ELECTRO-DYNAMICS

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TO THE READER

Mir Publishers would be grateful for your comments on the content, translation and design of this book.

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PREFACE TO THE RUSSIAN EDITION

This book originated from our experience in teaching electrodynamics as part of a series of lectures in theoretical physics. The lectures were read at the Leningrad State University for all students of the physics faculty, both future theoreticians and experimenters. The subject matter follows from what the students learned about electricity and magnetism in the general physics course. On the other hand, an electrodynamics course must serve as the basis for many special disciplines, such as plasma physics, propagation of radiowaves, electromagnetic methods in geophysics, the accelerator theory, and others. These factors have determined the content of this book and the manner of presentation.

With the current high level of instruction in the general physics course the students starting their study of electrodynamics have a considerable knowledge of the facts that are generalized in the Maxwell equations. This has made it possible for us to proceed from the equations and set the objective of showing how different problems in electrodynamics follow from the equations when the properties of the material media are taken into account. The approach via the Maxwell equations enables the reader to come closer to formulating in the most direct and modern way quite a number of problems currently under intensive discussion in scientific literature. Of these, this book considers some aspects of magnetohydrodynamics, the motion of charged particles in a nonuniform electromagnetic field, and the basis for the phenomenological equations of superconductivity. We have also included the concept of spatial dispersion and stated the basic ideas of nonlinear optics. Understandably, the presentation of these topics cannot claim to be complete.

In accordance with the purpose of this book, we assume that the mathematical grounding corresponds to that of a student who has completed his second year in the physics faculty of a university. This should be sufficient to understand all of the material of the book. The Appendices contain the basic facts about vector and tensor analyses, which are used throughout the book, and also some properties of the Dirac delta function.

Particular attention is paid to the application of the special theory of relativity. In Chapter 2 we briefly consider the fundamentals of special relativity, while in other sections of the book we apply this theory to specific problems. Notably, the relativistic theory of radiation by a point charge is treated in great detail. The reader more interested in nonrelativistic aspects of electrodynamics can skip these sections, which are marked with asterisks.

We have found it impossible to incorporate topics that need the quantum theory and/or statistical physics for their interpretation. Consequently, we de not discuss the electrodynamics of material media from the viewpoint of their microscopic structure. But the thermodynamical aspect of the interaction of electromagnetic fields with media is brought in wherever possible.

To keep the size of the book within limits we have found it necessary to exclude the specific problems of mathematical physics that originate in electrodynamics. For the same reason the book contains no exercises. The reader can find appropriate problems on classical electrodynamics in the well-known book by V. V. Batygin and I. N. Toptygin: *Problems in Electrodynamics* (2nd edition, Academic Press, New York, 1978). We should also like to note that the presentation is concise and hence the material requires attentive reading. The reader is advised to do all the intermediate calculations himself.

The overall number of topics discussed here is rather limited, since the book corresponds to the course taught at the Leningrad State University. It is our hope, however, that the material covered will show how diversified the field of electrodynamics is and how the different branches are connected with the common source, the Maxwell equations.

We express our gratitude to our science editor Prof. S. V. Izmailov and to the reviewers Profs. V. I. Grigor'ev and V. G. Solov'ev. Their many critical remarks and helpful suggestions greatly improved our manuscript.

In style and subject matter this book reflects the pedagogical principles of our teacher, Academician Vladimir A. Fock, to whom it is dedicated.

Yu. V. Novozhilov and Yu. A. Yappa

PREFACE TO THE ENGLISH EDITION

In preparing the original Russian edition for translation the text has been carefully checked and the necessary corrections have been made. We hope that this book will help those studying electrodynamics to go on from the general physics course of electricity and magnetism to the special literature on the subject and to more advanced textbooks, such as the acclaimed *Classical Electrodynamics* by J. D. Jackson, *Classical Electricity and Magnetism* by W.K.H. Panofsky and M. Phillips, and *The Classical Theory of Fields* and *Electrodynamics of Continuous Media* by L. D. Landau and E. M. Lifshitz. Many ideas from these books have been used in our exposition.

A note about units. In Chapter 1 the electrodynamic equations are written in a form that ensures an easy transition from Heaviside-Lorentz units to SI units and back. Next, in dealing with the electrodynamics of isotropic, homogeneous media we use Heaviside-Lorentz units (Chapters 2 to 6). In the remaining chapters we keep to SI units except for some special cases.

This book may, and should, prompt critical remarks concerning both the subject matter and the manner of presentation. Comments of this kind will be appreciated.

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Indices. In Chapters 1-6 Latin-letter indices i, j, ... assume values 0, 1, 2, 3, and Greek-letter indices α, β, ... assume values 1, 2, 3. In Chapters 7-9 Latin-letter indices assume values 1, 2, 3. In § 33 Greek-letter indices enumerate linear contours. In § 34 Latin-letter indices enumerate generalized forces and generalized currents.

Latin letters

A vector potential; $A_i^{i'}$—coefficients of linear transformations.

a 3-dimensional acceleration; a_{ij}—strain tensor (§ 32), Onsager coefficients (§ 34).

B magnetic induction.

$\vec{b} \equiv d\vec{w}/d\tau$.

C capacitance of a capacitor, eikonal (§ 21).

c velocity of light in vacuo; c_{ik}—capacitance coefficients (§ 30); c_{ijkl}—elasticity moduli (§ 32).

D electric induction.

$d_{i,kl}$ piezoelectric coefficients (§ 32).

E electric field strength; $\mathscr{E}$ —energy of a pointlike mass (§ 6), total radiated energy (§ 16).

e electron charge; $\mathbf{e}_i$—basis vectors in linear space; $\vec{e}_i$—basis vectors in Minkowski space.

F mechanical force; $\mathbf{F}_{\text{rad}}$—force of radiative reaction; $\mathbf{F}_{\text{ext}}$—external force applied to a charge; $\vec{F}$, F^i—4-dimensional Minkowski force; F^{ik}—stress tensor of electromagnetic field; F—free energy density (§§ 22, 31); $\tilde{F}$—thermodynamic potential (§ 31); $\mathscr{F}$—total free energy.

f bulk density of force; f^{ik}—induction tensor of electromagnetic field (§ 7).

G Green's function (§ 11); $\mathscr{G}$—Gibbs' potential (§ 32).

g momentum density of electromagnetic field; g_{ij}—metric tensor.

H magnetic field strength; $\mathscr{H}$—Hamiltonian (§§ 8, 28).

h Planck constant (§ 22); $\mathbf{h} \equiv d\mathbf{a}/dt$ (§ 23).

I total current (§ 12); I_1, I_2—invariants of electromagnetic field (§ 7); $I(\omega)$ — energy of radiation with frequency ω.

i surface current density.

$\mathscr{J}_i$ generalized currents (§ 34).

j current density; $\mathbf{j}^{\text{ext}}$—density of current produced by external electromotive forces; j_n, j_s—ordinary and superconducting currents (§ 37).

K inertial reference frame, radiance (§ 22), kinetic energy, $\vec{\mathscr{K}}$, $\mathscr{K}^i$—4-dimensional Newtonian force.

k wave number, Boltzmann constant; $\hat{k}$—complex wave number; $\mathbf{k}, \vec{k}$—wave vector.

L self-inductance of a coil; $L_{\alpha\beta}$—induction coefficients; $\mathscr{L}$—Lagrangian.

M magnetization; $\mathbf{M}_0$—residual magnetization; M^{ik}—total 4-dimensional angular momentum.

m magnetic moment; $\tilde{\mathbf{m}}$—angular momentum density (§ 3); $\tilde{m}_{kmj}$—density of 4-dimensional angular momentum (§ 10); m_0—rest mass of a particle.

N mechanical moment of forces; N—Nernst coefficient (§ 34), number of particles (§ 39).

n unit vector of outward normal; n—refractive index (§§ 19, 39); $\hat{n}$—complex refractive index (§ 39).

P electric polarization; $\vec{P}$, P^i—energy-momentum vector; P_λ—canonical momentum of field oscillator (§ 22).

p mechanical momentum density, momentum of a pointlike mass, electric dipole moment (§ 11); p—pressure (§ 22); p_m—magnetic pressure (§ 35); p_λ—canonical momentum of field oscillator (§ 22).

Q quantity of heat; $Q_{\alpha\beta}$—quadrupole moment (§§ 11, 16); Q_α—canonical coordinate of field oscillator (§ 22).

q electric charge; $\mathbf{q}$—heat flux density (§ 34); q_λ—canonical coordinate of field oscillator (§ 22).

R active resistance of linear contour (§ 33); $\mathbf{R}$—radius vector, $\mathbf{R} \equiv \mathbf{r} - \mathbf{r}'$.

r radius vector, $\mathbf{r}'$—radius vector of a source; r_L—Larmor radius; r_{ik}—tensor of resistance of anisotropic medium (§ 34).

S Poynting vector; S—entropy density; $\mathcal{S}$—action integral (§ 9), total entropy (§ 22).

s propagation vector (§ 39); s_{ik}—potential coefficients (§ 30); s_{ijkl}—elasticity coefficients (§ 32).

T temperature, time; T^{ik}—4-dimensional energy-momentum tensor of electromagnetic field; $T_{\alpha\gamma}$—total stress tensor.

t time, t'—source time.

U potential energy (§ 28); internal energy density (§§ 22, 31).

u 3-dimensional velocity; $\vec{u}$, u^i—4-dimensional velocity.

V 3-dimensional volume; electromotive force in a circuit (§ 33).

v 3-dimensional velocity; $\mathbf{v}_\infty$—velocity of electric drift in a medium with infinite electric conductivity (§ 35); $\mathbf{v}_{su}$—velocity of superconducting electrons (§ 37); $\mathbf{v}_{gr}$—group velocity (§ 39).

$\mathcal{W}$ radiative energy.

w total energy density; w_ν—spectral energy density; $\vec{w}$, w^i—4-dimensional velocity.

X_i generalized forces (§ 34).

$\vec{x}$, x^i 4-dimensional radius vector in Minkowski space.

Remark. **§ 15 and Appendix D operate with auxiliary symbols $\vec{p}$, W, B, V, defined differently.**

Greek letters

α dimensional coefficient in the Maxwell equations.
$\beta \equiv v/c$, $\boldsymbol{\beta} \equiv \mathbf{v}/c$.
$\gamma \equiv (1 - \beta^2)^{-1/2}$.
Γ natural width of spectral line; $\tilde{\Gamma}$—total width of spectral line.
δ_i^k Kronecker delta; $\delta(x - \xi)$—delta function.
ε dielectric permittivity of a medium, infinitesimal parameter (§ 27); ε_0—electric constant; ε'—relative dielectric permittivity (§ 29); $\boldsymbol{\varepsilon}$—polarization vector of a plane wave; $\varepsilon_{\alpha\beta\gamma}$—unit pseudoscalar (Levi-Civita symbol).
ζ chemical potential (§ 31); azimuthal angle.
η differential thermo-e.m.f. (§ 34).
ϑ polar angle in spherical system of coordinates.
$\varkappa$ mass density (§§ 35 and 40); $\varkappa \equiv dT/dt'$ (§ 14); $\varkappa_0$—invariant density of rest mass (§ 8).
λ surface charge density, wavelength; λ_L—London penetration depth (§ 37).
μ magnetic permeability of a medium; μ'—relative magnetic permeability (§ 29); μ_0—magnetic constant; $\boldsymbol{\mu}$—magnetic moment due to orbital motion (§ 26).
ν frequency; $\boldsymbol{\nu}$—normal to interface between two media (§ 18).
$\boldsymbol{\Pi}$ Hertz vector (§§ 2, 38); Π—Peltier coefficient (§ 34).
$\vec{\pi}$, π^k 4-momentum of charge.
ρ bulk density of charge, 4-dimensional distance between observer and source (§ 15); ρ_0—invariant charge density.
σ 2-dimensional surface, electric conductivity, Stefan-Boltzmann constant (22.13), scattering cross section (§ 25); σ_{abs}—absorption cross section (§ 25); σ_{kmj}—spin tensor of electromagnetic field (§ 10).
Σ 3-dimensional hypersurface; $\Delta\Sigma$—cross-sectional area of a contour (§ 33).
τ proper time, relaxation time (§ 35); $\boldsymbol{\tau}$—Thompson coefficient (§ 34); τ_{ij}—stress tensor of anisotropic medium (§ 32); $\boldsymbol{\tau}$—double layer density (§ 36).
Φ magnetic flux; $\vec{\Phi}$, Φ^k—4-dimensional potential.
φ scalar potential; $\boldsymbol{\varphi}$—surface force density.
χ_{el} electric susceptibility; χ_m—magnetic susceptibility.
ψ magnetic scalar potential, function defining gauge transformation (§ 2), any of the Cartesian components of a vector (§ 20).
Ω 4-dimensional volume in Minkowski space; $d\Omega$—element of solid angle (identical to $d\omega$ in § 15 and Appendix D).
ω cyclic frequency; ω_L—Larmor frequency (§ 26).
ω_{ij} coefficients of the infinitesimal Lorentz transformation.

CHAPTER 1

THE BASICS OF MAXWELL'S ELECTRODYNAMICS

§ 1. The Maxwell equations. Electromagnetic units

1.1. The experimental and theoretical study of physical phenomena led to the idea of the electromagnetic field as a physical reality (object), something with definite properties. It is created by sources—electric charges, currents, permanent magnets—and is the cause of the interaction of sources. The field created by a source can be measured by the effect it produces on other sources. To define a field quantitatively, it is necessary to measure the force with which the field acts on specific sources, called test sources. A *test source* is a source whose dimensions are negligibly small and whose field is so weak that it does not affect the results of the measurement. Hence a field can be measured at any point in space. Moreover, a field can, generally speaking, be time dependent. The force measured at time t with the help of a test source that is placed at a point with radius vector $\mathbf{r}$ will be denoted $\mathbf{F}(\mathbf{r}, t)$.

The properties of an electromagnetic field manifest themselves in "pure form" when the action of its sources is studied in vacuo. A *material medium* consists of the simplest (but not in the sense of their internal structure!) entities—atoms, electrons, molecules. These entities always possess definite electromagnetic properties. The properties of these entities together with their arrangement in space in relation to each other, and the state of their motion with respect to each other produce a specific reaction of the medium to the "external" electromagnetic field.

From the macroscopic viewpoint, the discreteness of matter can usually be ignored and matter can be described as a continuous distribution of field sources. The distribution may change if there is an electromagnetic field created by outside sources and/or if the thermodynamic properties of the medium change. The electromagnetic properties of material media vary greatly, but all are described, as the reader will soon see, by only two macroscopic quantities: electric polarization and magnetization.

The fundamental laws of the electromagnetic field with due regard for the macroscopic properties of material media are formulated mathematically in the Maxwell equations, which will be studied in this section. This very general formalism must, if possible, be independent of any specific assumptions about the microscopic structure of the media. It stands to reason that one of the main tasks of a physical theory is to explain observed facts from the microscopic viewpoint. More than that, it is the microscopic theory that often makes it possible to predict new physical phenomena and ways to observe them. However, the phenomenological method of description, our choice for this book, has its advantages. It uses none but those characteristics of phenomena that can be measured, at least in principle, by macroscopic instruments. Any microscopic theory, for its part, must inevitably lead to definite conclusions about the phenomenological characteristics and foretell and explain their behavior. This is true not only of the classical electron theory but of the present-day quantum theory of matter. To quote Niels Bohr,

> ...the unambiguous interpretation of any measurement must be essentially framed in terms of the classical physics theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time.[1]

Any mathematical reflection of the laws of nature, including the Maxwell equations, must be invariant with respect to certain groups of transformations of the physical quantities interconnected by this reflection. Above all, the principle of relativity must hold, that is, with fixed initial and boundary conditions the Maxwell equations must bring the same results in any inertial reference frame. The very concept of the electromagnetic field as a physical object, which does not depend on the choice of the inertial reference frame, may be defined only when the relativity principle is explicitly taken into account. In each given inertial frame the electromagnetic field "splits" into two fields, quite distinct in their properties, an electric and a magnetic. It is these fields that are measured. We will leave the study of the relativity principle and its corollaries for Chapter 2 and confine ourselves here to examining the Maxwell equations in an arbitrary inertial frame. In an inertial reference frame the equations must be independent of the orientation of the spatial axes. We will see that the Maxwell equations are indeed relations between three-dimensional vectors (3-vectors), so that the invariance with respect to the rotations of the spatial axes in three-dimensional space does hold.[2]

Thus, in what follows we will establish the main laws governing electric and magnetic fields in an inertial reference frame, which we

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[1] N. Bohr: "Maxwell and modern theoretical physics", *Nature*, **128** (1931), p. 692.

[2] A brief outline of the elements of vector analysis is given in Appendices A and B.

choose arbitrarily but which, after the choice is made, remains fixed. (An inertial reference frame is one of a class of frames which move with respect to each other with constant velocities.) But first we must agree on the units of measurement and on the dimensions of the quantities used in electrodynamics. This problem will also be considered.

1.2. We assume first that the field under study is *static*. By definition, in such a field the force created by a given distribution of sources and acting on the test source is time independent, that is, it can be represented by a function **F** (**r**) of the three-dimensional radius vector **r**. (The vector gives the position of the pointlike test source in space.) Static electric and magnetic fields are generated by sources with different physical properties and have different structures. To determine them it is important to consider the case when these sources are themselves pointlike. Here also the fields may have different symmetry, which means that the functions **F** (**r**) may differ, but at least the electric field on the one hand and the magnetic on the other are created by sources that ensure the simplest possible configurations of the fields. For the electric field this source is the point charge. In vacuo the field is spherically symmetric.

Experiment shows that if various pointlike charges are placed at some point in space near an arbitrary pointlike source (of an electric field), then to each of these charges there can be assigned a number q such that for any two such charges the following relationship holds: $F_\alpha^{(1)} : F_\alpha^{(2)} = q_1 : q_2$ $(\alpha = 1, 2, 3)$. It so happens that the number q (called *quantity of charge*, or simply *charge*) characterizes the physical properties of the point charge under consideration: it does not depend either on the properties of the source that produces the field acting on the charge or on the point in space where the charge is situated. For different point charges this "label" can take on positive or negative values.

Such a definition of the quantity of charge says nothing about the dimensions it must have. It is obvious, however, that any charge can be chosen as the unit charge. The force that a given source of an electric field exerts on a unit point charge is called the *electric field strength* **E** (**r**):

$$\mathbf{F}(\mathbf{r}) = q\mathbf{E}(\mathbf{r}) \tag{1.1}$$

The results of experimental studies of the properties of static electric fields generated by a variaty of sources can be expressed by equations involving vector **E** (**r**). We will now turn to these equations.

First, let s be an arbitrary closed circuit. Then

$$\oint_s \mathbf{E} \cdot d\mathbf{s} = 0 \tag{1.2}$$

In other words, when a point charge is moved along an open circuit in a static electric field, the work spent on, or produced by, transferring the charge does not depend on the form of the circuit but only on its initial and final points. Equation (B.25) makes it possible to find a differential equation for the vector function $\mathbf{E}(\mathbf{r})$:

$$\operatorname{curl} \mathbf{E} = 0 \tag{1.3}$$

($\mathbf{E}(\mathbf{r})$ must satisfy this equation at any point in space). Equations (1.2) and (1.3) hold for a static field both when the charges are in vacuo and when material media are present.

If the source of the electric field is not pointlike, the field can, in general, be represented as produced by a continuous distribution of charges with a density $\rho(\mathbf{r})$.[3] The total electric charge q of the source inside volume V is then given by the equation

$$q = \int_V \rho(\mathbf{r})\, dV \tag{1.4}$$

The field $\mathbf{E}$ created by a charge q in vacuo satisfies

$$\varepsilon_0 \oint \mathbf{E} \cdot \mathbf{n}\, d\sigma = q \tag{1.5}$$

irrespective of how this charge is distributed in a volume V. (Here σ is any closed two-dimensional surface bounding V, and $\mathbf{n}$ is the unit outward normal at the area element $d\sigma$.) The factor ε_0 is introduced so as to account for the different dimensions and units of measurements of electric quantities.

The definition (1.1) for the electric field strength shows that the product of the dimensions of q and $\mathbf{E}$ must be equal to the dimensions of force $\mathbf{F}$. If we multiply both sides of (1.5) by a quantity q' with the dimensions of charge, we get a condition which the dimensions of charge must satisfy (the brackets denote the dimensions of the quantity inside them):

$$[q]^2 = [F]\,[L]^2\,[\varepsilon_0] \tag{1.6}$$

Later in our exposition the reader will see that the choice of dimensions for ε_0, called the *electric constant* (also the *permittivity of empty space*), may be different, and so may be the dimensions of charge, $[q]$. In certain experiments (for instance, involving electrolysis), the basic unit (standard) of electric charge may be established, at least in principle, independently of the measurement of the field strength by mechanical means. The dimensions of ε_0 can then be determined via (1.6), and its numerical value will depend on the choice of the basic units of mechanical quantities and electric charge.

[3] In what follows we will also consider charge distributions over two-dimensional surfaces with a surface density λ; we will not dwell on this possibility here, since it does not alter our reasoning to any extent.