STATISTICS: textbooks and monographs volume 20

HANDBOOK OF STATISTICAL DISTRIBUTIONS

JAGDISH K. PATEL
C. H. KAPADIA
D. B. OWEN

Handbook of Statistical Distributions

JAGDISH K. PATEL
Department of Mathematics
University of Missouri
Rolla, Missouri

C. H. KAPADIA and D. B. OWEN
Department of Statistics
Southern Methodist University
Dallas, Texas

MARCEL DEKKER, INC. New York and Basel

MARCEL DEKKER, INC.

270 Madison Avenue, New York, New York 10016

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 76-48474

ISBN: 0-8247-6362-9

Current printing (last digit)
10 9 8 7 6 5 4 3 2 1

PRINTED IN THE UNITED STATES OF AMERICA

Preface

This handbook is the outcome of an attempt to collect, in as concise a form as is possible, many results which statisticians find useful. In so doing the authors have not always quoted original sources of the discoveries given, but rather the most readily available sources.

This text is not intended as a book of statistical theory but instead as a source book where concise statements and references may be found.

Throughout the text, all expressions given are regarded as being implicitly restricted to the case where all quantities contained therein exist and are finite. In cases where questions might arise, the authors have noted these restrictions.

It is hoped that students in graduate statistics courses will find this summary particularly useful. Research workers should also find the book to be helpful in tracing references to material they desire.

In the preparation of the manuscript the authors have checked and rechecked the information given. However, considering the diversity of the material, one hundred percent accuracy cannot be achieved. Readers are cordially invited to contact the authors regarding any errors or suggestions for improvements in presentation.

The authors are particularly grateful to Mr. William Parr who read the manuscript and gave valuable comments and suggestions. They also wish to acknowledge the initial typing of the manuscript by Jeannie Alley and Mary Surovik, the final production typing by Beverly Mullins and Christy Brawand and the copy editing by Molly Petri.

August, 1976

J. K. Patel
C. H. Kapadia
D. B. Owen

CONTENTS

Chapter 1

MOMENTS, CUMULANTS, AND GENERATING FUNCTIONS

1.1 MOMENTS AND CUMULANTS

The moments and cumulants are a set of constants of a distribution which are useful for measuring its properties and, under certain circumstances, for specifying it.

1.1.1 DEFINITIONS

(1) Expectation (One-Dimensional Case). Let X be a random variable and h(•) be a function with both domain and range the real line. The expectation or expected value of the function h(•) of the random variable X, denoted by E[h(X)], is defined by

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x)\, dx$$

if X is continuous with probability density function (pdf) f(x) provided the integral converges absolutely (i.e., $\int_{-\infty}^{\infty} |h(x)| f(x)\, dx < \infty$); and by

$$E[h(X)] = \sum_x h(x)f(x)$$

if X is discrete with probability function (pf) f(x) provided the sum converges absolutely (i.e., $\sum_x |h(x)|f(x) < \infty$). The summation is over all possible values of x.

If the integral or the sum in the above definition does not converge absolutely, we say E[h(X)] does not exist.

(2) Expectation (Two-Dimensional Case). Let (X,Y) be a bivariate random variable. The expected value of a function h(•,•) of the bivariate random variable, denoted by E[h(X,Y)], is defined to be

$$E[h(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)f_{X,Y}(x,y)\ dx\ dy$$

if the random variable is continuous with pdf, $f_{X,Y}(x,y)$; and is defined to be

$$E[h(X,Y)] = \sum_x \sum_y h(x,y)f_{X,Y}(x,y)$$

if the random variable is discrete with pf $f_{X,Y}(x,y)$. The summation is over all possible values of (x,y).

If the integral or the sum in the above definition does not converge absolutely, we say E[h(X,Y)] does not exist.

(3) Conditional Expectation. Let (X,Y) be a bivariate random variable and g(•,•) a function of two variables. The conditional expectation of g(X,Y) given X = x, denoted by $E_{Y|X}[g(X,Y)|X = x]$, is defined to be

$$E_{Y|X}[g(X,Y)|X = x] = \int_{-\infty}^{\infty} g(x,y)f_{Y|X}(y|x)\ dy$$

if the random variable is continuous with $f_{Y|X}(y|x)$ as the conditional probability density function of Y given X = x and

$$E_{Y|X}[g(X,Y)|X = x] = \sum_y g(x,y)f_{Y|X}(y|x)$$