


volume 19

lecture notes in pure and applied mathematics



nonlinear functional analysis
and differential equations

Lamberto Cesari
Rangachary Kannan
Jerry D. Schuur

Proceedings of the Michigan State University Conference

NONLINEAR FUNCTIONAL ANALYSIS AND DIFFERENTIAL EQUATIONS

Edited by

LAMBERTO CESARI

*Department of Mathematics
University of Michigan
Ann Arbor, Michigan*

RANGACHARY KANNAN

*Department of Mathematical Sciences
University of Missouri
St. Louis, Missouri*

JERRY D. SCHUUR

*Department of Mathematics
Michigan State University
East Lansing, Michigan*

MARCEL DEKKER, INC. New York and Basel

COPYRIGHT © 1976 by MARCEL DEKKER, INC. ALL RIGHTS RESERVED.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage and retrieval system, without permission in writing from the publisher

MARCEL DEKKER, INC.

270 Madison Avenue, New York, New York 10016

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 76-1797

ISBN: 0-8247-6452-8

Current printing (last digit):
10 9 8 7 6 5 4 3 2

PRINTED IN THE UNITED STATES OF AMERICA

FOREWORD

In recent years there has been considerable research in nonlinear functional analysis and its applications to differential equations. Hence it seemed appropriate, in the spring of 1975, to plan a conference which would bring together workers in this field to exchange thoughts, lay out a view of the present state of the field, and explore areas of future research.

This conference was held June 9 to 12, 1975 at Michigan State University with R. Kannan, then a visitor at Michigan State, and J. D. Schuur as its directors. Its format was six one-hour lectures by the principal speaker, L. Cesari, who developed in depth some aspects of the general topic; nine one-hour lectures by other invited speakers, who reviewed different aspects of the general topic; and research reports by participating mathematicians. This volume contains the lectures of the principal speaker and of the one-hour speakers and constitutes therefore the proceedings of the conference.

For his topic, Professor Cesari chose "Functional Analysis, Nonlinear Differential Equations, and the Alternative Method." He emphasized points at present under investigation, in particular the use of Banach's fixed point theorem, a priori bounds and topological arguments, monotone operators, and Schauder's fixed point theorem.

The one-hour talks ranged from problems in bifurcation theory to measurability of solutions of differential equations to current aspects of degree theory.

In such a large and active area, a survey will necessarily be limited by the choice of topics and a cutting point in time. However, we hope that this volume will prove useful in furthering the aims of the conference.

The editors are grateful to the Michigan State University Mathematics Department and its chairman, Professor J.E. Adney, for financial support, advice, and encouragement.

We also appreciate the work of Marcel Dekker in producing this volume, and of Mary Reynolds in typing it.

Lamberto Cesari

Rangachary Kannan

Jerry D. Schuur

CONFERENCE PARTICIPANTS

Walter Becker, Department of Physics, Michigan State University

Tom Bowman, Department of Mathematics, University of Florida

Russell Caflisch, Department of Mathematics, Michigan State University

Alfonso Castro, Department of Mathematics, University of Cincinnati

Lamberto Cesari, Department of Mathematics, University of Michigan

Shannon Coffey, Department of Mathematics, University of Cincinnati

Peter Dunkelberg, Department of Mathematics, University of Florida

Dennis Dunninger, Department of Mathematics, Michigan State University

Eric Grosse, Department of Mathematics, Michigan State University

Clark Givens, Department of Mathematics, Michigan Technical University

Carl Ganster, Department of Mathematics, Michigan State University

Tom Haigh, Department of Mathematics, Rose Hulman Institute

David Hale, Department of Mathematics, Michigan State University

Dan Henry, Department of Mathematics, Northwestern University and University of Kentucky

William Hall, Department of Mathematics, University of Pittsburgh

Rangachary Kannan, Department of Mathematics, Michigan State University and University of Missouri at St. Louis

Gene Klaasen, Department of Mathematics, University of Tennessee

George Knightly, Department of Mathematics, University of
Massachusetts

J. C. Kurtz, Department of Mathematics, Michigan State University

Alan Lazer, Department of Mathematics, University of Cincinnati

Chi Lo, Department of Mathematics, Michigan State University

John Locker, Department of Mathematics, Colorado State
University

John Mallet-Paret, Department of Mathematics, Brown University

P.J. McKenna, Department of Mathematics, University of Michigan

David McSweeney, Department of Mathematics, University of Kansas

R. Kent Nagle, Department of Mathematics, University of Michigan

Aubrey Poore, Department of Mathematics, Colorado State
University

Erich Rothe, Department of Mathematics, University of Michigan

Habib Salehi, Department of Mathematics, Michigan State
University

Jerry Schuur, Department of Mathematics, Michigan State
University

Howard Shaw, Department of Mathematics, University of Michigan

Harlan Stech, Department of Mathematics, Michigan State
University

M.B. Suryanarayana, Department of Mathematics, Eastern Michi-
gan University

Daniel Sweet, Department of Mathematics, University of Maryland

Russell Thompson, Department of Mathematics, Northern Illinois
University

Curtis Travis, Department of Mathematics, University of
Tennessee

Nancy Waller, Department of Mathematics, Michigan State
University

Richard Weinacht, Department of Mathematics, University of
Delaware

P.K. Wong, Department of Mathematics, Michigan State University

Dave Zachmann, Department of Mathematics, Colorado State
University

CONTENTS

Foreword	iii
Conference Participants	v
FUNCTIONAL ANALYSIS, NONLINEAR DIFFERENTIAL EQUATIONS, AND THE ALTERNATIVE METHOD	1
Lamberto Cesari	
Section 1. The Alternative Method	1
1. Introduction	1
2. The alternative scheme	2
3. The selfadjoint case. General reduction to an alternative problem by contraction maps	11
4. An analysis of auxiliary and bifurcation equations for the problem of periodic solutions	15
5. General reduction to an alternative problem by contraction maps in the nonselfadjoint case	23
Section 2. Problems of Perturbation for Ordinary Differential Equations	29
6. Boundary value problems for ordinary differential equations	29
7. The linear problem. Basic statements	32
8. The projection operators P and Q , and the partial inverse operator H	34
9. The perturbation problem	39
10. Examples	44
11. Some results for periodic solutions of certain typical systems	47
12. Symmetries and families of solutions	49

Section 3.	Problems of Perturbation for Partial Differential Equations	56
13.	Boundary value problems for partial differential equations	56
14.	The linear problem. Sobolev spaces. Basic statements	57
15.	The operators P , Q and H	60
16.	Problems of perturbation: the auxiliary equation	62
17.	The bifurcation equation, and examples	66
Section 4.	A Numerical Viewpoint and Error Bound Estimates	69
18.	Error estimates and existence of an exact solution	69
19.	Examples, and a method of successive approximations	75
Section 5.	Solutions in the Large of Liénard Systems with Forcing Terms	80
20.	Introductory remarks	80
21.	Formulation of the main theorems	83
22.	The canonic decomposition	86
23.	The operators J , J^*	88
24.	A topological degree argument and the Borsuk-Ulam theorem	90
25.	Proof of parts (a) and (b) of Theorem (21.i)	94
26.	Proof of part (c) of Theorem (21.i)	101
27.	Proof of parts (d) and (e) of Theorem (21.i)	103
Section 6.	The Alternative Method With Monotone Operators	104
28.	Monotone and maximal monotone operators	104
29.	The auxiliary equation and monotone maps	107

30.	The bifurcation equation and monotone maps	110
31.	Two statements for the nonresonance case	116
32.	Schauder's principle of invariance of domain	117
33.	The auxiliary equation and Schauder's principle	118
Section 7.	Elliptic Problems at Resonance in the Large	122
34.	Two abstract alternative statements	122
35.	One more existence theorem at resonance	129
36.	Existence of solutions across a point of resonance	131
37.	Extensions to nonselfadjoint problems	138
38.	Some theorems in the uniform topology	141
39.	Applications to problems of oscillation at resonance	143
40.	Linear elliptic operators in R^v	148
41.	A nonlinear Dirichlet problem at resonance	152
42.	More on nonlinear Dirichlet problems at resonance	154
Section 8.	Hyperbolic Partial Differential Equations	163
43.	Work of Cesari on doubly periodic solutions of the canonic hyperbolic equation and related plane wave equation	163
44.	Work of Hale on doubly periodic solutions of the plane wave equation	168
45.	Periodicity in only one variable	172
46.	Work of Petrovanu on periodic solutions of the canonic hyperbolic equation in R^3 and the Tricomi problem	174
47.	Work of Haimovici on problems whose unknowns are set functions	176

48. The work of Hecquet	177
49. The work of Hall	178
50. Other work	180
Bibliography	182
Selected references on the alternative method [1-125]	182
Other references [126-207]	191
APPLICATIONS OF GENERIC BIFURCATION	199
Shui-Nee Chow and John Mallet-Paret	
PERTURBATION METHODS FOR NONLINEAR PROBLEMS AT RESONANCE	209
Rangachary Kannan	
1. Introduction	209
2. The general problem of nonlinear perturbation at resonance	210
3. Brief survey of preliminary results	211
4. Landesman and Lazer's result	213
5. Outline of proof by "alternative problems"	215
6. Hess's proof of Theorem 1	216
7. An abstract generalization due to De Figueiredo	218
8. Remarks	221
References	224
MEASURABILITY OF SOLUTIONS OF NONLINEAR EQUATIONS	227
Rangachary Kannan and Habib Salehi	
1. Introduction	227
2. Preliminaries	228
3. Joint measurability	232
4. Stochastic models	234
References	243

SOME MATHEMATICAL PROBLEMS FROM PLATE AND SHELL THEORY 245

George H. Knightly

1. Introduction 245
2. Formulation of the operator equation 246
3. Existence 252
4. Branching 255
5. Branching in the plate and shell problems 260
- References 266

SOME RESONANCE PROBLEMS FOR ELLIPTIC BOUNDARY VALUE PROBLEMS 269

Alan C. Lazer

1. Introduction 269
2. An introductory example 271
3. A functional analytic approach and the role of the alternative method 273
4. An elliptic boundary value problem 277
5. A critical point principle 281
6. Further related results 286
- References 288

EXPOSITORY INTRODUCTION TO SOME ASPECTS OF DEGREE THEORY 291

Erich H. Rothe

1. Introduction 291
2. The finite dimension case 292
3. The generalized mapping degree 300
4. The theorem of Poincaré-Bohl and its consequences 306

5. Application to monotone mappings	311
References	316
BOUNDARY VALUE PROBLEMS FOR $x'' = f(t, x, x')$ AND NONLINEAR FUNCTIONAL ANALYSIS	319
Jerry D. Schuur	
1. Introduction	319
2. The Picard problem in $C^1[0,1]$	320
3. The periodic problem in $C^1[0,1]$	326
4. The periodic problem in $L^2[0,1]$	330
Bibliography	336
A NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEM	339
Howard Shaw	
1. Introduction and preliminaries	339
2. Proof of the theorem	340
3. Remarks	343
Bibliography	346
CHOOSING PROJECTION OPERATORS FOR THE ALTERNATIVE METHOD	347
Daniel Sweet	
1. Introduction	347
2. The general theory	348
3. Periodic solutions for ordinary differential equations	349
Bibliography	352

FUNCTIONAL ANALYSIS, NONLINEAR DIFFERENTIAL
EQUATIONS, AND THE ALTERNATIVE METHOD

Lamberto Cesari
University of Michigan

Section 1. The Alternative Method

1. Introduction. In this presentation of the alternative, or bifurcation method emphasis is given to certain points of recent research, in particular in connection with the use of Banach's fixed point theorem (nos. 6-19), a priori bounds and topological arguments (nos. 20-27), monotone operators (nos. 28-33), and Schauder's fixed point theorem (nos. 34-42).

We are interested in solving an operator equation $Ax = 0$ in a space X . We shall think of A as the difference $A = E - N$ of a linear operator E , not necessarily bounded, and of an operator N , not necessarily linear, and thus the equation takes the form

$$Ex = Nx, \quad x \in X.$$

If E^{-1} exists, or equivalently the null space of E is trivial, then this equation can be written as $x - E^{-1}Nx = 0$, or $(I + KN)x = 0$. This is a Hammerstein equation and very extensive work has been done in this direction. For references to the wide literature one can see [147].

Whenever KN is compact, concepts and methods of topological degree theory in function spaces are relevant, and we

should refer to the work of Leray-Schauder [167], Rothe [184], and Nagumo [177, 178].

We are particularly interested in the case where E has a nontrivial null space, the case which is often mentioned as the "problem at resonance." Thus we shall not exclude that E may have a nonzero null space. This will be actually the most interesting case.

In the line of the bifurcation process of Poincaré [180], Lyapunov [171], and Schmidt [187] we should decompose the equation $Ex = Nx$ into a system of two equations, possibly in different spaces. Much work has been done in this direction, for which we refer to [191] and [192].

The injection of ideas of functional analysis in the last years has made the process a remarkably fine tool of analysis, particularly in the difficult "problem at resonance" in the usual terminology.

The general theory which has ensued, with all its variants and ramifications through the works of many authors, is often referred to as the bifurcation theory, or the alternative method ([6], [7], [65], and recent expositions, e.g., [21-23], [63]). It is being used in theoretical existence analysis of the solutions, in methods of successive approximations of the solutions, in estimating the error of approximate solutions, in problems of perturbations, and in problems of bifurcation of eigenvalues.

2. The Alternative Scheme. To be specific, let us consider an equation of the form

$$Ex = Nx, \tag{2.1}$$

where $E: \mathcal{B}(E) \rightarrow Y$ is a linear operator, and $N: \mathcal{B}(N) \rightarrow Y$ is an operator, not necessarily linear, both E and N have domains $\mathcal{B}(E)$, $\mathcal{B}(N)$ in a Banach space X and ranges $\mathcal{R}(E)$, $\mathcal{R}(N)$ in a Banach space Y , $\mathcal{B}(E) \cap \mathcal{B}(N) \neq \emptyset$.

Let us assume for a moment that the following holds (we shall see in no. 3 that these assumptions are quite natural, and they are also easily verified in the so-called selfadjoint case). Let us assume that there are projection operators $P: X \rightarrow X$, $Q: Y \rightarrow Y$ (that is, linear, bounded, idempotent, or $PP = P$, $QQ = Q$), and a linear operator H , which we shall consider as a partial inverse of E satisfying

$$H(I - Q)Ex = (I - P)x \quad \text{for all } x \in \mathcal{B}(E), \quad (k_1)$$

$$QEx = EPx \quad \text{for all } x \in \mathcal{B}(E), \quad (k_2)$$

$$EH(I - Q)Nx = (I - Q)Nx \quad \text{for all } x \in \mathcal{B}(E) \cap \mathcal{B}(N). \quad (k_3)$$

Under these assumptions, the equation $Ex = Nx$ is equivalent to the system of two equations

$$x = Px + H(I - Q)Nx, \quad (2.2)$$

$$Q(Ex - Nx) = 0. \quad (2.3)$$

Indeed, if (2.1) is satisfied, then by applying Q to equation (2.1) we obtain (2.3). By applying $H(I - Q)$ to (2.1) we obtain $H(I - Q)Ex = H(I - Q)Nx$, and by using (k_1) we obtain (2.2). Conversely, if (2.2) and (2.3) are satisfied, then, by applying E to (2.2) we have $Ex - EPx = EH(I - Q)Nx$, and by using (k_2) , (k_3) , also $Ex - QEx = (I - Q)Nx$, or $Ex - Nx = Q(Ex - Nx) = 0$.

Equations (2.2) and (2.3) are usually denoted as the auxiliary and the bifurcation equations, respectively.

Let $X_0 = PX$, $X_1 = (I - P)X$, $Y_0 = QY$, $Y_1 = (I - Q)Y$, so that X and Y have the decompositions $X = X_0 + X_1$, $Y = Y_0 + Y_1$ (direct sums), $X_0 \cap X_1 = \{0\}$, $Y_0 \cap Y_1 = \{0\}$. The linear subspace of X of all elements $x \in X$ for which $Ex = 0$ is often denoted as the null space of E , or kernel of E , and denoted by $\ker E$. When there are elements $x \neq 0$ in X with $Ex = 0$, that is, $\ker E$ is not trivial, then we say that we have a resonance case. We shall always assume that $\ker E \subset X_0$. Also, we shall assume that $Y_1 \subset \mathcal{R}(E)$, $\mathcal{B}(H) = Y_1$, $\mathcal{R}(H) = \mathcal{B}(E) \cap X_1$. Thus $EHy = y$ for all $y \in Y_1$, $HEx = x$ for all $x \in \mathcal{B}(E) \cap X_1$. Often, we may assume $\ker E = X_0$, $Y_1 = \mathcal{R}(E)$. In this particular situation the bifurcation equation reduces to $QNx = 0$.

If we write $x^* = Px$, then the auxiliary equation can always be written in the form of a fixed point statement:

$$x = Tx, \quad Tx = x^* + H(I - Q)Nx, \quad (2.4)$$

where T is easily seen to map the fiber $P^{-1}x^*$ into itself. The auxiliary equation can also be written in the form of a Hammerstein equation:

$$x + KNx = x^*, \quad K = -H(I - Q). \quad (2.5)$$

Finally the auxiliary equation, written in the equivalent form

$$Ex - Nx = Q(Ex - Nx),$$

is often called the relaxed equation, or relaxed problem.

There are certain rather general situations (N locally Lipschitzian, N monotone, etc.) under which the operators P , Q , H can be so chosen that the ensuing map T is a contraction map on suitable sets, or has other suitable