MECHANICS OF VIBRATION

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PREFACE

This book has arisen out of a course in vibration analysis which has been offered for several years by the Department of Engineering Mechanics of the University of Michigan. This course is the first of a series which treat the theory of mechanical vibrations and its application to engineering problems. The aim of this first course has been to present the fundamentals and basic theory in a manner readily understood by the undergraduate, and yet, at the same time, on a plane acceptable to the graduate student.

Many excellent treatises are available which treat the theory of vibrations and its applications to engineering problems, and they serve as invaluable references for a student of this subject. However, with the increased demand by industry for engineers trained in the technique of vibration analysis and the resulting increase in the number of universities offering training in this subject, there appears to be a need for a treatment of vibrations designed primarily as a textbook; particularly a textbook that covers the all-important basic principles in a thorough fashion and yet is suitable for a student who has had nothing more than an elementary course in dynamics and the standard instruction in mathematics offered to undergraduate engineering students today. It is our sincere hope that this book will help to fill the need for a suitable textbook in this expanding field of applied mechanics.

A conscious effort has been made to present the theory in such a manner that it can be extended with ease to all the various and diverse vibration problems which the practicing engineer has come to know. An equally sincere effort has been made to avoid the extension in detail to specific problems, which is counter to the purpose of this book. Specific applications therefore have been considered only as a vehicle in demonstrating the theory and general technique. Considerable effort has been made to insure that the content of the book will be as broad as space permits. The necessity of keeping a textbook within reasonable size has forced us to select the material carefully. The inevitable decision as to the methods and topics to be included and those to be omitted has been made as judiciously as our experience in teaching will permit.

We firmly believed that it is essential for the student to obtain some confidence in his ability to set up a problem from its basic elements as well as to know how to solve the equations that arise in vibration analysis. The confidence on the part of the student that "he can get started," more than anything else, removes the "mystery" from vibration problems. Since the student is frequently able to picture the results better through a sketch or graph than by the manipulation of an equation, rather extensive use has been made of figures. The student is urged to portray his results graphically whenever it is appropriate. Many problems have been included on the theory that the student learns best through application. The problems have been graduated in difficulty to increase their value as a vehicle with which to master the theory.

The book is divided into three parts. The first deals with steady-state vibrations of systems of one degree of freedom. As these systems are of fundamental significance in most forms of vibration, a considerable amount of space has been devoted to their treatment. The second part extends the theory to systems of several degrees of freedom. The theory has been discussed from the classical standpoint, although emphasis has been placed on the extremely useful "mobility" concept. The third part consists of an introduction to special topics which are an essential part in a more general and more refined analysis of vibration problems. Although a thorough discussion of these topics is beyond the scope of this book, we believe that an introduction to these subjects is desirable to form a link between the idealized and more precise theories. These topics include nonlinear systems, systems with distributed physical characteristics, and systems subjected to transient motions.

A textbook of an elementary nature cannot be expected to include any new theories or specialized applications; however, the following items have been treated in a manner which is either new or considered to be an improvement over the usual presentation.

1. The concept of relaxation frequency and its physical meaning has been introduced and the concept used throughout.

2. The use of equivalent springs, dampers, and masses as well as dimensionless parameters has been stressed in the analysis of complex problems.

3. Energy methods in general and Rayleigh's method in particular have been discussed and used extensively in certain applications.

4. The mobility method has received extensive treatment. The use of velocity as a parameter, preferable, when dealing with problems of sound transmission, fluid flow, and electric currents, has been replaced

by a displacement parameter, which has a more direct significance in mechanical vibrations.

5. The solution of the frequency equation has been given considerable attention.

In the course of the development of this book we have inevitably been influenced by the classic works of Timoshenko, den Hartog, and others. These works and the teachings of these pioneers in vibration analysis have furnished the prime motivation for the present volume. A particular tribute is due to Mr. F. A. Firestone, whose paper, "Mobility Method," in the *Journal of Applied Physics*, June, 1938, represents the initial incentive for the development and adaptation of this method in this book. We are especially thankful to Mr. T. A. Hunter, who read the manuscript and made many valuable suggestions. Much credit is also due to Professor E. L. Eriksen, who lent constant encouragement. We are further indebted to Mr. R. E. Peterson of the Westinghouse Electric Corporation for the frontispiece.

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Paul F. Chenea

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TABLE 1

Symbol	*Quantity*	*Dimensions* FLT	MLT
A	Area	L^2	L^2
A	Amplitude	L	L
a	Acceleration	LT^{-2}	LT^{-2}
a	Amplitude per unit force	$F^{-1}L$	$M^{-1}T^2$
a	Wave velocity	LT^{-1}	LT^{-1}
c	Damping constant in translation	$FL^{-1}T$	MT^{-1}
c	Damping constant in rotation	FLT	ML^2T^{-1}
c	Speed of sound	LT^{-1}	LT^{-1}
c	Distance to centroid	L	L
c_{cr}	Critical damping constant	*See* Damping constant	
D	Diameter	L	L
d	Diameter	L	L
E	Young's modulus	FL^{-2}	$ML^{-1}T^{-2}$
E	Energy	FL	ML^2T^{-2}
e	Base of natural logarithms = 2.71828	Dimensionless	
e	Eccentricity	L	L
F	Dissipation function	FLT^{-1}	ML^2T^{-3}
F	Force	F	MLT^{-2}
F	Elliptic integral of the first kind	—	—
f	Frequency	T^{-1}	T^{-1}
G	Shear modulus	FL^{-2}	$ML^{-1}T^{-2}$
g	Acceleration gravity = 386 inches per second²	LT^{-2}	LT^{-2}
$\mathcal{H}$	Dynamic product of inertia (cross product)	FL	ML^2T^{-2}
h	Height or depth	L	L
I	Moment of inertia (mass)	FLT^2	ML^2
I	Moment of inertia (area)	L^4	L^4
$\mathcal{I}$	Dynamic moment of inertia	FL	ML^2T^{-2}
$\mathcal{I}$	Impulse	FT	MLT^{-1}
i	Index number	Dimensionless	
J	Moment of inertia (mass)	FLT^2	ML^2
J	Polar moment of inertia (area)	L^4	L^4
j	Imaginary unit $j^2 = -1$	Dimensionless	
K	Complete elliptic integral of first kind	—	—
k	Spring constant in translation	FL^{-1}	MT^{-2}
k	Spring constant in rotation	FL	ML^2T^{-2}
L	Length	L	L
l	Length	L	L
M	Mass	$FL^{-1}T^2$	M
M	Moment of a force	FL	ML^2T^{-2}
$\mathfrak{M}$	Momentum	FT	MLT^{-1}
m	Mass	$FL^{-1}T^2$	M
n	Index number	Dimensionless	
n	Gear ratio	Dimensionless	
P	Force	F	MLT^{-2}
p	Natural circular frequency	T^{-1}	T^{-1}
p	Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Q	Force	F	MLT^{-2}
Q	Discharge	L^3T^{-1}	L^3T^{-1}
q	Relaxation frequency	T^{-1}	T^{-1}
q_i	Generalized coordinates	—	—
R	Reaction	F	MLT^{-2}
R	Radius	L	L
r	Radius	L	L
$\bar{r}$	Radius of gyration	L	L
S	Tension	F	MLT^{-2}
T	Kinetic energy	FL	ML^2T^{-2}
T	Torque	FL	ML^2T^{-2}
T	Time	T	T
T	Tension	F	MLT^{-2}
t	Time	T	T
U	Energy	FL	ML^2T^{-2}

TABLE 1 (*Continued*)

Symbol	*Quantity*	*Dimensions*	
u	Displacement	L	L
V	Potential energy	FL	ML^2T^{-2}
V	Velocity	LT^{-1}	LT^{-1}
V	Volume	L^3	L^3
V	Shear force	F	MLT^{-2}
v	Velocity	LT^{-1}	LT^{-1}
W	Total complex displacement	—	—
W	Work	FL	ML^2T^{-2}
$W\sim$	Work per cycle	FL	ML^2T^{-2}
W	Weight	F	MLT^{-2}
$\dot{W}$	Power	FLT^{-1}	ML^2T^{-3}
w	Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
w	Complex displacement per unit force	—	—
X	Force in x direction	F	MLT^{-2}
x	Coordinate	L	L
x	Displacement in x direction	L	L
Y	Force in y direction	F	MLT^{-2}
y	Coordinate	L	L
y	Displacement in y direction	L	L
Z	Total impedance	—	—
Z	Force in z direction	F	MLT^{-2}
z	Coordinate	L	L
z	Displacement in z direction	L	L
z	Impedance	—	—
α	Angular acceleration	T^{-2}	T^{-2}
α	Angle	Dimensionless	
α	Frequency ratio	Dimensionless	
β	Angle	Dimensionless	
γ	Specific gravity	Dimensionless	
γ	Angle	Dimensionless	
Δ	Frequency function	—	—
Δ	Determinant	—	—
δ	Displacement	L	L
δ_{st}	Static deformation	L	L
ϵ	Strain	Dimensionless	
Θ	Angular amplitude	Dimensionless	
θ	Angle	Dimensionless	
θ	Angular amplitude per unit torque	$F^{-1}L^{-1}$	$M^{-1}L^{-2}T^2$
μ	Coefficient of friction	Dimensionless	
μ	Absolute viscosity	$FL^{-2}T$	$ML^{-1}T^{-1}$
ν	Poisson's ratio	Dimensionless	
π	3.14159	Dimensionless	
ρ	Density	$FL^{-4}T^2$	ML^{-3}
σ	Stress	FL^{-2}	$ML^{-1}T^{-2}$
τ	Period	T	T
τ	Time	T	T
Φ	Angular amplitude	Dimensionless	
φ	Friction angle	Dimensionless	
φ	Angle	Dimensionless	
Ψ	Angular amplitude	Dimensionless	
ψ	Angle	Dimensionless	
Ω	Angular velocity	T^{-1}	T^{-1}
ω	Angular velocity	T^{-1}	T^{-1}
ω	Circular frequency	T^{-1}	T^{-1}

CONTENTS

PROBLEMS

Chapter 1

GENERAL CONCEPTS

1.1 Introduction

Vibratory motions occur to some degree in practically every structural and mechanical device known to man, such as vehicles, machinery, bridges, and buildings. In the great majority of these instances, the vibration is of too small a magnitude to cause any concern. There are, however, numerous examples where vibrations are sufficiently dangerous to cause failure of structures which otherwise would have operated satisfactorily. Examples of such failures are found in broken crankshafts, failure of turbine blades, fracture of springs (Fig. 1–1), destruction of buildings due to earthquakes, and destruction of bridges due to vibrations induced by the wind.

Vibration is sometimes objectionable because of its effect on human comfort or its interference with the operation of delicate instruments. In these cases, structural failure may never occur, but the human discomfort or the inability of instruments to function properly make it mandatory that some form of vibration control be utilized.

In still other instances, vibrations may be essential to the operation of machines, shaking devices such as grain separators, or musical instruments. Such machines and instruments must be designed so that the finished product has the proper periodic motion. Not infrequently, the design problem is one of eliminating a particular vibratory motion while another is amplified. Many instruments designed to measure frequency are based upon properly tuned vibratory motions, and in other machines, such as fatigue-testing apparatus, vibration is employed to produce stress reversals in the test specimen, with a minimum of power input.

In all of these mechanical systems, regardless of whether the vibration is to be eliminated because of its objectionable stresses or human discomfort, or whether it is to be merely altered as a desirable feature, the first step is an analysis of the vibratory motion and an understanding of its characteristics. Only after an investigation of the pertinent properties of the vibration can an intelligent procedure be formulated to accomplish the desired change. It is the analysis of these fundamental properties of vibratory motion with which this book is concerned.

1.2 Examples of Vibratory Systems

As in other engineering topics, the introduction to a new subject is most easily made through the study of idealized elementary examples. This is a justifiable procedure because many of the more complex problems of vibration analysis can be replaced by a combination of elementary systems without appreciable sacrifice in accuracy. A vibratory system is usually understood to be a combination of elements which, either by interaction, or through the action of external forces, are able to sustain a periodic oscillating motion. In mechanical vibration these elements can be divided into three characteristic types which may be referred to as masses or inertia elements, springs or elastic elements, and resistors or damping elements. Of these, the first two, masses and springs or their equivalents, are able, by interaction, to produce and sustain oscillation while the third type, the damping elements, act as a deterent on the motion. In reality, all these characteristics are present in all mechanical parts, as any such part possesses mass which is to some degree elastic and which will in addition absorb some energy by being deformed. However, it is usually possible in practice to deal with a mechanical vibratory system as if it were made up of a number of idealized elements, each of which represents only one of the characteristic types which effect the energy distribution of the system in a specific manner.

The three characteristic elements as used in vibration analysis may be defined as follows:

Mass element

The mass element is assumed to be an inelastic solid or an incompressible and non-viscous fluid. It is therefore able to act only as an inertia and as such can gain or lose kinetic energy according to the manner in which its velocity is changed.

Spring element

The spring or elastic element is assumed to be without inertia and to resist deformation or displacement in such a manner that the work done in producing the deformation or displacement is conserved by the element in the form of potential energy until the work stored is recovered by a return to the initial shape or position.

The mass and spring elements together constitute in this idealized form a conservative system in which any energy stored in the mass due to its state of motion (kinetic energy) and any energy contained in the spring element or its equivalent due to its deformation or displacement (potential energy) can be completely recovered. The constant interchange of kinetic energy from the mass element to potential energy in the spring element is fundamental in most vibratory systems.

The spring may take the form of any elastic body. Common examples are a bent beam, a coiled spring, a twisted shaft, and an air or rubber cushion. The pull of gravity or the buoyancy exerted by a fluid on a floating body are likewise equivalent to a spring.

Damper

The third element common to all vibratory systems is a damper. A damper is any device that dissipates energy from the vibratory system. The damper gives rise to a force called the damping force which at all times resists the motion. The damping force dissipates mechanical energy which is usually converted to heat, thus depleting the mechanical energy of the system. Dampers that employ dry friction are called "friction dampers" and dampers that employ fluid friction are generally denoted as "viscous dampers."

The object of much modern engineering is to decrease frictional resistance in machines and instruments, as this resistance wastes energy and hinders performance. This same frictional resistance is the most common form of damping, and therefore many machines have extremely small damping forces associated with their operation. For this reason, the damping is frequently of small effect, and the vibration analysis is simplified by neglecting damping forces.

With these three basic elements in mind, all vibratory systems may be constructed. Consider the simple case of a mass suspended by a common coil spring, as shown in Fig. 1–2. The spring and mass in this system are easily recognized. The damping forces arise from the resistance of the air to the motion of the mass and spring as well as the internal resistance in the spring called hysteresis damping. The damping forces are usually small in this system.

As a second example, consider the simple or mathematical pendulum, as shown in Fig. 1–3. The ball constitutes the mass element whereas the equivalent of the spring in this example is the pull of gravity. The action of the force of gravity is always such as to restore the mass to the equilibrium position in which the pendulum hangs vertical. The damping forces are produced by the resistance of the air to the motion of the mass and string.

Another example is a vessel bobbing up and down in the water (Fig. 1–4). The mass is that of the vessel and the spring equivalent is

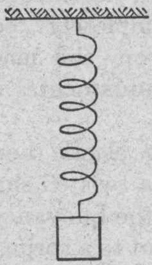

Fig. 1–2

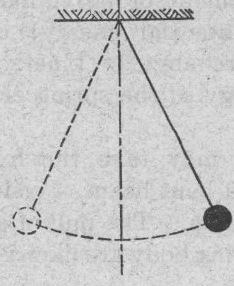

Fig. 1–3

Fig. 1–4

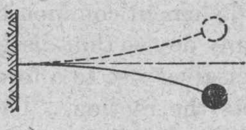

Fig. 1–5

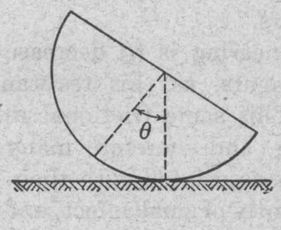

Fig. 1–6

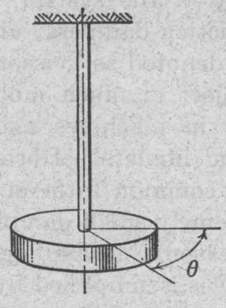

Fig. 1–7

the buoyancy of the water and the force of gravity acting together. The damping is partly air resistance and partly fluid friction.

Other simple systems are shown in Figs. 1–5, 1–6, and 1–7. The systems shown in Figs. 1–8 and 1–9 involve a combination of springs and dampers in the form of dashpots. By suitably combining masses or inertias with springs and dampers, complex systems may be constructed which, to a high degree of approximation, represent the actual mechanical system to be analyzed. It is through the use of these idealized systems that many engineering problems are solved. In those instances where idealized systems do not yield results to a sufficient degree of accuracy, more advanced methods are required. Some of these advanced methods are discussed in Part 3 of this book.

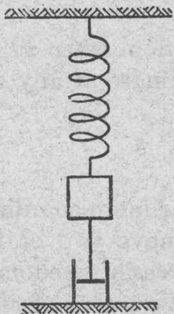

FIG. 1-8

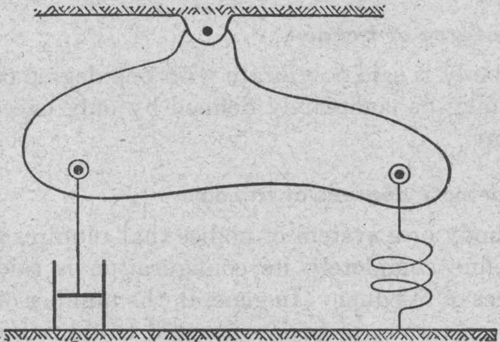

FIG. 1-9

1.3. Fundamental Definitions

As with any well-cultivated subject, there is a large number of special terms and phrases used in discussing vibration analysis which have been given precise meanings. It is essential that the meanings of the more important words and phrases be known well as they will be used throughout the text. Some of the more common terms together with their definitions follow.

Vibration

A vibration or a vibratory motion may be defined as a motion that is periodic. The motion consists of an oscillation about an equilibrium position in such a manner that it repeats itself in definite intervals of time. The graph of such a motion is shown in Fig. 1-10.

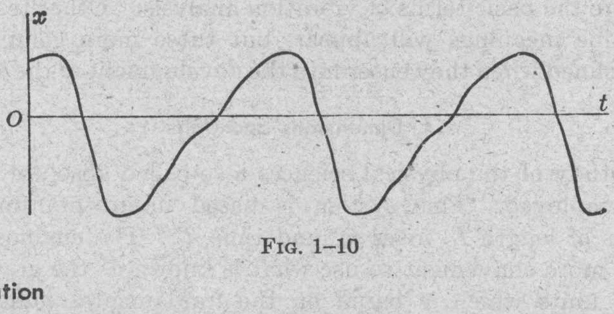

FIG. 1-10

Free vibration

A vibration that is independent of outside forces is said to be free. Free vibrations are frequently called natural vibrations.

Forced vibration

Vibrations that are caused and maintained by a periodic disturbing force are called forced vibrations.

One degree of freedom

A body is said to vibrate with one degree of freedom when its position may be completely defined by only one coordinate at any given instant.

Two or more degrees of freedom

A body or a system of bodies that requires two or more coordinates to define completely its configuration is said to have two or more degrees of freedom. In general, the number of degrees of freedom of a body or system of bodies is equal to the minimum number of coordinates required to define completely the configuration of the body, or the system of bodies, at any given time.

Amplitude

The amplitude of a vibration is the maximum linear or rotational displacement from the equilibrium position that occurs during a complete cycle of the motion.

Period

The period of a vibration is the time required to execute one complete cycle or oscillation.

Frequency

The reciprocal of the period is called the frequency. The frequency represents the number of cycles completed in a unit of time.

These are the basic terms of vibration analysis. Other terms which have specific meanings will appear, but these more complex terms are best-defined when they enter into the development of the theory.

1.4. Dimensions and Units

In the study of the physical sciences a so-called absolute system of units is employed. This system is based upon the fundamental dimensions of length L, mass m, and time T. The engineer finds it somewhat more convenient to use what is known as the gravitational system of units which is based on the fundamental dimensions of length L, force F, and time T. In each system all of the other quantities that arise in the study of mechanics may be expressed in terms of the three basic dimensions. The two systems of units are related through Newton's second law of motion,

$$F = ma$$

where a is the acceleration. This may be written dimensionally as

$$F = m\frac{L}{T^2} = mLT^{-2} \tag{1.4-1}$$

This basic law of mechanics gives another fundamental dimensional equality when solved for m,

$$m = \frac{FT^2}{L} = FL^{-1}T^2 \tag{1.4-2}$$

When the "engineer's" or gravitational system of units is employed, the pound is taken as the unit of force, the foot as the unit of length, and the second as the unit of time. The unit of mass becomes pounds second^2 per foot. It is natural to express the unit of mass in terms of the weight W of the body through the relation

$$W = mg$$

where g is the acceleration of gravity, whence

$$m = \frac{W}{g}$$

Since the acceleration of gravity has a value of 32.2 ft per sec^2, the unit mass is a body weighing 32.2 lb. This unit of mass has been denoted as the "slug." More often the gravitational acceleration is stated in terms of an inch unit of length giving $g = 386$ in. per sec^2. The corresponding unit of mass has a weight of 386 lb and may be called the "inch slug."

Certain significant quantities are involved in vibration analysis. These quantities are in part characteristics of the vibratory motion and in part characteristics of the elements of the particular vibrating system. The main characteristics of the vibratory motion are defined in the previous section. The two principal characteristics of the elements of a vibrating system other than the mass are the spring constant and the damping constant.

The spring constant is a measure of the stiffness of the spring. If the spring is designed to be deformed by changing its length, the spring constant k is defined as the force required for a unit change in length. Thus

$$k = \frac{F}{\delta} \approx \frac{F}{L} = FL^{-1} \tag{1.4-3}$$