

# Mathematical Models and Marketing Management

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"I think my REAL trouble is that I don't BELIEVE in arithmetic."

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#### Foreword

Shortly after World War II, a move began to adapt some of the mathematical approaches developed for military operations to the problems of business management. This movement, known generally as "operations research," focused largely on such problems as production scheduling, physical distribution, capital budgeting, and inventory control during its rapid rise in the 1950's. Only in recent years has there been substantial interest among marketers in operations research and a significant effort to apply mathematical models and techniques to marketing management.

In this volume Professor Buzzell seeks to describe the nature of some of the applications of mathematical models to marketing problems, to outline some of the limitations and potentialities of this new development, and to analyze some of the problems marketing men will encounter in utilizing these techniques. He sets forth in nonmathematical terms the underlying concepts involved in the use of mathematical models in a manner designed to assist marketing management to work effectively with technical specialists. Perhaps of even greater importance, he deals with the use of mathematical models in the context of the marketing process as it exists in practice. The case studies are not abstract descriptions of problems; they present the full backgrounds of the marketing situations in which the models were employed. As such they will also be of value to staff specialists in providing a better appreciation of the context in which their mathematical skills are to be used.

If the application of models is to bear real fruit in the future, there must be a blending of the marketing manager's understanding of the complex reality of the marketing problems with the analytical skill of the mathematically trained researcher. An important objective of this study is to highlight the necessity for this blend and to contribute to its development by providing materials relevant to the activities of both parties in this joint task.

The study was initiated with the financial support of an allocation from funds contributed to the School by The Associates of the Harvard Business School. In the course of the research Professor

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Buzzell established close working relationships with the Marketing Science Institute, and the Institute at a later stage made a specific grant to the School for the partial support of this research. May I express for the School its gratitude for these two sources of financial support.

BERTRAND FOX
Director of Research

Soldiers Field Boston, Massachusetts May 1964

#### Acknowledgments

This book summarizes the results of an investigation of the applications of mathematical models to problems of marketing management. Unlike some other recently published books with similar titles, it is not a textbook on "mathematics for nonmathematicians." The purpose of the study was to evaluate the uses and limitations of mathematical concepts and techniques from the standpoint of the marketing manager.

The idea for the study grew out of my "discovery" of mathematics as a participant in the Institute of Basic Mathematics for Application to Business in 1959–1960. This program, sponsored by the Ford Foundation, gave some 40 teachers of business and economics a concentrated exposure to mathematics and its applications. I owe my limited understanding of the subject to the faculty of the Institute, which included Professor Howard Raiffa of Harvard University, Professor Samuel Goldberg of Oberlin College, Professor Allan Spivey of the University of Michigan, and Professor Ronald Howard of the Massachusetts Institute of Technology. Each of these men has that rare combination of mathematical competence and teaching skill which made the Institute a memorable experience.

Professor Howard also introduced me to the field of operations research through his work at Arthur D. Little, Inc. At ADL I was exposed to many and varied applications of mathematical models to practical business problems. In the course of this study, I learned much from discussions with members of the ADL staff, including John Magee, Dr. Jerome D. Herniter, and Dr. Charles C. Slater (now at Michigan State University).

Since the study deals with applications of a body of techniques to business practice, it was essential to have full cooperation from competent practitioners of marketing management and marketing research. The names of some of the companies and associations which provided help are listed in Chapter 1. Certain individuals should also be singled out: these include David B. Hertz of Mc-Kinsey and Company; Gordon Hughes of Scott Paper Company;

David Learner of Batten, Barton, Durstine, & Osborn, Inc.; Dr. Charles Ramond of the Advertising Research Foundation; Dudley Ruch of the Pillsbury Company; and Robert Weinberg of International Business Machines Corporation. To each of these men, and many others who were generous with their time and ideas, I am indebted.

The Marketing Science Institute provided valuable support for the study, financial and otherwise. Members of the MSI staff, including Michael Halbert, Dr. Wendell Smith, Dr. David Luck, and Patrick Robinson, read portions of the manuscript and made valuable suggestions. I hope that the result will contribute to their goal of developing more scientific approaches to marketing.

Several of my colleagues at the Harvard Business School gave me help and encouragement. Professors Raymond A. Bauer, Howard Raiffa, and Donald Cox all read the first draft and made substantial improvements in it.

Having acknowledged the help I received from various people, I naturally hasten to absolve them of any responsibility for misconceptions or errors in the finished product.

Finally, I want to express my continuing debt to my father, whose slide rule always fascinated me. He may get some small satisfaction from learning that I finally understand how it works.

ROBERT D. BUZZELL

Boston, Massachusetts May 1964

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### PART I

Introduction to Mathematical Models