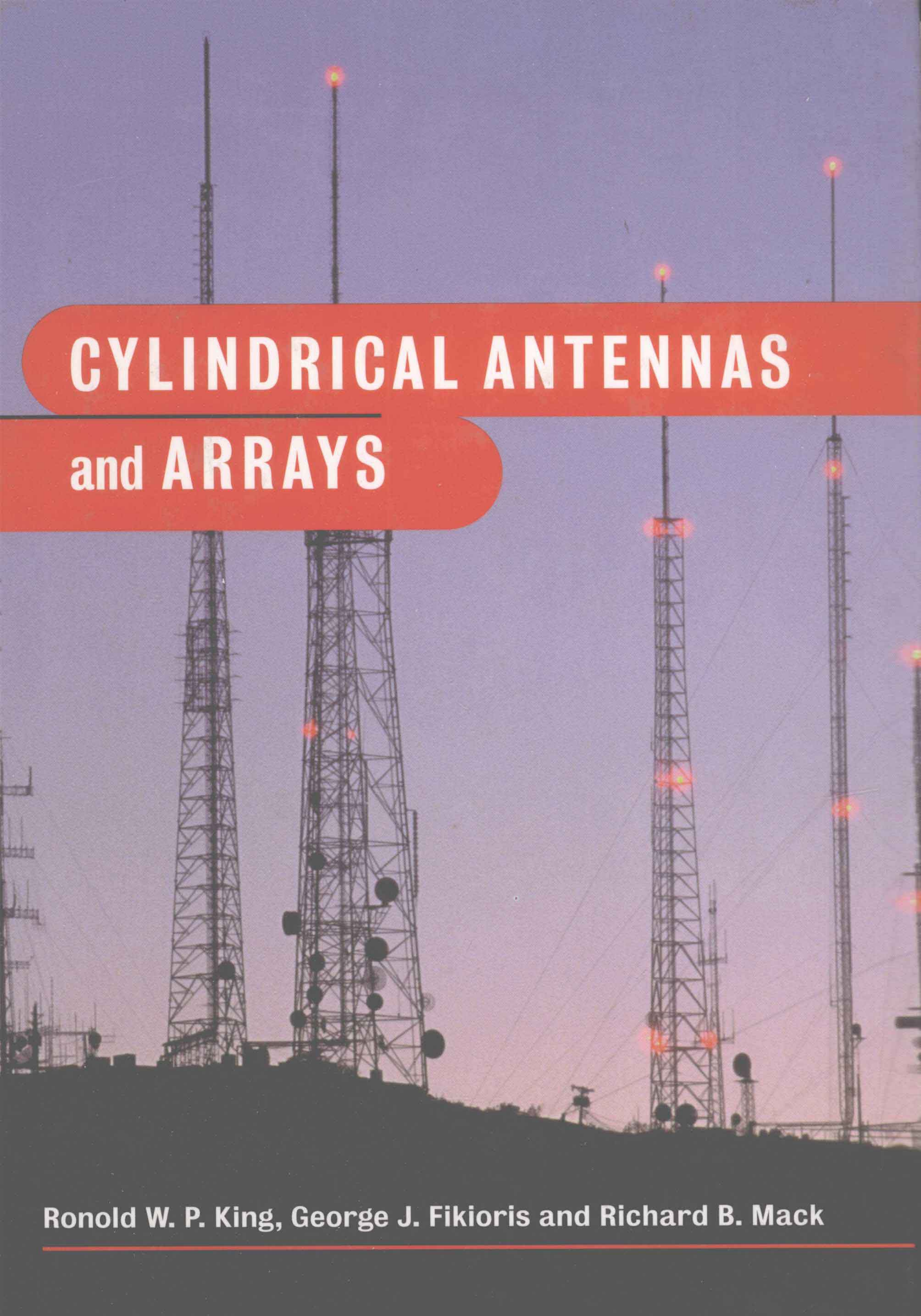

CYLINDRICAL ANTENNAS and ARRAYS

Ronold W. P. King, George J. Fikioris and Richard B. Mack

Cylindrical Antennas and Arrays

Revised and enlarged 2nd edition of 'Arrays of Cylindrical Dipoles'

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Preface

Over three decades have passed since the publication, in 1968, of "Arrays of Cylindrical Dipoles" by R. W. P. King, R. B. Mack, and S. S. Sandler. The present volume is a revised and enlarged second edition of that work. The objectives of "Cylindrical Antennas and Arrays" are similar to those of "Arrays of Cylindrical Dipoles": to present approximate but efficient theoretical methods for determining current distributions, input admittances, and field patterns of arrays of cylindrical dipoles; to use such methods to analyze particular types of arrays; to describe experimental methods for determining current distributions, input admittances, and field patterns; and to correlate and compare theoretical and experimental results.

The most fundamental quantities, and the ones most difficult to determine theoretically, are the current distributions on the array elements. Rather than postulating the current distributions, perhaps the most common treatment in the literature of the 1960s, "Arrays of Cylindrical Dipoles" sought to determine the current distributions on the array elements by solving integral equations. Today's antenna and engineering literature is quite different from that of the 1960s: even elementary textbooks include discussions on determining current distributions from integral or similar equations.

As an example, consider the simplest configuration discussed in this book, the single, isolated cylindrical antenna of given length and radius. The integral equation treated in Chapter 2 is but one of the several integral or integro-differential equations that are encountered in the present-day literature. Although such equations were derived many years ago, the reasons for their increased popularity are the easy accessibility to high-speed computers and the availability of a large number of numerical methods. As a result of these developments, application of general-purpose numerical methods to the aforementioned equations is today much more common than in the 1960s.

Rather than discretizing the integral equations, the "two-term" and "three-term" theories developed in this book treat them by analytical means. These theories, which apply to elements that are not too long, are here presented as powerful alternatives to applying general-purpose numerical methods. Because the final two- and three-term formulas are quite simple in form, they require less running time when programmed in a computer. In addition, the analytical methods present a physical basis for understanding changes in the characteristics of the antenna as the parameters are changed. Furthermore, applying numerical methods to the integral equations in this book presents difficulties that are often overlooked in the literature.

Chapter 1, which has been completely rewritten, has the same purpose as the first chapter of the 1968 edition: to introduce the reader to the theory of antennas and arrays presented in subsequent chapters by discussing some commonly used methods of studying a single antenna in free space. The chapter concludes with an introductory discussion of integral equations and the application of numerical methods, a subject discussed in greater depth in Chapter 13.

Chapters 2–5 develop the two- and three-term theories for the single isolated antenna, the two-element array, the circular array, and the curtain array, respectively. In Chapters 3–5, the array elements are assumed to be identical, parallel, and non-staggered. In Chapters 6 and 7, the two- and three-term theories are extended and applied to certain types of arrays that do not satisfy the aforementioned conditions. Apart from editing changes, Chapters 2–7 are the same as those in the first edition.

Chapters 8–13 have no counterparts in "Arrays of Cylindrical Dipoles". Chapters 8 and 9 analyze vertical and horizontal dipoles and arrays over and on the surface of the conducting and dielectric earth or sea. Included are asphalt-coated earth and ice-coated water. A major new addition is long-distance propagation over the spherical surface of the sea.

In Chapter 10, arrays of identical, parallel, non-staggered elements are discussed once more, from the point of view of computer implementation of the two-term formulas. Some restrictions placed on the arrays of Chapters 3–5 are removed in this chapter, which also serves as an introduction to the study of the large circular array in Chapters 11 and 12.

In Chapters 11 and 12, a novel type of circular array is studied. The arrays under consideration differ from conventional circular arrays in that only one or two of the many array elements are excited and the entire array is tuned to spatial resonance. Both the integral equations and the two-term theory must be modified and extended to deal with the phenomena studied in this chapter. The modified theory is used to show that such arrays possess new, unusual and potentially useful resonant and directive properties. The analytical nature of the underlying theory is an important advantage for this study.

Although the two- and three-term theories are analytical in nature, they are not claimed to be mathematically rigorous. (Nonetheless, very good agreement between theory and experiment is obtained in Chapters 1–12.) Until shown otherwise [1], the lack of rigor is a necessity rather than a convenience: most of the integral equations dealt with in this book have no exact solutions, even in principle. From a theoretical point of view, then, one cannot do better than to find "reasonable solutions" that satisfy the integral equations approximately, and the aforementioned lack of rigor must be true for any method used to "solve" the integral equations. The main purpose of Chapter 13 is to discuss the consequences of this rather peculiar situation when general-purpose numerical methods are applied to the integral equations in question. The case of an isolated antenna is discussed in detail, and extensions to arrays are pointed out. The special numerical difficulties associated with the circular arrays of the previous two chapters are also discussed here.

The concluding Chapter 14 discusses experimental methods, with emphasis on the measurement of antenna impedance. This chapter corresponds to Chapter 8 of the first edition; it has been significantly revised to discuss modern measurement techniques.

R. B. Mack wrote Chapter 14, G. Fikioris wrote Chapters 1 and 10–13; R. W. P. King wrote Chapters 8 and 9, and organized the present edition as a whole. In addition to the contributions of the several individuals named in the Preface to the first edition, the authors gratefully acknowledge the contribution of Chapter 5 by Sheldon S. Sandler. Without the extensive and very thorough work of Margaret Owens, both in preparing and correcting the new manuscripts and in editing Chapters 2–7, the present arrangement would not be possible. Finally, we thank Tai Tsun Wu for providing the initial and most fundamental ideas for the work in Chapters 11–13, and for guiding and inspiring the ensuing researches.

R.W.P.K.
G.J.F.
R.B.M.

Preface to first edition

Studies of coupled antennas in arrays may be separated into two groups: those which postulate a single convenient distribution of current along all structurally identical elements regardless of their relative locations in the array and those which seek to determine the actual currents in the several elements. Virtually all of the early and most of the more recent analyses are in the first group in which both field patterns and impedances have been obtained for elements with assumed currents. Pioneer work in the determination of field patterns of arrays of elements with sinusoidally distributed currents was carried out for uniform arrays by Bontsch-Bruewitsch [1] in 1926, by Southworth [2] in 1930, by Sterba [3] and by Carter *et al.* [4] in 1931. Early studies of non-uniform arrays are by Schelkunoff [5] in 1943, by Dolph [6] in 1946, and by Taylor and Whinnery [7] in 1951. The self- and mutual impedances of arrays of elements with sinusoidally distributed currents were studied especially by Carter [8] in 1932, by Brown [9] in 1937, by Walkinshaw [10] in 1946, by Cox [11] in 1947, by Barzilai [12] in 1948, and by Starnecki and Fitch [13] in 1948. A thorough presentation of the basic theory of antennas with sinusoidal currents was given by Brückmann [14] in 1939. Actually, the current in any cylindrical antenna of length $2h$ and finite radius a is accurately sinusoidal only when it is driven by a continuous distribution of electromotive forces of proper amplitude and phase along its entire length. It is approximately sinusoidal in an isolated very thin antenna ($a \ll h$) driven by a single lumped generator primarily when the antenna is near resonance. When antennas are coupled in an array with each driven by a single generator or excited parasitically, it is generally assumed that (1) the phase of the current along each element is the same as at the driving point and (2) the amplitude is distributed sinusoidally. Both of these assumptions are reasonably well satisfied only for very thin antennas ($a \ll \lambda$) that are not too long ($h \leq \lambda/4$). Nevertheless, a very extensive theory of arrays has been developed based implicitly on one or both of these assumptions. Evidently it is correspondingly restricted in its generality.

The analysis of coupled antennas from the point of view of determining the actual distributions of current was studied for two antennas by Tai [15] in 1948 and extended to the N-element circular array by King [16] in 1950. A general analysis of arrays of coupled antennas has been given by King [17]. Unfortunately, the rigorous solution of the simultaneous integral equations for the distributions of current in the elements of an array of parallel elements is very complicated and no simple and practically useful set of formulas was obtained. As a consequence, the extensive study of the electromagnetic fields of antennas and arrays in this earlier work (Chapters 5 and 6 in King [17]) was limited to arrays with currents in the elements that satisfied the assumptions of constant phase angle and sinusoidal amplitude. Similar restrictions are implicit in the fields calculated, for example, by Aharoni [18], Stratton [19], Hansen [20] and many others.

A practical method for obtaining solutions of the simultaneous integral equations for the distributions of current in the elements of a parallel array in a form that combines simplicity with quantitative accuracy was proposed by King [21] in 1959. In this analysis an approximate procedure was developed which provided simple, two-term trigonometric formulas for the currents in all of the arbitrarily driven or parasitic elements in a circular array of N elements in a manner that took full account of the effects of mutual interactions on the distributions of current. These formulas applied to elements up to one and one-quarter wavelengths long. The application of this new procedure to actual arrays and the experimental verification of the results were carried out in an extensive series of investigations by Mack [22]. The generalization of the method to curtain arrays was developed by King and Sandler [23, 24] in 1963 and 1964. The extension of the method to parasitic elements in arrays of the Yagi type was verified experimentally by Mailloux [25] in 1966. A modification of the theory and its application to the optimization of Yagi arrays by the use of a high-speed computer were devised by Morris [26] in 1965. In 1967 Cheong [27] extended the theory to unequal and unequally spaced elements. (The several researches were supported in part by Joint Services Contract Nonr 1866(32), Air Force Contract AF19(604)-4118 and National Science Foundation Grants NSF-GP-851 and GK-273.)

A further improvement in the simplified trigonometric representation of the current in an isolated antenna was introduced by King and Wu [28] in 1965 and extended to arrays in the present work.

This book begins with an introductory chapter that reviews the foundations and limitations of conventional antenna theory. It then proceeds to derive the new two- and three-term formulas for the isolated antenna in Chapter 2 and for two coupled antennas in Chapter 3. Chapter 4 provides the complete formulation of the new theory for the N-element circular array; Chapter 5 for the N-element curtain array of identical elements. The more difficult problem of treating elements of different lengths—notably in the Yagi array and the log-periodic antenna—is treated in Chapter 6. Chapter 7 is devoted to planar and three-dimensional arrays that include staggered and collinear elements. Chapter 8[1] is concerned with the broad problems of measurement—currents, impedances, field patterns and the correlation of theory with experiment. In the appendices summaries of programs are given for the computational analysis of circular, curtain, and Yagi arrays.[2]

[1] The original Chapter 8 corresponds to Chapter 14 of the present (2nd) edition.

[2] Much of this material is omitted from the appendices in the 2nd edition.

In the preparation of the manuscript, S. S. Sandler was responsible for Chapters 1 and 5, R. B. Mack for Chapters 4 and 8, and R. W. P. King for Chapters 2, 3, 6, and 7 and for the co-ordination of the several parts.

The authors are happy to acknowledge the important contributions of Drs Robert J. Mailloux, I. Larry Morris, and W.-M. Cheong whose researches form the basis of Chapter 6; and of V. W. H. Chang whose work underlies Chapter 7. They are grateful to Professor Tai Tsun Wu for many valuable suggestions and to Mrs Dilla G. Tingley for continuing painstaking assistance with the preparation of the manuscript, the graphical representation, the computations, and the programs. Mrs Barbara Sandler, Mrs Evelyn Mack, and Mr Chang also assisted with the programs, Mrs S. R. Seshadri with the preparation of the manuscript. Miss Margaret Owens contributed greatly to the accuracy of the presentation with her meticulous reading of the proofs. She also had a major share in the preparation of the index.

R.W.P.K.
R.B.M.
S.S.S.

Cambridge, Mass.
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Contents

1 Introduction

1.1 Linear antennas

Wireless communication depends upon the interaction of oscillating electric currents in specially designed, often widely separated configurations of conductors known as antennas. Those considered in this book consist of thin metal wires, rods or tubes arranged in arrays. Electric charges in the conductors of a transmitting array are maintained in systematic accelerated motion by suitable generators that are connected to one or more of the elements by transmission lines. These oscillating charges exert forces on other charges located in the distant conductors of a receiving array of elements of which at least one is connected by a transmission line to a receiver. Fundamental quantities which describe such interactions are the electromagnetic field, the driving-point admittance, and the driving-point impedance. These can be easily determined if the distributions of current on the array elements are known. The determination of the currents on the array elements is the main concern of this book. In this first chapter, the basic electromagnetic equations are formulated and applied to a single antenna in free space. The simplest approach of assuming the current rather than actually determining it is reviewed first. Then, integral equations for the current distributions are derived, and determining the current by numerical methods is discussed. These discussions serve as an introduction to the analytical theory of antennas and arrays based on the solution of integral equations that is presented in subsequent chapters.

Figures 1.1a and 1.1b show two simple practical radiating systems. In Fig. 1.1a, a section at the open end of a two-wire transmission line has been bent outward to form a dipole antenna. In Fig. 1.1b, the inner conductor of a coaxial transmission line is extended above a ground plane. In both cases, the transmission lines are connected to generators which oscillate at a frequency $f = \omega/2\pi$. In a small region (comparable in extent with the distance between the two conductors of the transmission line), the antenna and line are coupled. Owing to the complications involved in this coupling, it is convenient to replace the actual generator/transmission line with an idealized so-called *delta-function* generator, which maintains an impressed electric field $\mathbf{E}^e(z) = \hat{\mathbf{z}}E_z^e(z) = V\delta(z)\hat{\mathbf{z}}$ at the surface of the antenna. This is the linear antenna of Fig. 1.1c. The impressed field is non-zero only at the center $z = 0$ of the cylindrical surface. The delta-function generator is an independent voltage source in the sense of ordinary

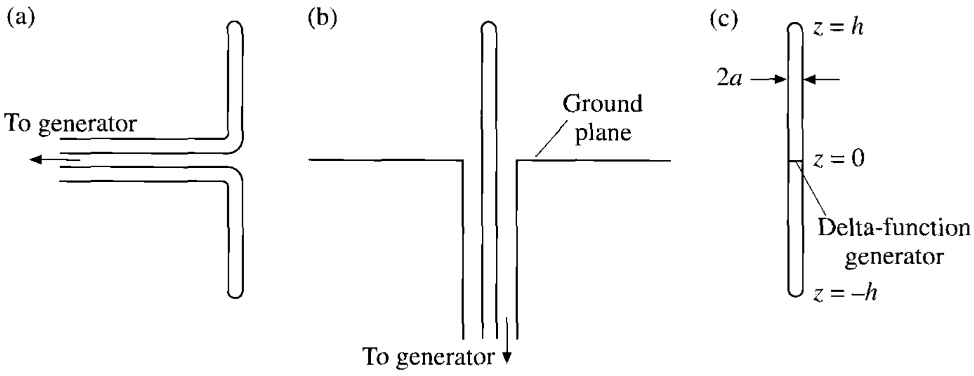

Figure 1.1 (a) Dipole antenna and two-wire transmission line. (b) Monopole antenna over a ground plane. (c) Simplified center-driven linear antenna.

circuit theory. The linear antenna of Fig. 1.1c can also serve as a model for other types of radiating systems. The simplifying assumption of studying the antenna in the absence of the connecting transmission line is particularly useful when the antenna is an array element.

The radius of the linear dipole antenna of Fig. 1.1c is a, and its half-length is h. It is assumed throughout this book that the radius is much smaller than both the wavelength λ and the length $2h$ of the antenna. Under such conditions, one can neglect the small currents on the capped ends of the antenna and assume that only a current $K_z(z) = I(z)/2\pi a$ is maintained on the cylindrical surface of the antenna. Other concepts of circuit theory can be introduced, and are particularly useful to the antenna engineer: the driving-point admittance Y_0 and driving-point impedance Z_0 are defined as

$$Y_0 = G_0 + jB_0 = \frac{I(0)}{V} = \frac{1}{Z_0}, \qquad Z_0 = R_0 + jX_0 = \frac{V}{I(0)} = \frac{1}{Y_0}. \tag{1.1}$$

G_0, B_0, R_0, and X_0 are respectively, the driving-point conductance, susceptance, resistance, and reactance. When h, a, and f are such that the antenna is at resonance, one has $X_0 = 0$ and $B_0 = 0$. As an example of the use of these quantities in a practical situation, consider the problem of designing the antenna so that, at a given frequency f, there is maximum power transfer from a transmission line of given characteristic impedance Z_c. With the assumption that the transmission line and the antenna can be studied separately, the problem is reduced to that of determining h and a so that Z_0 is equal to Z_c^*, the complex conjugate of Z_c.

The delta function $\delta(z)$ is zero except when $z = 0$. Additional, well-known properties of the delta function are

$$\delta(z) = \begin{cases} 0, & \text{if } z \neq 0 \\ \infty, & \text{if } z = 0 \end{cases}, \qquad \int_{-b}^{b} \delta(z)\, dz = 1 \tag{1.2a}$$

$$\delta(kz) = \frac{1}{|k|}\delta(z), \qquad f(z)\delta(z) = f(0)\delta(z) \tag{1.2b}$$

$$\int_{-b}^{b} f(z)\delta(z)\,dz = f(0) \tag{1.2c}$$

$$\frac{d}{dz}H(z) = \delta(z) \quad \text{where} \quad H(z) = \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{if } z < 0. \end{cases} \tag{1.2d}$$

In (1.2), b is any positive constant, k is any real constant, $f(z)$ is any smooth function of z, and $H(z)$ is the step function.

The next section introduces the fundamental equations of electromagnetic theory that are useful in the antenna problems considered in this book. More details can be found in [1], and in more concise form in [2, Chapter 1].

1.2 Maxwell's equations and the potential functions

The interaction of charges and currents is governed by Maxwell's equations which define the electromagnetic field. With an assumed time dependence $e^{j\omega t}$, they are

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J} + j\omega\epsilon_0\mathbf{E}), \qquad \nabla \cdot \mathbf{B} = 0 \tag{1.3a}$$

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}, \qquad \nabla \cdot \mathbf{E} = \rho/\epsilon_0, \tag{1.3b}$$

where the electric vector $\mathbf{E}$ is in volts per meter (V/m), the magnetic vector $\mathbf{B}$ in tesla (T). SI units are used throughout this book. The volume density of current $\mathbf{J}$ in amperes per square meter (A/m^2) is the charge crossing unit area per second. The volume density of charge ρ is in coulombs per cubic meter (C/m^3). $\mathbf{J}$ and ρ satisfy the equation of continuity,

$$\nabla \cdot \mathbf{J} + j\omega\rho = 0. \tag{1.3c}$$

In the interior of perfect conductors, $\mathbf{J} = 0$ and $\rho = 0$. In (1.3), ϵ_0 and μ_0 are the absolute permittivity and permeability of free space. They have the numerical values $\epsilon_0 = 8.854 \times 10^{-12}$ farads per meter (F/m) and $\mu_0 = 4\pi \times 10^{-7}$ henrys per meter (H/m), and are related to the velocity c of light and the characteristic impedance ζ_0 of free space by

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}, \qquad \zeta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}. \tag{1.4}$$

Transmission lines and antennas are made from highly conducting materials such as brass or copper. In most cases, it is an excellent approximation to assume that the