

**ECONOMIC THEORY  
AND  
OPERATIONS ANALYSIS**

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**fourth edition**

**WILLIAM J. BAUMOL**

# ECONOMIC THEORY AND OPERATIONS ANALYSIS

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Princeton and New York Universities*

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# Prefaces

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## ***Preface to the First Edition***

The last few years have brought with them a happy increase in rapport between the economic theorist and the managerial economist. This development has involved their simultaneous realization that business practice can be a fertile source of more abstract analytical ideas and that the theorist's rigorous tools can make an important contribution to the analysis of applied problems. That, in essence, is the spirit in which this book was written.

The subject of this book is economic theory, *not* operations research. The volume is intended to offer the reader both a systematic exposition of received microeconomic analysis, and an intuitive grasp of the many recent developments in mathematical economics that have too long remained a mystery in the private possession of the specialists (who, it must be admitted, have always been willing and anxious to share their secrets). The discussions of applications of economic theory to the tools of operations research and to business analysis are primarily illustrative, and though a considerable portion of the body of operations research equipment is described, the result can by no means be considered to constitute a survey of the field. As one reader has suggested, this book is intended to be more helpful to an operations researcher who wishes to learn economics than to an economist who desires a systematic education in operations research.

For their helpful comments and suggestions on all or part of the manuscript I must thank Forman Acton, Wroe Alderson, S. T. Beza, W. W. Cooper, Robert Dorfman, Ralph Gomory, Herman Karreman, Robert Kuenne, Harold Kuhn, Don Patinkin, Gardner Patterson, Maurice Peston, and, above all, Alvaro Lopez and Richard Quandt. The devoted labors of my research assistant, Charles Frisbie, and the extraordinary workmanship of my secretary, Mrs. C. B. Brown, were of immeasurable help. The role of my several years' experience with the management consulting firm of Alderson Associates, Inc., will be apparent in many parts of the book. I must also acknowledge my sincere gratitude to the Ford Foundation, whose grant to the Department of Economics at Princeton helped to finance both the research involved in the more original portions of this volume and the typing of the manuscript.

Finally, I must thank the editors of the several journals involved, as well as my co-authors, Ralph Gomory and Philip Wolfe, who graciously permitted me to reprint portions of the following articles: "On the Role of Marketing Theory," *Journal of Marketing*, Vol. XXI (April, 1957); "Selecting an Appropriate Model for an Operations Research Problem," Vol. VIII (November, 1955), "Solution of Management Problems Through Mathematical Programming," Vol. IX (May, 1956), "Operations Research Applied to Marketing Problems," Vol. X (March, 1957) and "A Guide to Operations Research Methods," Vol. X (April, 1957), all in *Cost and Profit Outlook*; "Community Indifference," *Review of Economic Studies*, Vol. XIV (1946-47); "On the Theory of Oligopoly," *Economica*, Vol. XXV (August, 1958); "Marginalism and the Demand for Cash in Light of Operations Research Experience," *Review of Economics and Statistics*, Vol. XL (August, 1958); (P. Wolfe co-author), "A Warehouse-Location Problem," *Operations Research*, Vol. 6, No. 2 (March-April, 1958); "Economic Theory and the Political Scientist," *World Politics*, Vol. VI (January, 1954); "Activity Analysis in One Lesson," *American Economic Review*, Vol. XLVIII (December, 1958); (R. Gomory co-author), "Integer Programming and Pricing," *Econometrica*, Vol. 28 (1960); and "The Cardinal Utility Which Is Ordinal," *Economic Journal*, Vol. LXVIII (December, 1958).

### **Preface to the Second Edition**

No doubt it is in the nature of things that revised versions of books appear as "second edition—*expanded*." This book is no exception. Though two chapters from the first edition have been expunged, on balance the book has grown at a rate not too dissimilar to the GNP.

There have been only a few minor changes in the text itself, most notably an attempt to improve the explanation of the basic theorem of

linear programming. A substantial number of exercises have been added, and brief discussions of the applications of differential calculus to standard economic theory have been inserted at the ends of Chapters 9, 11, 13, and 14. The five new chapters include one on duality, one on linear programming and production, one on statistical problems in demand estimation, and two on capital theory and its applications. The duality chapter and the chapter on demand estimation and its appendix provide elementary materials that are, I believe, particularly difficult to find elsewhere. This is especially true of the latter, which discusses some of the econometric techniques for dealing with simultaneous equation problems and treats such subjects as least squares bias, identification, and simultaneous equation estimates in an intuitive manner.

As usual I find myself deeply indebted to a number of persons for their very substantial help in the preparation of this second edition—to my colleagues Harold Kuhn, Burton Malkiel, Richard Quandt, and Frederic Scherer for their many suggestions, to Robert Bushnell for his revision of the chapter on computers, to Edward Pearsall for proofreading and supervising the preparation of the new diagrams, and to Mrs. C. B. Brown for her superb workmanship in the preparation of the manuscript. To all of these I am most grateful. To those others whose assistance has momentarily slipped my mind, I can only apologize.

### *Preface to the Third Edition*

This edition differs from its predecessor largely in the addition of some fairly extensive materials on the Kuhn-Tucker Theorem, including a discussion of a number of its important applications in economics. Several exercises using these materials are intended to demonstrate how this powerful theorem can be applied to obtain qualitative results in economic analysis. In addition, the discussion of the simplex method has been modified and, I hope, improved, at several points.

Professors A. W. Tucker and H. W. Kuhn were extremely generous in helping me at various stages in the revision. Thus they must get the credit not only for the substance of the new material, but for drawing, most gently, to my attention several weaknesses in the presentation in an earlier draft.

I am also heartily indebted to the many students who over the years have made a sport of catching errors in the book. I do not delude myself, however, that it is even yet nearly free of mistakes. I grow increasingly convinced that the species has evolved to a point where it reproduces itself and multiplies, so that no sooner has one generation of errors been brought under control than it is replaced by a host of successors sprung forth apparently from nowhere.

Finally I want to thank for their help in the final task of preparation of the new edition the several very competent secretaries, who were more patient with me than I deserved, and Mr. Stephen E. Kagann, who conducted so capably the latest hunt for inaccuracies.

### ***Preface to the Fourth Edition***

If this book had feelings, no doubt—like GBS on his 90th birthday—it would be somewhat surprised to find itself alive and apparently well in its fourth metamorphosis. I, too, have been surprised and delighted by the increasing frequency with which younger colleagues throughout the profession tell me that they have been subjected at some time to its materials; happily, as far as I could tell, the experience did not elicit their lasting resentment.

When it was first written, the book was intended to guide readers to the frontiers of economic analysis. This new edition represents continued dedication to that goal. Frontiers have a way of moving, and the contents of the volume have had to change accordingly. I have added discussions of a variety of what I believe to be important materials on topics such as the duality analysis of consumption and production (including Shepherd's lemma), the Ramsey-Boiteaux theorem on quasi-optimal pricing under a budget constraint, properties of quasi-concave utility functions and their relationship to ordinal theory, and the reswitching debate in the Cambridge-Cambridge controversy. Many of these have never before appeared in a textbook or have been dealt with only cursorily. Although some of the new materials are, in the nature of the case, somewhat more difficult than the discussions of the standard theory, they have been tested in classes by myself and others and, as far as I can judge from both written and oral comments and from examination results, they have passed the test of comprehensibility.

The new edition contains two essentially new chapters—one on comparative statics (Chapter 13) and the other on duality theory in consumption and production (Chapter 14). In addition, five chapters have been revised extensively—those on the neoclassical theory of consumption and production (Chapters 9 and 11), the chapter on welfare theory (21), and the chapters on distribution (24) and capital theory (26). Finally, the organization of the book has been revised on the basis of economic area covered, rather than on the degree of novelty of the materials.

As always, my debts are great, and words are the only coin I have to offer in repayment.

My greatest debts, for painstaking reading and detailed and invaluable comments, are to Elizabeth Bailey, David Folkerts-Landau, Lester Lave,

and Jerome Hass. Their suggestions added to my labors, but that additional effort was thoroughly worthwhile.

In addition, I received very useful comments on all or part of the manuscript from Sebastian Arango, Alan Blinder, Michael Rothschild, Vu Viet, and the members of my first-year graduate class in microeconomics at Princeton in the Fall of 1974.

I was helped in the task of revising the reading lists by Roger Klein, Wassily Leontief, Charles McCallum, Janusz Ordover, Richard Quandt, and Andrew Schotter. To all of them I offer my sincere thanks.

Finally, and most strongly, I must express my appreciation to Sue Anne Batey, my research assistant and secretary at Princeton, for her intelligent assistance, her ingenuity in grasping the intent of my unintelligible intentions, her ability to bring order out of chaos, and, above all, her qualities as a human being.

W.J.B.

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# **ANALYTIC TOOLS OF OPTIMIZATION**

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