

**FRANCIS J.
MUELLER**

**SECOND
EDITION**

ARITHMETIC

ITS STRUCTURE AND CONCEPTS

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By Francis J. Mueller

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PRENTICE - HALL, INC. ENGLEWOOD CLIFFS, N.J.

TO MY WIFE

PREFACE

The singular objective of both the previous and current editions of this book has been to broaden and deepen the reader's appreciation of the oldest and most fundamental branch of mathematics, arithmetic. However, those who were familiar with the previous edition will find little in the present one that is exactly the same. All the mathematics is not new, of course; but the context, the framework in which it is presented, is new—adapted to reflect the swiftly changing face of mathematics instruction.

In the process of constructing this revision I have studied the latest modern elementary textbook series and have weighed carefully the writings and pronouncements of such important professional groups as the National Council of Teachers of Mathematics, the School Mathematics Study Group, the Committee on the Undergraduate Program in Mathematics of The Mathematical Association of America, and The Report of The Cambridge Conference on School Mathematics. While there exists no unanimity among these forces certain to influence tomorrow's curriculum in elementary mathematics, they do seem to point in the same general direction. This book, then, is one author's understanding of where that direction lies, and what others need to know to move along its course.

As with the first edition, I have intended the book to be primarily an instrument of instruction. The selection and blending of modern and traditional material, the moderate path and tempo of its development, the careful, simplified writing, the many illustrations and examples, the refined, yet challenging set of problems at the end of each unit, and the solutions to many of these problems at the back of the book—all have been retained in this edition and, to the best of my ability, improved.

Now, as before, users of the book will most likely include college students preparing for the absorbing life of teaching, practicing teachers, high school or junior college students in quest of a more mature understanding of arithmetic, and, perhaps more frequently now, perplexed parents mystified by their youngster's "new fangled" mathematics.

The manuscript for this revision had the benefit of a critique by Dr. J. Fred Weaver of Boston University, to whom I am indebted. As with the previous edition, the considerable patience of my wife and children during the writing and editing deserves recognition as a material contribution to whatever good purpose this book may serve.

F. J. M.

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