GRAPH THEORY WITH APPLICATIONS TO ALGORITHMS AND COMPUTER SCIENCE

Y. Alavi
•
G. Chartrand
•
L. Lesniak
•
D. R. Lick
•
C. E. Wall

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Edited by

Y. ALAVI

G. CHARTRAND

L. LESNIAK

D R ICK

C. E. WALL

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The Fifth Conference and these Proceedings are dedicated to

Paul Erdös

and

Ronald L. Graham

and the editors herewith recognize and laud their outstanding contributions to mathematics and their promotion of Graph Theory around the world.

PREFACE

This Volume constitutes the Proceedings of the Fifth Quadrennial International Conference on the Theory and Applications of Graphs with special emphasis on Algorithms and Computer Science Applications, held at Western Michigan University in Kalamazoo, Michigan, June 4-8, 1984. Conference participants included research mathematicians and computer scientists from colleges, universities, and the industry, as well as graduate and undergraduate students. Altogether 29 states and 15 countries were represented. The contributions to this Volume include many topics in current research in both the theory and applications in the areas of graph theory and computer science.

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Y.A.
G.C.
L.L.
D.L.
C.W.

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TILING FINITE FIGURES CONSISTING OF REGULAR POLYGONS

Jin AKIYAMA
Tokai University

Mikio KANO
Akashi Technological College

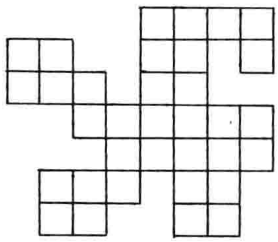
Mari-Jo RUIZ
Ateneo de Manila University

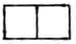
ABSTRACT

We give some sufficient conditions for a finite plane figure consisting of squares or hexagons to be covered with tiles of specific shapes.

1. Introduction

Tiling problems examine the possibility of covering plane figures with tiles of specific shapes, where covering a figure with tiles means to lay tiles over the figure so that it is completely covered, but such that no tiles are stacked on each other and no tiles exceed the edges of the figure. For example, the problem of deciding whether the defective chessboard of Figure 1.1 (a) can be covered with dominoes (Figure 1.1 (b)) is a tiling problem. In this paper, we shall present some results on tiling plane figures consisting of a finite number of squares or regular hexagons.





(a) A defective chessboard (b) A domino

Figure 1.1

Consider a plane figure consisting of some polygons. Two polygons in the figure are said to be adjacent if they have a common edge. For any figure, we can associate a graph as follows: Represent each polygon in the figure by a vertex and join two vertices by an edge if the polygons represented by these vertices are adjacent. There is a close relationship between the solution of a tiling problem and the existence of a certain component factor (i.e. a spanning subgraph with a given component) in the graph associated with the figure. For example, the graph associated with the defective chessboard of Figure 1.1 (a) is shown in Figure 1.2 (a). Moreover, it is clear that the existence of tiling a defective chessboard with dominoes is equivalent to the existence of a 1-factor in the graph associated with the defective chessboard. Note that the graph given in Figure 1.2 (b) cannot be the graph associated with any defective chessboard since two vertices x and y are not joined by an edge.

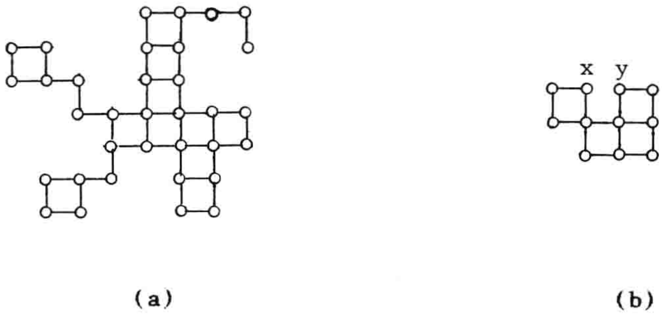


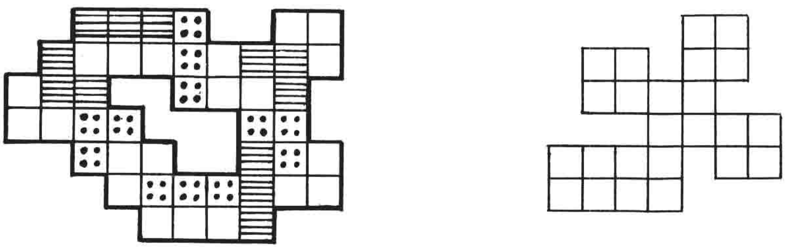
Figure 1.2

We use the following notation. By P_n, C_n and $K_{1,n-1}$, we denote the path, the cycle and the star with n vertices, respectively. For a graph G and a set $\{A,B,\cdots,K\}$ of graphs, a spanning subgraph F of G is called an $\{A,B,\cdots.K\}$-factor of G if each component of F is isomorphic to one of $\{A,B,\cdots,K\}$. In particular, each component of an H-factor (i.e. an $\{H\}$-factor) is H.

We conclude this section by mentioning that some papers on tiling problem are concerned with the infinite plane, the infinite half plane and others, rather than finite planes. Golomb [7] studied the problem of tiling the infinite plane, the half plane, a quadrant of the plane, infinite strips, and other infinite figures with polyominoes of one shape. He also gave a classification of the capability of each polyomino, consisting of up to six unit squares, tiling each of the figures mentioned above. The papers [5], [6], [8] discuss various ways of tiling the plane.

2. Tiling defective chessboards

A figure obtained from an $m \times n$ chessboard (m rows and n columns) by removing a certain number of unit squares is called a defective chessboard, where m and n are arbitrary integers. The order of a defective chessboard is the number of unit squares in it. A defective chessboard B is said to be <u>tough</u> if every pair of adjacent dominoes in B is contained in a 2×2 square of B. Observe that the defective chessboard in Figure 2.1 (a) is tough, while the one in Figure 2.1 (b) is not. The graph associated with a defective chessboard is called a <u>square graph</u>, and a square graph is said to be <u>tough</u> if its associated defective chessboard is tough, that is, a square graph is tough if every edge is contained in some C_4.



(a) A tough defective chessboard that covered with triminoes.

(b) A defective chessboard is not tough and cannot be covered with triominoes.

<u>Figure 2.1</u>

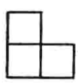
We say that a defective chessboard is <u>connected</u> if the corresponding square graph is connected. Generally, a plane figure is said to be <u>connected</u> if the corresponding graph is connected.

Tutte's 1-factor theorem [11] provides a solution to tiling a defective chessboard with dominoes. Moreover, Edmonds [4] gives a polyominal time algorithm for determining the existence of tiling a defective chessboard with dominoes. We now turn our attention from dominoes to other tiles. There are two kinds of triominoes, namely L-shaped and I-shaped triominoes (Figure 2.2 (a) and (b)).

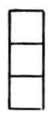
Theorem 2.1 [2] Every connected tough defective chessboard of order 3p can be covered with triomiones (Figure 2.1 (a)).

We omit the proof of this theorem, which is similar to that of Theorem 2.2, presented next. Note that both I-shaped and L-shaped triomiones are represented by P_3 in the corresponding graph, and so the above theorem also gives a local sufficient condition for a square graph to have a P_3-factor.

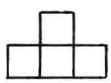
There exists a connected tough square graph of order 4p which has neither P_2-factors nor $\{P_4, K_{1,3}\}$-factors for any $p \geq 7$ (Figure 2.3). However, we have the next theorem, which says that every connected tough chessboard of even order can be tiled with dominoes and tetrominoes given in Figure 2.2 (c).



(a) The L-shaped triomino

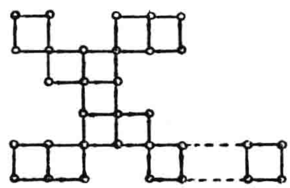


(b) The I-shaped triomino



(c) The tetriomino corresponding to $K_{1,3}$

Figure 2.2



A tough square graph having neither P_2-factors nor $\{P_4, K_{1,3}\}$-factors

Figure 2.3

Theorem 2.2 Every connected tough square graph of even order has a $\{P_2, K_{1,3}\}$-factor (Figure 2.4).