

**GRAPH THEORY
WITH APPLICATIONS
TO ALGORITHMS AND
COMPUTER SCIENCE**

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GRAPH THEORY WITH APPLICATIONS TO ALGORITHMS AND COMPUTER SCIENCE

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JOHN WILEY & SONS

New York • Chichester • Brisbane • Toronto • Singapore

The Theory and Applications of Graphs
with Special Emphasis on Algorithms
and Computer Science Applications

Fifth International Conference
June 4–8, 1984
Western Michigan University
Kalamazoo, Michigan

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Library of Congress Cataloging in Publication Data

Main entry under title:

Graph theory with applications to algorithms
and computer science.

"Proceedings of the Fifth Quadrennial International
Conference on the Theory and Applications of Graphs
with special emphasis on Algorithms and Computer
Science Applications, held at Western Michigan
University in Kalamazoo, June 4–8, 1984"—Pref.
"A Wiley-Interscience publication."

1. Graph theory—Congresses. 2. Algorithms—
Congresses. 3. Graph theory—Data processing—
Congresses. I. Alavi, Y. II. International
Conference on the Theory and Applications of
Graphs (5th : 1984 : Western Michigan University)

QA166.G733 1985 511'.5'024 85-9565
ISBN 0-471-81635-3

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

The Fifth Conference and these Proceedings are dedicated to

Paul Erdős

and

Ronald L. Graham

and the editors herewith recognize and laud their outstanding contributions to mathematics and their promotion of Graph Theory around the world.

PREFACE

This Volume constitutes the Proceedings of the Fifth Quadrennial International Conference on the Theory and Applications of Graphs with special emphasis on Algorithms and Computer Science Applications, held at Western Michigan University in Kalamazoo, Michigan, June 4-8, 1984. Conference participants included research mathematicians and computer scientists from colleges, universities, and the industry, as well as graduate and undergraduate students. Altogether 29 states and 15 countries were represented. The contributions to this Volume include many topics in current research in both the theory and applications in the areas of graph theory and computer science.

ACKNOWLEDGMENTS

The Editors take special pleasure in thanking the many people who contributed to the success of the Fifth Conference as well as the preparation of these Proceedings. In addition to the Conference speakers and participants, and the contributors and the referees for this Volume, we gratefully acknowledge

- The outstanding support of Western Michigan University,
Dr. John T. Bernhard, President and
Dr. Philip S. Denenfeld, Vice President and
Dr. L. Michael Moskovis, Associate Vice
President, Academic Affairs
- The excellent and overall support of the College of
Arts and Sciences, Dr. A. Bruce Clarke, Dean and
Dr. Clare Goldfarb, Associate Dean
- The generous support of the Graduate College,
Dr. Laurel Grotzinger, Dean and Chief Research
Officer
- The enthusiastic and overall support of the Department
of Mathematics, Dr. James H. Powell, Chairman
and Dr. Joseph T. Buckley, Associate Chairman
- The continuing encouragement and fine assistance of
our esteemed Graph Theory colleagues Professors
Donald L. Goldsmith, S.F. Kapoor, and
Arthur T. White
- The extraordinary efforts with these Proceedings of many
of our graph theory colleagues from around the
world, particularly F.R.K. Chung, R.J. Gould,
S.T. Hedetniemi, S.F. Kapoor, A.J. Schwenk and
A.T. White.
- The special administrative and overall assistance of
Darlene Lard and general secretarial assistance
of Margo Johnson
- The outstanding work of the latter two and Karen
Schaaf and Myrl Helwig for their skillful typing
and assistance with the manuscript
- The extensive work of Vicky Koski, senior Mathematics
and Computer Science student, as Conference
Secretary covering a span of two years
- The dedicated and superb work of our Conference
Assistants: Afsaneh Behdad, Virginia Mangoyan,
Nacer Hedroug, Susan Hollar, Ortrud Oellermann,
Nancy Otten, Joan Rahn, Reza Rashidi, Farrokh
Saba, Farhad Shahrokhi, Siu-Lung Tang, Zahra
Tavakoli, Barbara Treadwell, Mary Wovcha,
Hung-Bin Zou, Ek Leng Chua, Andrea Kempfer,
and Kim Higgins.

Finally, we wish to thank Ken MacLeod, editor, and in particular Rose Ann Campise, Senior Production Supervisor, John Wiley & Sons, for their interest and outstanding assistance with these Proceedings.

The Editors apologize for any oversights in the acknowledgments or any errors in the manuscript,

Y.A.
G.C.
L.L.
D.L.
C.W.

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TILING FINITE FIGURES CONSISTING OF
REGULAR POLYGONS

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Tokai University

Mikio KANO
Akashi Technological College

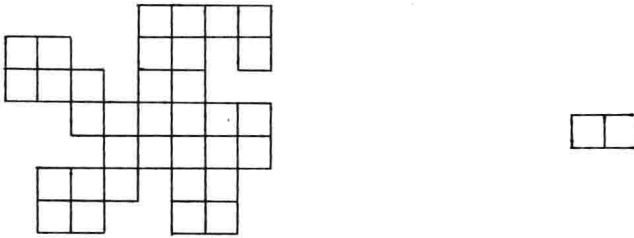
Mari-Jo RUIZ
Ateneo de Manila University

ABSTRACT

We give some sufficient conditions for a finite plane figure consisting of squares or hexagons to be covered with tiles of specific shapes.

1. Introduction

Tiling problems examine the possibility of covering plane figures with tiles of specific shapes, where covering a figure with tiles means to lay tiles over the figure so that it is completely covered, but such that no tiles are stacked on each other and no tiles exceed the edges of the figure. For example, the problem of deciding whether the defective chessboard of Figure 1.1 (a) can be covered with dominoes (Figure 1.1 (b)) is a tiling problem. In this paper, we shall present some results on tiling plane figures consisting of a finite number of squares or regular hexagons.



(a) A defective chessboard

(b) A domino

Figure 1.1

Consider a plane figure consisting of some polygons. Two polygons in the figure are said to be adjacent if they have a common edge. For any figure, we can associate a graph as follows: Represent each polygon in the figure by a vertex and join two vertices by an edge if the polygons represented by these vertices are adjacent. There is a close relationship between the solution of a tiling problem and the existence of a certain component factor (i.e. a spanning subgraph with a given component) in the graph associated with the figure. For example, the graph associated with the defective chessboard of Figure 1.1 (a) is shown in Figure 1.2 (a). Moreover, it is clear that the existence of tiling a defective chessboard with dominoes is equivalent to the existence of a 1-factor in the graph associated with the defective chessboard. Note that the graph given in Figure 1.2 (b) cannot be the graph associated with any defective chessboard since two vertices x and y are not joined by an edge.

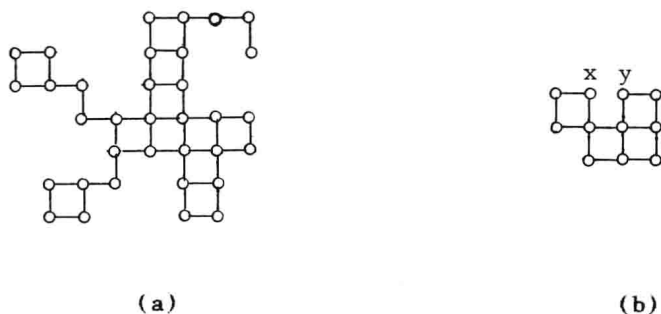


Figure 1.2

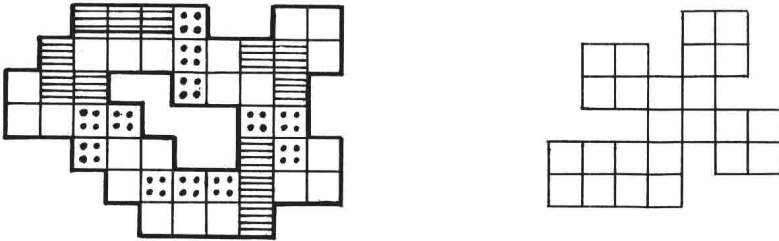
We use the following notation. By P_n , C_n and $K_{1,n-1}$, we denote the path, the cycle and the star with n vertices, respectively. For a graph G and a set $\{A, B, \dots, K\}$ of graphs, a spanning subgraph F of G is called an $\{A, B, \dots, K\}$ -factor of G if each component of F is isomorphic to one of $\{A, B, \dots, K\}$. In particular, each component of an H -factor (i.e. an $\{H\}$ -factor) is H .

We conclude this section by mentioning that some papers on tiling problem are concerned with the infinite plane, the infinite half plane and others, rather than finite planes. Golomb [7] studied the problem of tiling the infinite plane, the half plane, a quadrant of the plane, infinite strips, and other infinite figures with polyominoes of one shape. He also gave a classification of the capability of each polyomino, consisting of up to six unit squares, tiling each of the figures mentioned above. The papers [5], [6], [8] discuss various ways of tiling the plane.

2. Tiling defective chessboards

A figure obtained from an $m \times n$ chessboard (m rows and n columns) by removing a certain number of unit squares is called a defective chessboard, where m and n are arbitrary integers. The order of a defective chessboard is the number of unit squares

in it. A defective chessboard B is said to be tough if every pair of adjacent dominoes in B is contained in a 2×2 square of B . Observe that the defective chessboard in Figure 2.1 (a) is tough, while the one in Figure 2.1 (b) is not. The graph associated with a defective chessboard is called a square graph, and a square graph is said to be tough if its associated defective chessboard is tough, that is, a square graph is tough if every edge is contained in some C_4 .



(a) A tough defective chessboard that covered with triminoes. (b) A defective chessboard is not tough and cannot be covered with triminoes.

Figure 2.1

We say that a defective chessboard is connected if the corresponding square graph is connected. Generally, a plane figure is said to be connected if the corresponding graph is connected.

Tutte's 1-factor theorem [11] provides a solution to tiling a defective chessboard with dominoes. Moreover, Edmonds [4] gives a polyominal time algorithm for determining the existence of tiling a defective chessboard with dominoes. We now turn our attention from dominoes to other tiles. There are two kinds of triominoes, namely L-shaped and I-shaped triominoes (Figure 2.2 (a) and (b)).

Theorem 2.1 [2] Every connected tough defective chessboard of order $3p$ can be covered with triomionies (Figure 2.1 (a)).

We omit the proof of this theorem, which is similar to that of Theorem 2.2, presented next. Note that both I-shaped and L-shaped triomionies are represented by P_3 in the corresponding graph, and so the above theorem also gives a local sufficient condition for a square graph to have a P_3 -factor.

There exists a connected tough square graph of order $4p$ which has neither P_2 -factors nor $\{P_4, K_{1,3}\}$ -factors for any $p \geq 7$ (Figure 2.3). However, we have the next theorem, which says that every connected tough chessboard of even order can be tiled with dominoes and tetromionies given in Figure 2.2 (c).

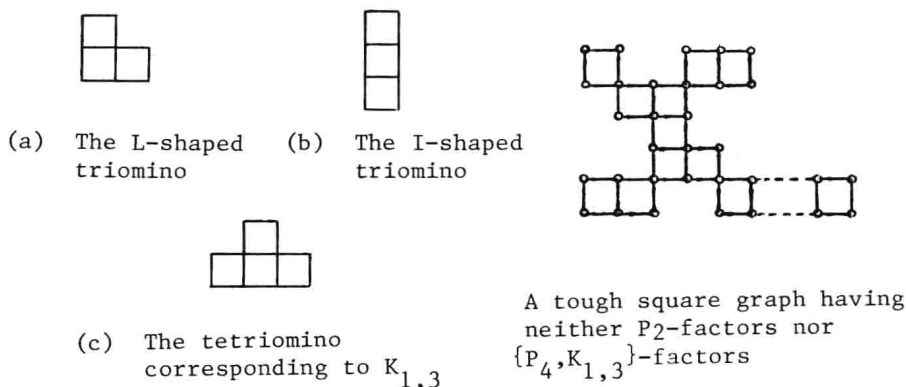


Figure 2.2

Figure 2.3

Theorem 2.2 Every connected tough square graph of even order has a $\{P_2, K_{1,3}\}$ -factor (Figure 2.4).