# Fundamentals of the Finite Element Method for Heat and Fluid Flow

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# Fundamentals of the Finite Element Method for Heat and Fluid Flow

To

Celia
Sujatha
and Uma

# Preface

In this text, we provide the readers with the fundamentals of the finite element method for heat and fluid flow problems. Most of the other available texts concentrate either on conduction heat transfer or the fluid flow aspects of heat transfer. We have combined the two to provide a comprehensive text for heat transfer engineers and scientists who would like to pursue a finite element–based heat transfer analysis. This text is suitable for senior undergraduate students, postgraduate students, engineers and scientists.

The first three chapters of the book deal with the essential fundamentals of both the heat conduction and the finite element method. The first chapter deals with the fundamentals of energy balance and the standard derivation of the relevant equations for a heat conduction analysis. Chapter 2 deals with basic discrete systems, which are the fundamentals for the finite element method. The discrete system analysis is supported with a variety of simple heat transfer and fluid flow problems. The third chapter gives a complete account of the finite element method and its relevant history. Several examples and exercises included in Chapter 3 give the reader a full account of the theory and practice associated with the finite element method.

The application of the finite element method to heat conduction problems are discussed in detail in Chapters 4, 5 and 6. The conduction analysis starts with a simple one-dimensional steady state heat conduction in Chapter 4 and is extended to multi-dimensions in Chapter 5. Chapter 6 gives the transient solution procedures for heat conduction problems.

Chapters 7 and 8 deal with heat transfer by convection. In Chapter 7, heat transfer, aided by the movement of a single-phase fluid, is discussed in detail. All the relevant differential equations are derived from first principles. All the three types of convection modes, forced, mixed and natural convection, are discussed in detail. Examples and comparisons are provided to support the accuracy and flexibility of the finite element method. In Chapter 8, convection heat transfer is extended to flow in porous media. Some examples and comparisons provide the readers an opportunity to access the accuracy of the methods employed.

In Chapter 9, we have provided the readers with several examples, both benchmark and application problems of heat transfer and fluid flow. The systematic approach of problem solving is discussed in detail. Finally, Chapter 10 briefly introduces the topic of computer implementation. The readers will be able to download the two-dimensional source codes from the authors' web sites. They will also be able to analyse both two-dimensional heat conduction and heat convection studies on unstructured meshes using the downloaded programs.

Many people helped either directly or indirectly during the preparation of this text. In particular, the authors wish to thank Professors N.P. Weatherill, K. Morgan and O. Hassan of the University of Wales Swansea for allowing us to use the 3-D mesh generator in some of the examples provided in this book. Dr Nithiarasu also acknowledges Dr N. Massarotti of the University of Cassino, Italy, and Dr J.S. Mathur of the National Aeronautical Laboratories, India, for their help in producing some of the 3-D results presented in this text. Professor Seetharamu acknowledges Professor Ahmed Yusoff Hassan, Associate Professor Zainal Alimuddin and Dr Zaidi Md Ripin of the School of Mechanical Engineering, Universiti Sains Malaysia for their moral support.

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# Contents

# 1

# Introduction

## 1.1 Importance of Heat Transfer

The subject of heat transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested in knowing the mechanisms of heat transfer involved in the operation of equipment, for example boilers, condensers, air pre-heaters, economizers, and so on, in a thermal power plant in order to improve their performance. Nuclear power plants require precise information on heat transfer, as safe operation is an important factor in their design. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage due to hot spots, developed by improper heat transfer design, in electric motors, generators and transformers. An *electronic engineer* is interested in knowing the efficient methods of heat dissipation from chips and semiconductor devices so that they can operate within safe operating temperatures. A *computer hardware engineer* is interested in knowing the cooling requirements of circuit boards, as the miniaturization of computing devices is advancing at a rapid rate. *Chemical engineers* are interested in heat transfer processes in various chemical reactions. A *metallurgical engineer* would be interested in knowing the rate of heat transfer required for a particular heat treatment process, for example, the rate of cooling in a casting process has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. A *civil engineer* would need to be aware of the thermal stresses developed in quick-setting concrete, the effect of heat and mass transfer on building and building materials and also the effect of heat on nuclear containment, and so on. An *environmental engineer* is concerned with the effect of heat on the dispersion of pollutants in air, diffusion of pollutants in soils, thermal pollution in lakes and seas and their impact on life. The global, thermal changes and associated problems caused by *El Nino* are very well known phenomena, in which energy transfer in the form of heat exists.

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The previously-mentioned examples are only a sample of heat transfer applications to name but a few. The solar system and the associated energy transfer are the principal factors for existence of life on earth. It is not untrue to say that it is extremely difficult, often impossible, to avoid some form of heat transfer in any process on earth.

The study of heat transfer provides economical and efficient solutions for critical problems encountered in many engineering items of equipment. For example, we can consider the development of heat pipes that can transport heat at a much greater rate than copper or silver rods of the same dimensions, even at almost isothermal conditions. The development of present day gas turbine blades, in which the gas temperature exceeds the melting point of the material of the blade, is possible by providing efficient cooling systems and is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat flux of the order occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is again a success story for heat transfer analysis.

Although there are many successful heat transfer designs, further developments are still necessary in order to increase the life span and efficiency of the many devices discussed previously, which can lead to many more new inventions. Also, if we are to protect our environment, it is essential to understand the many heat transfer processes involved and, if necessary, to take appropriate action.

## 1.2 Heat Transfer Modes

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to a temperature difference (Bejan 1993; Holman 1989; Incropera and Dewitt 1990; Sukhatme 1992). The three modes of heat transfer are

1. Conduction
2. Convection
3. Radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another, without the actual motion of the molecules, or because of the motion of the free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion, and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another, due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called *heat transfer* by convection. Convection may be free, forced or mixed. When fluid motion occurs because of a density variation caused by temperature differences, the situation is said to be a free, or natural, convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being one of forced convection. A mixed convection state is one in which both natural and forced convections are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode that does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part is transmitted and the remaining part is absorbed.

All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are identifying the significant modes and deciding whether the heat transferred by other modes can be neglected.

## 1.3 The Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time and for that we require the use of rate equations.

For heat conduction, the rate equation is known as *Fourier's law*, which is expressed for one dimension as

$$q_x = -k\frac{\mathrm{d}T}{\mathrm{d}x} \tag{1.1}$$

where $q_x$ is the heat flux in the $x$ direction (W/m$^2$); $k$ is the thermal conductivity (W/mK, a property of material, see Table 1.1)and $\mathrm{d}T/\mathrm{d}x$ is the temperature gradient (K/m).

For convective heat transfer, the rate equation is given by *Newton's law of cooling* as

$$q = h(T_\mathrm{w} - T_\mathrm{a}) \tag{1.2}$$

where $q$ is the convective heat flux; (W/m$^2$); $(T_\mathrm{w} - T_\mathrm{a})$ is the temperature difference between the wall and the fluid and $h$ is the convection heat transfer coefficient, (W/m$^2$K) (film coefficient, see Table 1.2).

The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids. We assume $h$ to be known in many such problems. In the analysis of thermal systems, one can again assume an appropriate $h$ if not available (e.g., heat exchangers, combustion chambers, etc.). However, if required, $h$ can be determined via suitable experiments, although this is a difficult option.

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan–Boltzmann Law*, that is,

$$q = \sigma {T_\mathrm{w}}^4 \tag{1.3}$$

where $q$ is the radiative heat flux, (W/m$^2$); $\sigma$ is the Stefan–Boltzmann constant ($5.669 \times 10^{-8}$), in W/m$^2$K$^4$ and $T_\mathrm{w}$ is the surface temperature, (K).

The heat flux emitted by a real surface is less than that of a black surface and is given by

$$q = \epsilon\sigma {T_\mathrm{w}}^4 \tag{1.4}$$

Table 1.1 Typical values of thermal conductivity of some materials in W/mK at 20 °C

| Material | Thermal conductivity |
|---|---|
| *Metals*: | |
| Pure silver | 410 |
| Pure copper | 385 |
| Pure aluminium | 200 |
| Pure iron | 73 |
| *Alloys*: | |
| Stainless steel (18% Cr, 8% Ni) | 16 |
| Aluminium alloy (4.5% Cr) | 168 |
| *Non metals*: | |
| Plastics | 0.6 |
| Wood | 0.2 |
| *Liquid*: | |
| Water | 0.6 |
| *Gases*: | |
| Dry air | 0.025 (at atmospheric pressure) |

Table 1.2 Typical values of heat transfer coefficient in W/m$^2$K

| | |
|---|---|
| Gases (stagnant) | 15 |
| Gases (flowing) | 15–250 |
| Liquids (stagnant) | 100 |
| Liquids (flowing) | 100–2000 |
| Boiling liquids | 2000–35,000 |
| Condensing vapours | 2000–25,000 |

where $\epsilon$ is the radiative property of the surface and is referred to as the *emissivity*. The net radiant energy exchange between any two surfaces 1 and 2 is given by

$$Q = F_\epsilon F_G \sigma A_1 (T_1^4 - T_2^4) \tag{1.5}$$

where $F_\epsilon$ is a factor that takes into account the nature of the two radiating surfaces; $F_G$ is a factor that takes into account the geometric orientation of the two radiating surfaces and $A_1$ is the area of surface 1.

When a heat transfer surface, at temperature $T_1$, is completely enclosed by a much larger surface at temperature $T_2$, the net radiant exchange can be calculated by

$$Q = qA_1 = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4) \tag{1.6}$$

With respect to the laws of thermodynamics, only the first law is of interest in heat transfer problems. The increase of energy in a system is equal to the difference between the energy transfer by heat to the system and the energy transfer by work done on the surroundings by the system, that is,

$$\mathrm{d}E = \mathrm{d}Q - \mathrm{d}W \tag{1.7}$$

where $Q$ is the total heat entering the system and $W$ is the work done on the surroundings. Since we are interested in the rate of energy transfer in heat transfer processes, we can restate the first law of thermodynamics as

'The rate of increase of the energy of the system is equal to the difference between the rate at which energy enters the system and the rate at which the system does work on the surroundings', that is,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}t} - \frac{\mathrm{d}W}{\mathrm{d}t} \tag{1.8}$$

where $t$ is the time.

## 1.4 Formulation of Heat Transfer Problems

In analysing a thermal system, the engineer should be able to identify the relevant heat transfer processes and only then can the system behaviour be properly quantified. In this section, some typical heat transfer problems are formulated by identifying appropriate heat transfer mechanisms.

### 1.4.1 Heat transfer from a plate exposed to solar heat flux

Consider a plate of size $L \times B \times d$ exposed to a solar flux of intensity $q_{\mathrm{S}}$, as shown in Figure 1.1. In many solar applications such as a solar water heater, solar cooker and so on, the temperature of the plate is a function of time. The plate loses heat by convection and radiation to the ambient air, which is at a temperature $T_{\mathrm{a}}$. Some heat flows through the plate and is convected to the bottom side. We shall apply the law of conservation of energy to derive an equation, the solution of which gives the temperature distribution of the plate with respect to time.

*Heat entering the top surface of the plate:*

$$q_{\mathrm{S}} A_{\mathrm{T}} \tag{1.9}$$

*Heat loss from the plate to surroundings:*

Top surface:

$$hA_{\mathrm{T}}(T - T_{\mathrm{a}}) + \epsilon\sigma A_{\mathrm{T}}(T^4 - T_{\mathrm{a}}^4) \tag{1.10}$$

Side surface:

$$hA_{\mathrm{S}}(T - T_{\mathrm{a}}) + \epsilon\sigma A_{\mathrm{S}}(T^4 - T_{\mathrm{a}}^4) \tag{1.11}$$