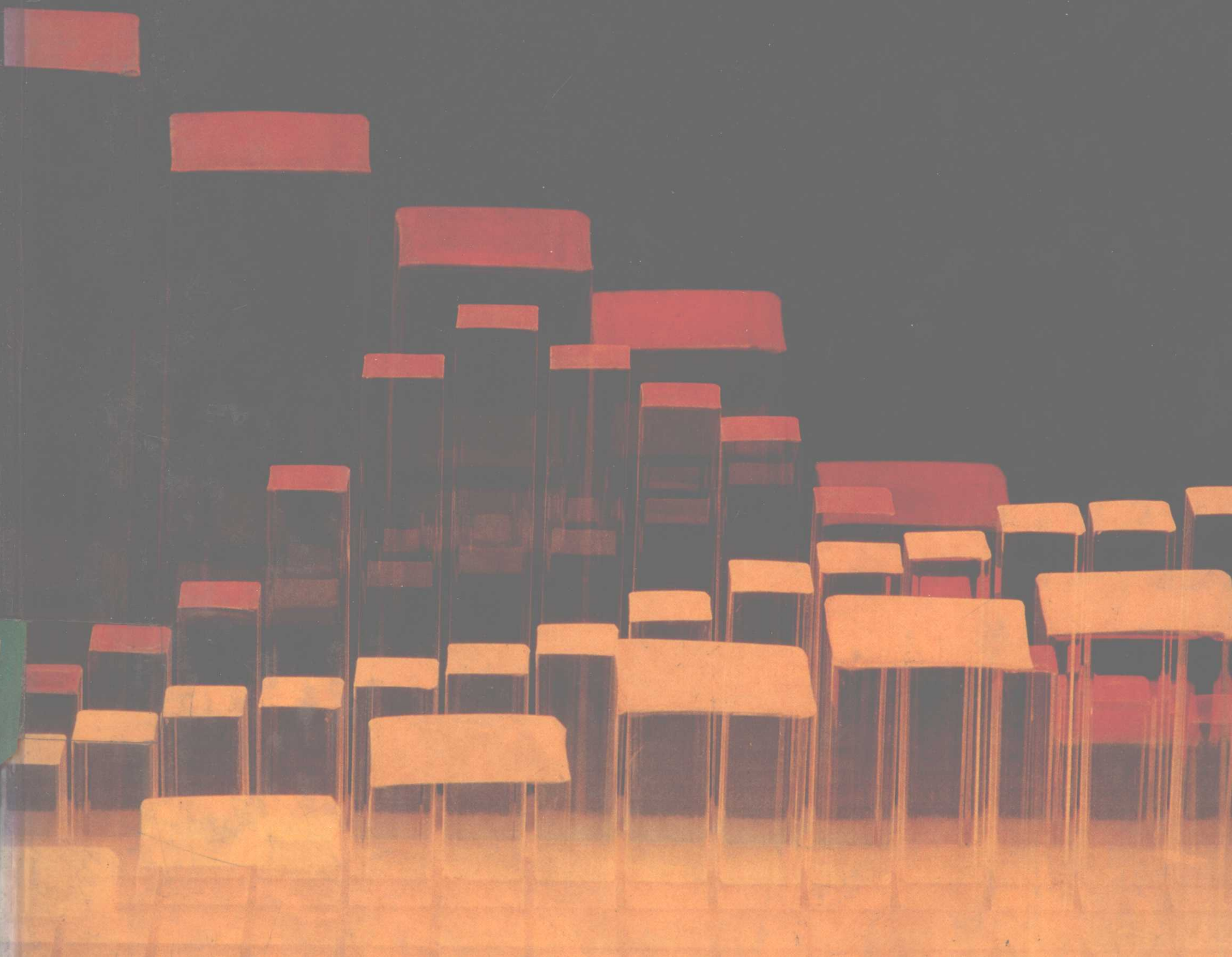
# Calculus
## WITH APPLICATIONS

KARL J. SMITH



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# Preface

*Calculus with Applications* can be used in a one-semester or a two-quarter calculus course for students in business, management, or the life or social sciences. Emphasis throughout is to enhance students' understanding of the modeling process and how mathematics is used in real world applications. The prerequisite for this course is intermediate algebra.

It seems as if every new book claims innovation, state-of-the-art production, supplementary materials, readability, abundant problems, and relevant applications. How, then, is one book chosen over another? More specifically, how does this book differ from other books for this course, and what factors were taken into consideration as it was written?

## Content

Every book, of course, must cover the appropriate topics, hopefully in the right order. I believe that it is important to introduce calculus as soon as possible, so Chapter 1 begins with a section called "What Is Calculus?" The rest of the chapter discusses the nature of mathematical modeling while reviewing the important types of functions that students need to know for calculus. This means that the derivative can be introduced in Chapter 2 so students can begin using calculus almost immediately in a variety of real world applications. Many books begin with a chapter reviewing algebra, but I have decided to put this chapter at the end of the text in an appendix. A class needing to review linear and quadratic equations and inequalities, as well as rational expressions, can begin with the appendix or can review these topics as needed.

Model building is one of the most difficult, yet most important skills we need to teach our students. It is a skill that cannot be learned in a single lesson, or even in a single course. Learning this skill must be a gradual process, and that is how I approach it in this book—in small steps with realistic applications. However, in order to give students experience with true model building for real life situations, I have included modeling applications at the end of each chapter. These applications are open-ended assignments that require a mathematical model-building approach for their development. An essay written in response to each of these applications is given in its entirety in the *Student Solutions Manual* to illustrate how model building can be developed. These model-building applications, even if not assigned, demonstrate, in a very real way, how the material developed in the rest of the chapter can be used to answer some nontrivial questions (see, for example, the Modeling Application on Cobb-Douglas Production Functions in Chapter 9). They also can help to answer the legitimate question, "Why are we doing this?"

There are two main concepts in calculus: the derivative and the integral. The derivative is introduced and defined in Chapter 2. Chapter 2 also presents a variety of differentiation techniques. Then Chapters 3 and 4 contain some meaningful applications. Notice that exponential and logarithmic functions (Chapter 4) precede integration. This is because I have found that students need a good review of logarithmic and exponential functions before using them in calculus. With this review, functions can be used in a natural and consistent manner to introduce the second principal concept of calculus, that of an integral (Chapter 6).

In 1986 the Sloan Foundation sponsored a four-day conference on the calculus curriculum. The conference's work was summarized in the following statement: "We propose a calculus syllabus which emphasizes intuition and conceptual understanding by giving priority to numerical and geometrical methods and approximation techniques." I follow the conference's recommendations in the development of integration in this book. Emphasis is placed on the use of tables and numerical integration rather than on integration techniques. In fact, as you can see in Chapter 7, there are only two main methods of integration discussed: integration by parts and by tables. This reduced emphasis on integration techniques allows extra time for meaningful applications. For example, probability models and probability density functions are discussed in Section 8.2. There is an entire section on business models (Section 8.1).

The text concludes with two optional chapters, multivariable calculus in Chapter 9 and differential equations in Chapter 10. These chapters are independent and can be covered in any order.

## Style

An author's writing style also distinguishes one textbook from another. My writing style is informal, and I write with the student always in mind. I offer study hints along the way and let the students know what is important. Frequent and abundant examples are provided so that students can understand each step before proceeding to the next. The chapter reviews list important terms and provide a sample test to help the student master the material for that chapter. In addition, a list of specific chapter objectives is given in the *Student Solutions Manual* as well as 50 to 100 additional practice problems for each chapter. The sample tests and supplementary problems (if necessary) give students a clear grasp of all important ideas in the course. There are also cumulative review problems at the end of Chapters 4 (for Chapters 1–4) and 8 (for Chapters 5–8).

## Problems

The third—and perhaps the most important—factor in deciding on a textbook is the number, quality, and type of problems presented. This is where I have spent a great deal of effort in developing this book. Problems should help to develop students' understanding of the material, not inhibit or thwart it by being obscure. Problems are presented in the problem sets in matched pairs with an answer for one provided in the back. There are about three times more problems than are needed for assignments, so students have the opportunity to practice additional problems, both for the midterm and for the final. Each problem set provides drill problems to develop necessary manipulative skills and a large number of applications to show how the material can be used in business, management, and the life and social sciences.

## Contemporary

This book is as up to date as possible. It was written with the realization that virtually every mathematics student has a calculator. This allows many more real life applications and situations to be included. For example, the optional Modeling Applications provide an in-depth model-building practice in a particular setting. Computers have affected the types of skills college graduates need, so I have not forgotten that many students now have access to a computer. For those, I have included a Modeling Application at the end of Chapter 6 that can serve as an introduction to muMath. (MuMath is a software package that enables the user to do algebraic manipulations on a computer in much the same way that a calculator can do arithmetic manipulations.)

## Supplementary Materials

The final factor often used in selecting a textbook is the type and quality of available supplements. In addition to the odd-numbered answers in the back of the book, there is a *Student Solutions Manual* that provides complete solutions to every odd-numbered problem in the book. The manual also includes essays that illustrate the modeling process and lists of objectives that define the necessary skills in each chapter. Finally, there is an extensive list of supplementary problems keyed to the listed objectives that provide additional opportunities for self-study. The *Instructor's Supplement* includes answers to all of the problems in the book and additional questions for the modeling applications. There is also a computerized test bank available with text-editing capabilities that allows you to create an almost unlimited number of tests or retests of the material. This test bank is also available in printed form for instructors without access to a computer. As the author, I am also available to help you create any other set of supplementary materials that you feel are necessary or worthwhile for your course.

## Acknowledgments

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I have put considerable effort into making sure that the problems and answers are correct; in addition to myself, others have worked all the problems and examples to ensure their accuracy, and I would like to thank Terry Shell, Donna Szott, Pat Bannantine, and Michael Anderson for their careful working of all the problems and examples. A textbook is produced through the collaborative efforts of many people, and I appreciate the work of many people: at Brooks/Cole, Jeremy Hayhurst, Maxine Westby, Joan Marsh, and Katherine Minerva; and Cece Munson, Jamie Sue Brooks, and Carl Brown. And, finally, I thank my wife, Linda, and my children, Missy and Shannon, for their help, love, and support.

Karl J. Smith
Sebastopol, California

# Contents



* Optional sections

# CHAPTER 1
# Functions

## CHAPTER CONTENTS



## APPLICATIONS

**Management** (*Business, Economics, Finance, and Investments*)

Purchasing power of the dollar (1.1, Problems 45–54)
Sales graph (1.2, Problem 44)
Supply and demand equations (1.3, Problems 58–59)
1985 Tax Rate Schedule (1.3, Problems 65–69)
Demand, revenue, and break-even points (1.4, Problems 49–50)
Maximum profit (1.4, Problems 51–53)
Minimum cost (1.4, Problem 54)
Cost and revenue functions (1.4, Problems 54–55)
Equilibrium point for supply and demand (1.5, Problem 40)

**Life sciences** (*Biology, Ecology, Health, and Medicine*)

Alcohol content in bloodstream (1.1, Problems 55–56)
Atmospheric pressure (1.2, Problem 43)
Response of a nerve (1.2, Problem 45)
Cost of removing a pollutant (1.2, Problem 46)
Cost-benefit model for removing pollutants (1.5, Problems 28–35; 1.6, Problems 19–20)
Radiology (1.5, Problems 36–37)
Pressure-volume relationship (1.5, Problems 38–39)

**Social sciences** (*Demography, Political Science, Population, Psychology, Society, and Sociology*)

U.S. population (1.1, Problems 57–58)
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Stopping distance for a car (1.4, Problem 58)

**Modeling application**—An Analysis of Two Car-Buying Strategies

**CHAPTER OVERVIEW** In Chapter 1 you are introduced to the building blocks for this course. We begin by asking the question, "What is calculus?" and then introduce the two main threads used for developing the material in this book: the concepts of mathematical models and of graphs.

**PREVIEW** We begin by reviewing the concept of a function and evaluating functions and functional notation. Then we discuss graphing functions, and in the remainder of the chapter focus on three very important types of functions: linear, quadratic, and rational.

**PERSPECTIVE** There are three main ideas in calculus; the notion of a limit, a derivative, and an integral. Each of these ideas requires a thorough understanding not only of the function concept but also of functional notation. The last part of the book considers functions of several variables. In order to focus your attention on the concepts at hand, you will need to have mastered the material on functions in this first chapter. For an algebra diagnostic test and a review of algebra, see Appendix A.

## 1.1 What Is Calculus?

You are about to begin a study of calculus specifically oriented for students in management, life sciences, or social sciences.

Before beginning, let us take a few moments to consider the nature of calculus. We begin by comparing the types of problems encountered in elementary mathematics with those encountered in calculus (see Table 1.1).

The ideas of calculus were first considered toward the end of the sixteenth century, but the theory wasn't developed until the second half of the seventeenth century when Gottfried Leibniz (1646–1716) and Isaac Newton (1642–1727) simultaneously, but independently, invented calculus as we know it today.

**TABLE 1.1**
**Problems and applications in elementary mathematics and calculus**

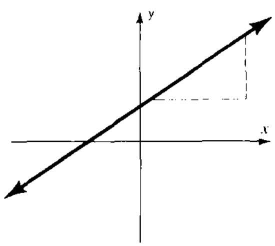
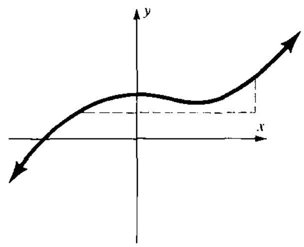
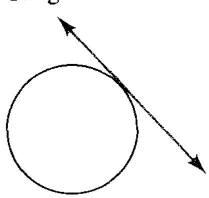
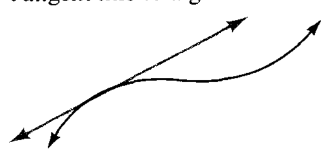
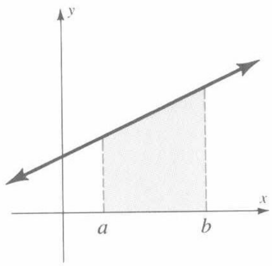
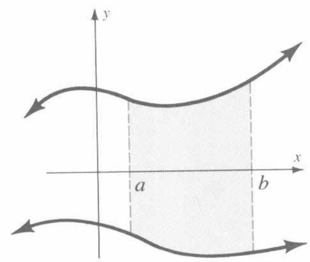
| Elementary mathematics | Calculus |
|---|---|
| Slope of a line  | Slope of a curve  |
| Tangent line to a circle  | Tangent line to a general curve  |

**TABLE 1.1** (*continued*)

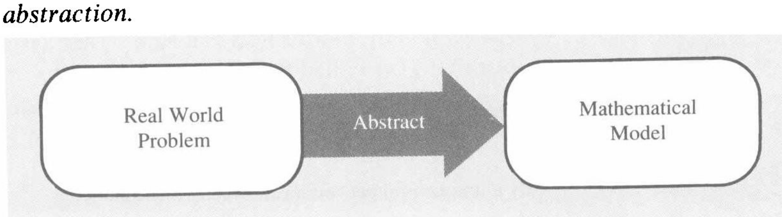
| Elementary mathematics | Calculus |
|---|---|
| Area of a region bounded by line segments  | Area of a region bounded by curves  |
| Average change | Instantaneous change |
| Average velocity | Instantaneous velocity |
| Average acceleration | Instantaneous acceleration |
| Average of a finite collection of numbers | Average value of a function on an interval |

Up to now, your study of mathematics has focused on the mechanics of mathematics (such as solving equations, drawing graphs, and manipulating symbols). In this course you will not only reinforce such skills but also apply them to particular real life situations.

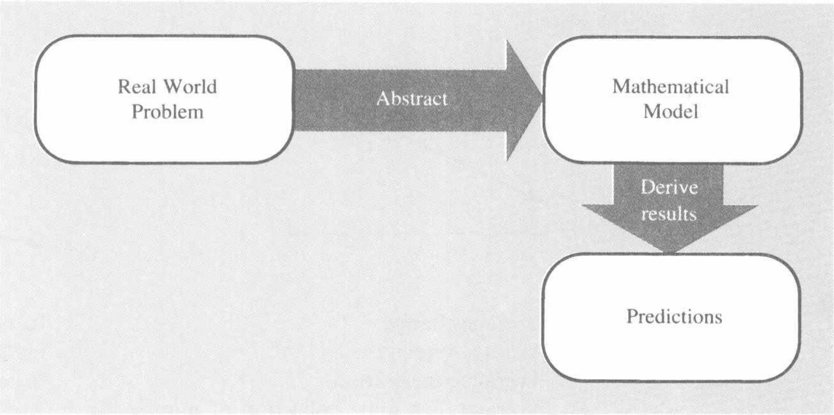
## Mathematical Models

A real life situation is usually far too complicated to be precisely and mathematically defined. It therefore was necessary to develop a body of mathematics based on certain assumptions about the real world. These **mathematical models** are then modified by experimentation and the accumulation of data and used to predict some future occurrence in the real world. Such mathematical models are neither static nor unchanging but are continually being revised and modified as additional relevant information becomes known.

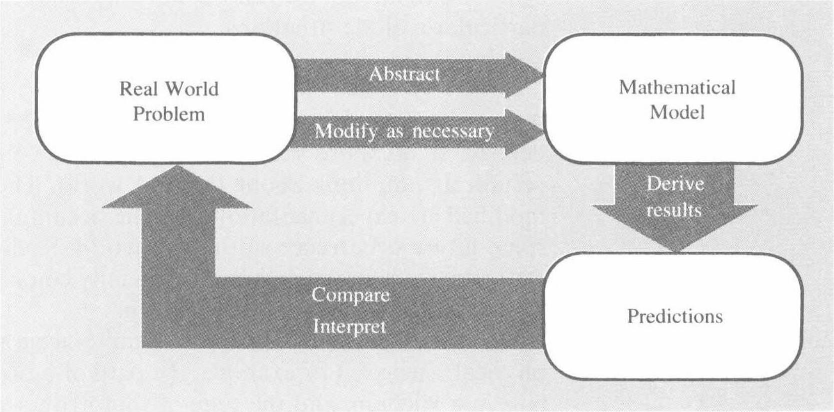
Some mathematical models are quite accurate, particularly those used in the physical sciences. For example, the path of a projectile, the distance that an object falls in a vacuum, and the time of tomorrow's sunrise have mathematical models that provide very accurate predictions. On the other hand, in the fields of management, life sciences, and social sciences, the models provide much less accurate predictions because they deal with situations that involve many unknowns or undefined variables that are often random in character. How can we go about constructing a model in management, life science, or social science? We observe a real world problem and make assumptions about influencing factors. This is called *abstraction.*



We must know enough about the mechanics of mathematics to *derive results* from the model.




The next step is to gather data. Does the prediction fit all the known data? If not, we use this data to *modify* the assumptions used in the model. This is an ongoing process.




## Mathematical Functions

We begin by reviewing an idea from algebra that is fundamental for the study of calculus, namely, the idea of a **function**.

***Function***

> A *function* of a variable $x$ is a rule $f$ that assigns to each value of $x$ a unique* number $f(x)$, called the *value of the function at x*. A function can be defined by a verbal rule, a table, a graph, or an algebraic formula.

* By a unique number, we mean exactly one number.

Remember:

1. $f(x)$ is pronounced "$f$ of $x$."
2. $f$ is a *function* while $f(x)$ is a *number.*
3. The set of replacements for $x$ is called the **domain** of the function.
4. The set of all values $f(x)$ is called the **range** of $f$.

**EXAMPLE 1** *A function defined as a verbal rule.*
Let $x$ be a year from 1980 to 1985, inclusive. We define the function $p$ by the rule that $p(x)$ is the closing price of Xerox stock on January 4 of year $x$. The domain is the set {1980, 1981, 1982, 1983, 1984, 1985}, and the range is the set of possible prices for Xerox stock.* For example, $p(1984) = 49\frac{1}{4}$. ■

**EXAMPLE 2** *A function defined by a table.*
Let $g$ be a function defined so that $g(x)$ is the price of gasoline in the year $x$ as given by the following table:

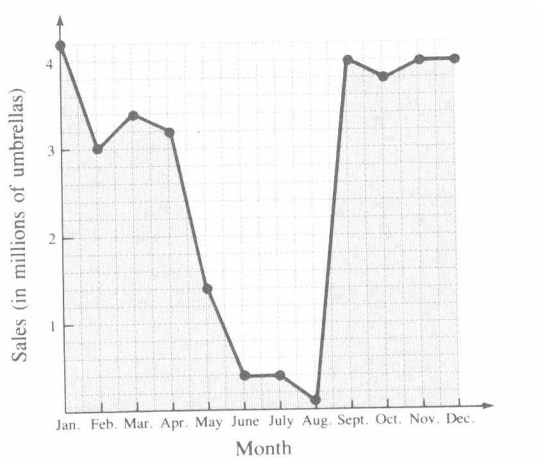
| Year | Average price per gallon of gasoline on January 4 |
|---|---|
| 1944 | \$ .21 |
| 1954 | \$ .29 |
| 1964 | \$ .30 |
| 1974 | \$ .53 |
| 1984 | \$1.24 |

The domain is {1944, 1954, 1964, 1974, 1984}

The range is {\$.21, \$.29, \$.30, \$.53, \$1.24}

For example, $g(1984) = \$1.24$. ■

**EXAMPLE 3** *A function defined by a graph.*




* If you are familiar with the stock market, you know that Xerox stock prices are quoted in eighths of a dollar. The actual prices of Xerox stock over the years 1980–1985 provide the elements of the range.

Let $s(x)$ be the sales of umbrellas in millions of units for $x$, a month in 1988. The domain is {Jan., Feb., . . . , Nov., Dec.}, and the range is $0.1 \le y \le 4.2$. (Remember, the units on the $y$-axis are in millions, so this means $100{,}000 \le y \le 4{,}200{,}000$.) For example, $s(\text{Jan.}) \approx 4.2$, $s(\text{Aug.}) \approx 0.1$. ■

**EXAMPLE 4** *A function defined by an algebraic formula.*
Let $f$ be a function defined by

$$f(x) = 2x - 5$$

This rule says take a number $x$ from the domain, multiply it by 2, and then subtract 5. For example,

$$\begin{aligned} f(3) &= 2(3) - 5 \\ &= 1 \end{aligned}$$ ■

In this book, unless otherwise specified, the *domain is the set of real numbers for which the given function is meaningful.* This means $x$ can be replaced by *any real number*. For example,

$$f(x) = 2x - 5$$

has a domain consisting of all real numbers, while

$$f(x) = \frac{1}{x-1} \quad \text{and} \quad g(x) = \frac{3}{5x-5}$$

have domains that exclude $x = 1$ because division by zero is not defined ($x - 1 = 0$ and $5x - 5 = 0$ if $x = 1$). We say $f(1)$ does not exist or we say $f(x)$ is not defined at $x = 1$. Finally, if

$$f(x) = \sqrt{x}$$

then $x \ge 0$ is implied since the square root of a number $x$ is real if and only if $x \ge 0$, so $f(x)$ is defined only if $x \ge 0$.

**EXAMPLE 5** Give the domain of the following functions.

**a.** $f(x) = \dfrac{x-8}{3}$ **b.** $f(x) = \dfrac{3}{x-8}$ **c.** $f(x) = \sqrt{x+2}$ **d.** $f(x) = \dfrac{1}{\sqrt{x+2}}$

***Solution*** **a.** The domain is all real numbers.

**b.** The domain is all real numbers except where $x - 8 = 0$ or $x = 8$. We state this domain by simply writing $x \ne 8$.

**c.** The domain is all real numbers for which

$$\begin{aligned} x + 2 &\ge 0 \\ x &\ge -2 \end{aligned}$$

This can be stated in **set-builder notation** $\{x \mid x \ge -2\}$, which is read

$$\underbrace{\text{“The set of}}_{\text{“}\{}\ \underbrace{\text{all } x,}_{x}\ \underbrace{\text{such that}}_{|}\ \underbrace{x \ge -2\text{”}}_{x \ge -2\}\text{”}}$$

However, set-builder notation is too formal for our work in this course, so we will just write $x \ge -2$ for the domain.

**d.** This is similar to part e, but division by zero must also be excluded, so we see that the domain is $x > -2$. ■

Example 4 illustrates a very important process called **evaluating a function**. To find $f(3)$ you substitute 3 for every occurrence of $x$ in the formula for $f(x)$. This process is illustrated in Examples 6 and 7.

EXAMPLE 6 Given $f$ and $g$ defined by $f(x) = 2x - 5$ and $g(x) = x^2 + 4x + 3$, find the indicated values:

**a.** $f(1)$ **b.** $g(2)$ **c.** $g(-3)$ **d.** $f(-3)$

***Solution*** **a.** The symbol $f(1)$ is found by replacing $x$ by 1 in the expression

$$\begin{aligned} f(x) &= 2x - 5 \\ &\updownarrow \quad \updownarrow \\ f(1) &= 2(1) - 5 \\ &= -3 \end{aligned}$$

*Warning: This Is a Very Important Example*

**b.** $g(2)$:

$$\begin{aligned} g(x) &= x^2 + 4x + 3 \\ &\updownarrow \quad \updownarrow \quad \updownarrow \\ g(2) &= (2)^2 + 4(2) + 3 \\ &= 4 + 8 + 3 \\ &= 15 \end{aligned}$$

**c.** $g(-3)$:

$$\begin{aligned} g(-3) &= (-3)^2 + 4(-3) + 3 \\ &= 9 - 12 + 3 \\ &= 0 \end{aligned}$$

**d.**

$$\begin{aligned} f(-3) &= 2(-3) - 5 \\ &= -11 \end{aligned}$$

■

A function may be evaluated by using a variable.

EXAMPLE 7 Let $F$ and $G$ be defined by $F(x) = x^2 + 1$ and $G(x) = (x + 1)^2$. Then:

**a.** $F(w) = w^2 + 1$

**b.** $G(t) = (t + 1)^2 = t^2 + 2t + 1$

**c.** $F(-a) = (-a)^2 + 1 = a^2 + 1$

**d.** $G(-a) = (-a + 1)^2 = a^2 - 2a + 1$

**e.** $F(w + h) = (w + h)^2 + 1 = w^2 + 2wh + h^2 + 1$

**f.** $F(w^3) = (w^3)^2 + 1 = w^6 + 1$

**g.** $[G(a)]^2 = [(a + 1)^2]^2$ since $G(a) = (a + 1)^2$
$= (a + 1)^4$

**h.**

$$\begin{aligned} F[G(x)] &= F[(x + 1)^2] \\ &= [(x + 1)^2]^2 + 1 \\ &= (x + 1)^4 + 1 \end{aligned}$$

**i.** $F(\sqrt{t}) = (\sqrt{t})^2 + 1 = t + 1$

**j.**

$$\begin{aligned} G(x + h) &= [(x + h) + 1]^2 \\ &= (x + h)^2 + 2(x + h) + 1 \\ &= x^2 + 2xh + h^2 + 2x + 2h + 1 \end{aligned}$$

■