CALCULATOR-BASED TRIGONOMETRY

WITH APPLICATIONS

GREENLEAF

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PREFACE

More than 2,000 years ago astronomers began to make tables of numbers to be used in computing the positions of the sun, moon, and planets, and in making astronomical and astrological predictions. These tables were the precursors of modern tables of trigonometric functions, which were, until recently, the primary tool for making trigonometric computations. Trigonometry developed as the science of *solving* triangles, of determining all of the sides and angles when some of them are given or measured. Trigonometric methods were increasingly applied in surveying, navigation, architecture, engineering, and physics, as well as in astronomy. They are all the more useful today because the necessary computations can be made quickly and easily with a hand-held scientific calculator. This subject is known as *classical trigonometry* and is covered in the first part of this book, Chapters 1–3.

About three hundred years ago the English mathematician and scientist Sir Isaac Newton (1642–1727) developed the calculus. This achievement is often considered the biggest "breakthrough" in the history of mathematics. Since then, mathematicians have combined the techniques of trigonometry and calculus to study periodic (repeating) phenomena such as vibrations and waves. This aspect of trigonometry, known as *analytic trigonometry*, forms the basis for the mathematics of sound, light, heat, and fluids, and now provides an indispensable background for the study of calculus. Analytic trigonometry, based on circles rather than on triangles, is covered in the second part of this text, Chapters 4–6.

Both classical and analytic trigonometry are used in many places in science and higher mathematics. Both should be studied, and it is possible to start with either area. There are advantages to each approach. In beginning with classical trigonometry, I follow the sound pedagogical principle of moving from the more concrete to the more abstract.

Classical trigonometry formerly required extensive use of tables and many tedious hand computations. But with the arrival of the hand-held scientific calculator as a standard classroom tool, the situation is dramatically changed. Serious problems can now be solved quickly, and the subject even becomes enjoyable.

The third part of the book, Chapters 7 and 8, considers the related topics of logarithms, exponential functions, and complex numbers.

This book is designed for flexibility of use. The following table shows the division between "core" material, which would generally be taught in any trigonometry class, and those sections and chapters that might be omitted or considered optional. Since the latter material can be added to the core in many different ways, the book is easily adapted to courses of one or two quarters, or of one semester.

Chapter	*Core Material*	*Additional Material*
1. Right Triangles	1.1–1.5	1.6
2. Polar Angles	2.1–2.4	2.5, 2.6
3. Solving Oblique Triangles	3.1, 3.2	3.3–3.6
4. Introduction to Analytic Trigonometry	4.1–4.7	4.8, 4.9
5. Graphical Methods	5.1, 5.2	5.3–5.5
6. Trigonometric Identities and Equations	6.1–6.4, 6.6	6.5, 6.7, 6.8
7. Logarithmic and Exponential Functions		All sections
8. Complex Numbers		All sections

The following specific aspects of this text may distinguish it from others:

1. It is assumed that every student possesses a scientific calculator. Choice of a calculator is discussed in the first section of the book, "Calculators."

For many students this will be the first time that they have used a calculator to solve complex problems. Although I assume a familiarity with the basic operation of a calculator, I often address the question of correct and efficient use of a calculator. Many worked-out examples include suggested sequences of keystrokes (programs) for both *algebraic* (AOS) and *reverse Polish* (RPN) calculators.

Formula entry (FOR) calculators, which allow formulas to be entered and examined before being evaluated, are the easiest to use. Although such calculators are more expensive than AOS models, I have found that those students who purchase them consider them to be a worthwhile investment.

Tables of trigonometric functions, like slide rules, may have instructional value but are no longer used in solving problems. Tables are discussed but are not used systematically. There are no tables in the back of the book.

2. Many texts obscure the differences between *classical* and *analytic* trigonometry. The organization of this book, however, emphasizes them. In addition to conceptual clarity, this approach has other advantages. For example, angles are measured in degrees in the first part of the book, while radian measure is employed consistently in the last two parts. Also, I concentrate on the sine, cosine, and tangent in Chapters 1–3. The cotangent, secant, and cosecant, which play no role in the calculator solution of triangles, are introduced in Chapter 4.

3. To emphasize that trigonometry is useful mathematics, one or two *applications* sections are included in every chapter (except Chapter 8). With a calculator it is often possible to solve realistic problems quite easily. Classical trigonometry applications include refraction, navigation, astronomical measurement, and force vectors. Applications of analytic trigonometry include satellite orbits and periods, vibrating springs, alternating current, and sound and radio waves. Chapter 7 contains a section on exponential growth and decay, applied to bacterial growth, radioactive decay, and compound interest.

If there is not enough time to cover the applications sections in class, they can be used for extra credit assignments or simply to illustrate the wide range of uses of trigonometry.

4. Because calculators generally display many nonsignificant digits, the questions of *accuracy* and *rounding* acquire new importance. Systematic discussion is properly left to courses in numerical analysis, but the basic concepts of exact and approximate numbers, and of significant digits and rounding, are covered in Section 1.5. In order to concentrate on trigonometry proper, I generally avoid questions of accuracy by simply specifying the way in which the result should be rounded. Answers from calculators are generally presented first in unrounded form (usually to four decimal places) and then the rounding is indicated by an arrow:

$$\sin 21.3^\circ = 0.3633 \rightarrow 0.36\,.$$

I often use the equality sign "=" for approximate equality. The value of $\sin 21.3^\circ$ is only approximately 0.3633.

5. *Polar coordinates* are introduced in Chapter 2, and are used systematically throughout the book. I describe the relationship between polar and rectangular coordinates as lying at the heart of trigonometry.

6. The standard treatment of "reference angles" is appropriate for use with tables but is not suited for use with calculators. Note the following way in which using a calculator differs from using tables. A calculator finds the trigonometric function of any angle without having to first find the reference angle, as must be done for tables. And, the calculator provides the correct sign for the value.

7. The *inverse trigonometric functions* are first introduced informally in Chapters 1 and 2 for use in solving triangles. The systematic treatment of the inverse trigonometric functions that follows in Chapter 4 is simple and concrete. First the domains of the trigonometric functions are restricted to make them increasing or decreasing, and hence, one-to-one. Inverses are defined only for one-to-one functions, and the concept "inverse relation" is not introduced.

8. Although the material in the last two chapters is not strictly part of trigonometry, there are good reasons for its inclusion. Logarithms are no longer used for trigonometric computations, but it is appropriate that all of the elementary "transcendental" functions be studied together. These are the trigonometric functions, the inverse trigonometric functions, the exponential functions, and the logarithmic functions.

I include complex numbers because of the key role that trigonometry plays in this field of mathematics. The rule for multiplying complex numbers in polar form is one of the most important applications of trigonometry in higher mathematics.

9. The objective of the book is not to produce a "mindless button pusher," but rather, someone with a solid understanding of trigonometry that can be applied in other contexts. Hence, many examples and exercises *cannot* be worked with a calculator.

Some numerical examples and exercises ask for answers in exact form. Such problems focus attention on the process by which the answer is obtained and on the relationship between the answer and the given data.

The construction of accurate graphs is emphasized, particularly in Chapter 5. Calculators make it possible to draw better graphs since the student can easily generate accurate tables of coordinates to be plotted.

Many exercises and examples deal with proving identities. While a calculator cannot be used to prove an identity, it can be used to show quickly that an equation is *not* an identity. Also, a calculator check *can* show that a given equation is likely to be an identity.

Some examples and exercises in Chapters 1–3 ask for a geometric construction with ruler and compass, and the answer is obtained by direct measurement of the constructed figure. Although these exercises may easily be omitted, I feel that solving a few problems in this way is very helpful in developing geometric intuition.

Most exercises can be solved by methods similar to those used in fully worked-out examples. The end of the solution of an example is indicated by ■ . Particularly challenging exercises are starred. (Two asterisks indicate a greater level of difficulty than a single asterisk.) Unstarred exercises are generally paired, with an odd-numbered exercise similar to the succeeding even-numbered one. Answers to most odd-numbered exercises are given in the answer section at the back of the book.

10. Each chapter ends with a "Chapter Review," a set of exercises that is designed to help the student organize and refine his or her understanding of the chapter. Chapter reviews are accompanied by "extra credit" computer projects for students who have programming skills and access to a computer.

The availability of the scientific calculator is transforming "trig" from everyone's least favorite subject into an interesting and exciting area of mathematics that has much room for student experimentation using a new tool, the calculator. It is my hope that this book will contribute to the transformation of trigonometry into a popular subject. Trig is fun! And what's more, it is very useful.

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CONTENTS

C A L C U L A T O R S

CHOOSING A SCIENTIFIC CALCULATOR

To study trigonometry from this text you will need a scientific calculator. It is essential that your calculator be able to compute the trigonometric functions and their inverses, with angles measured in both degrees and radians. On the keypad of the calculator, look for buttons labeled

[sin] [cos] [tan]

Then carefully check the calculator's manual to make sure that it has the desired capabilities. For Chapter 7 you will also need to be able to compute logarithms and exponentials.

The most common type of calculator is the AOS (Algebraic Operating System) type. Most of the less expensive models are of this type. The sequence in which buttons are pushed generally corresponds to standard algebraic notation.

It is more difficult to learn to use an RPN (Reverse Polish Notation) calculator. The order of entry is often reversed from that of algebra, which accounts for the "reverse" in the title. This notation originated in a system of logic developed by Polish mathematicians in the 1920s. Many users insist that the RPN system is easier to use, once you become accustomed to it. Most RPN calculators are made by the Hewlett-Packard Company.

To add 2 and 5 on an AOS calculator you would enter

2 [+] 5 [=]

On an RPN calculator you would press

2 [enter] 5 [+]

Both AOS and RPN calculators carry out the computations as the keys are pressed. With a FOR (formula entry) calculator, the formula to be evaluated is displayed as it is entered, and then the entire computation is carried out when [=] is pressed. This type is a more recent development, made possible by the availability of low-cost LCDs (liquid crystal displays). While the formula is being entered, it is possible to correct and edit it. After the answer has been obtained, the [PB] (playback) key restores the formula to the display, where it can be checked or modified. This type of calculator is *by far* the easiest to use for the evaluation of complicated expressions. Speed is increased and the probability of error is greatly reduced.

Other calculators have capabilities that go beyond those mentioned here. Programmable calculators have been with us for more than a decade now, but lately they have been largely supplanted by personal computers. Recently, calculators with graphic displays have been introduced. You might want to look at some of these; they have many uses. For this course we feel that the formula-entry calculator is the best choice if one can spend more than the minimum amount of money. But even the cheapest scientific calculator will do.

You will probably use your scientific calculator for many years. Make a careful choice to obtain the instrument most suited to your needs.

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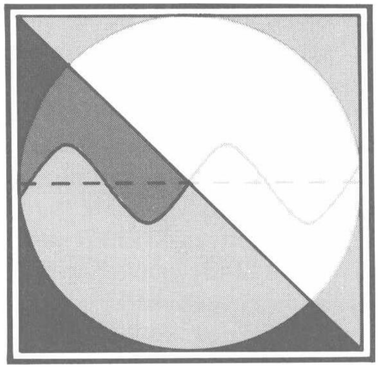

RIGHT TRIANGLES

1.1 SOLVING TRIANGLES

Trigonometry gives us the tools to solve triangles. This chapter introduces trigonometry through the study of right triangles, and in this section we review the geometry of triangles that will be used.

DIRECT AND INDIRECT MEASUREMENTS

How tall are you? To answer this question you might use a tape measure or a yardstick to make a *direct measurement*. The measurement is *direct* because no computation is required. A measurement is called *indirect* if computation and reasoning are used to obtain it. The width of a lake, the height of a mountain or of a satellite orbiting the earth, the distance from the earth to the moon—all must be measured indirectly, generally by "solving" some triangle. To *solve* a triangle, you must find, directly or indirectly, all of its sides and angles. The main mathematical tool for solving triangles is trigonometry. Let us consider an example.

Suppose that you are a surveyor making a map of little Lake Wobegon (Figure 1.1). You need an accurate measurement of the distance between the points A and B. Since they are on opposite sides of the lake, your measurement must be indirect. You first choose a point C from which you can see both A and B, and from which you make direct measurements of the distances to A, 475 yards, and to B, 207 yards. At C you use a "surveyor's compass" to measure directly the angle between A and B and find it to be 83.2°. Now, with your knowledge of trigonometry (the Law of Cosines from Chapter 3), and your scientific calculator, you quickly and easily compute the distance between A and B. To the nearest yard, it is 508 yards from A to B.

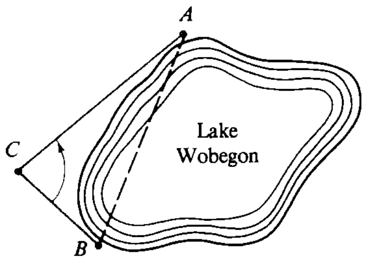

FIGURE 1.1

THE SIX PARTS OF A TRIANGLE

The three sides and three angles of a triangle are often known as the *parts* of the triangle. You have solved a triangle when you have measured, directly or indirectly, all of its parts. We will commonly label the sides a, b, and c, and the angles A, B, and C, as in Figure 1.2. Note that side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C. Sometimes all of the parts of a triangle can be measured directly. For instance, you could measure all of the parts of the triangle of Figure 1.2 with a ruler and protractor.

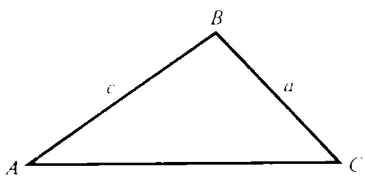

FIGURE 1.2 A triangle has six parts: A, B, C, a, b, c.

UNITS

Whenever an angle or a length is measured, a *unit* must first be decided upon. Two systems of units are commonly used for length. There are the units of the *U.S. Customary System:* inch, foot, yard, and mile. These units originated in England and are the same as the units of length of the British Imperial System. In addition, there are the *metric* units: meter, centimeter, kilometer, and others, which are used in scientific work and in daily life throughout much of the world. There are also special units of length such as the nautical mile, used in navigation, and the light-year, used in astronomy. In this book we will use both customary and metric units, as seems appropriate.

Often no unit of length is indicated, which means that the choice of unit is irrelevant for that problem.

There are two units that are commonly used for measuring angles, the *degree* and the *radian*. Degrees are generally used for measuring the angles of triangles. We will use degrees in the first part of this book (Chapters 1–3).

A degree is $\frac{1}{90}$ of a right angle. Hence there are 180 degrees in a straight angle and 360 degrees in a full circle (see Figure 1.3). For

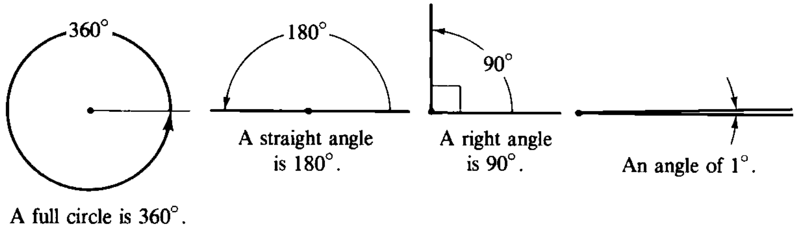

FIGURE 1.3

more accuracy, the degree has traditionally been divided into 60 *minutes,* and a minute into 60 *seconds:*

$$1^\circ = 60' = 3600'' \qquad 1' = 60'' = \frac{1^\circ}{60} \qquad 1'' = \frac{1'}{60} = \frac{1^\circ}{60^2}$$

While minutes and seconds are still extensively used in applications such as surveying and astronomy, in this book we follow the modern practice of using decimal degrees, which are much easier to use with scientific calculators. The following examples show how conversions can be made between these two systems.

EXAMPLE 1

Convert 58°39′2″ into decimal degrees. Round to three decimal places.

Solution

First convert this to the exact form

$$\left(58 + \frac{39}{60} + \frac{2}{60^2}\right)^\circ \tag{1}$$

which can also be written as

$$\left[58 + \frac{1}{60} \times \left(39 + \frac{2}{60}\right)\right]^\circ \tag{2}$$

Now evaluate either of these expressions with a calculator. It is helpful to break longer computations into a sequence of shorter steps in each of which a particular quantity is computed and displayed, as in Table 1.1. Sequences of keystrokes for evaluating (2) are given for AOS and RPN calculators. Such a sequence of operations is called a *program.*

TABLE 1.1 Programs for converting 58°39′2″ to decimal degrees

AOS	RPN	*Quantity*	*Display*
2 [÷] 60	2 [enter] 60		
[+] 39 [=]	[÷] 39 [+]	39 + 2/60	39.0333
[÷] 60 [+] 58 [=]	60 [÷] 58 [+]	$58 + 39/60 + 2/60^2$	58.6506

The programs of Table 1.1 follow the important general strategy of starting the computation inside as many parentheses as possible.

The answer on your calculator should be

$$58.6506° \rightarrow 58.651°$$

where the arrow represents rounding from four to three decimal places. If you consistently get a different answer, you must "debug" your program. That is, you must discover why it is not producing the correct result, modify it, and try again. ■

You may also find it helpful to practice executing a program on your calculator. A calculator is like a typewriter in that you need to press the right keys in the right order, quickly and accurately. But while a single typing error often does not make much difference, a single error in executing your program can make total nonsense of the calculator result.

EXAMPLE 2

Convert 85.732° to DMS (degree, minute, second) form.

Solution

The number of whole degrees is 85, and we need to convert 0.732° to minutes and seconds. Since there are 60 minutes in a degree,

$$0.732° = (0.732 \times 60)' = 43.9200'$$

In the same way, 0.9200′ converts to 55.2000″, which rounds to 55″, yielding the answer

$$85.732° = 85°43'55''$$

Suggested programs are given in Table 1.2. You can shorten these programs if your calculator has a key for taking the fractional part of the number on display. ■

Your calculator may have special keys for converting between decimal degrees and degrees, minutes, and seconds (look for a [DMS]

TABLE 1.2 Programs for converting 85.732° to degrees, minutes, and seconds

AOS	RPN	*Quantity*	*Display*
0.732 [×] 60 [=]	0.732 [enter]		
	60 [×]	0.732°	43.9200′
[−] 43 [=]	43 [−] 60 [×]		
[×] 60 [=]		0.9200′	55.2000″