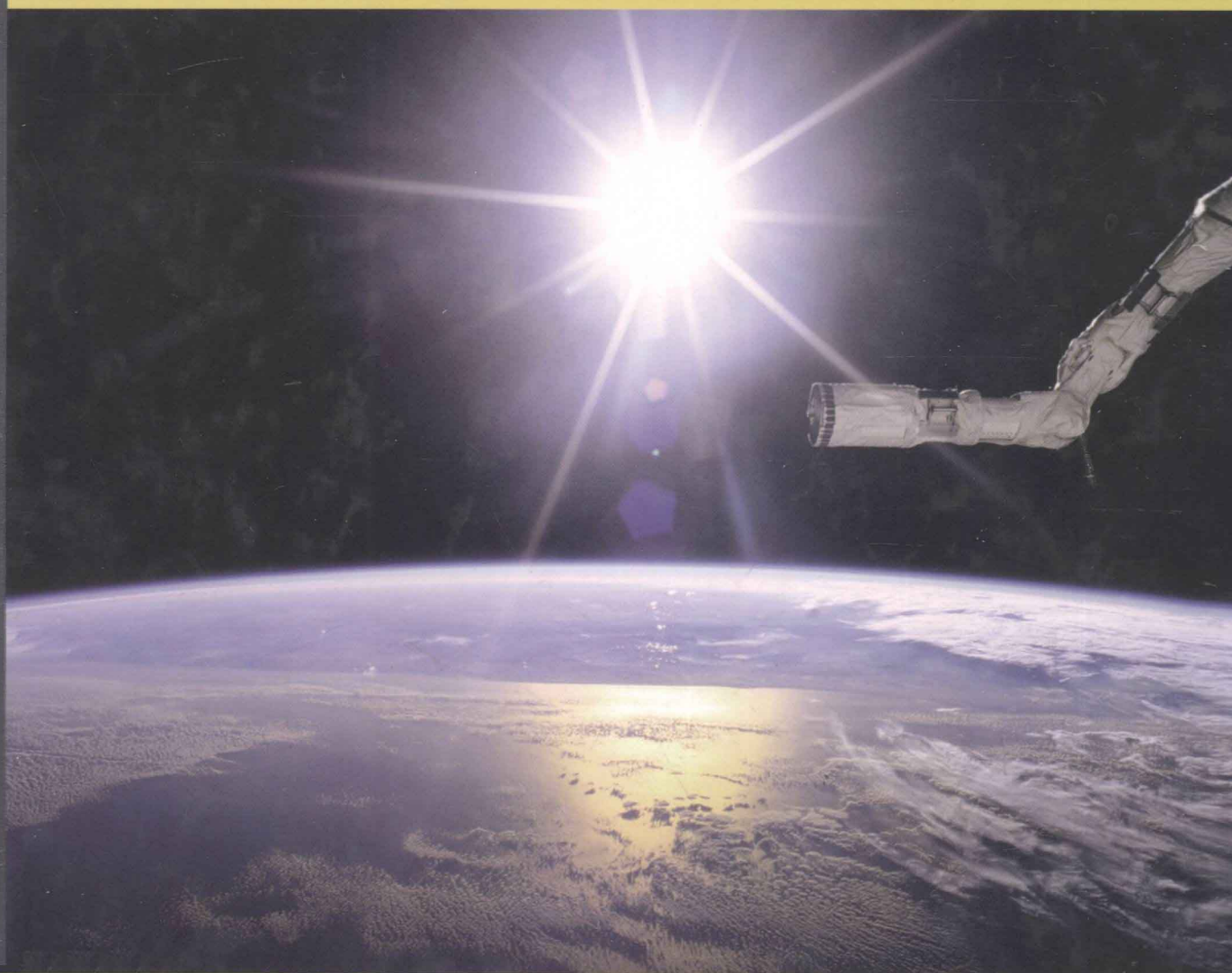


ROBOT MODELING AND CONTROL



Mark W. Spong | Seth Hutchinson | M. Vidyasagar

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To the women in our lives

Lila - MWS

Cynthia - SH

Shakunthala and Aparna - MV

Preface

The field of robotics has changed in numerous and exciting ways since the early 1980's when robot manipulators were touted as the ultimate solution to automated manufacturing. Early predictions were that entire factories of the future would require few, if any, human operators. Some predicted that even electric lighting would be unnecessary as robots would "happily" carry out their work in total darkness. These predictions seem naive today but it is nevertheless interesting to examine some of the reasons why they failed to materialize. The first reason can be stated very simply: robotics is difficult or, somewhat equivalently, humans are very good at what they do. Automated manufacturing is not simply a matter of removing a human worker from an assembly line and installing a robot. Rather, it involves complex systems integration problems. Often, the entire workcell must be redesigned, beginning with an analysis of the assembly process itself and leading to part and fixture redesign, workcell layout, sensor development, control system design, software verification, and a host of other interconnected issues. The result is that any savings in labor costs often did not outweigh the development costs, except for relatively simple tasks like spot welding, spray painting, and palletizing.

As a result, robotics fell out of favor in the late 1980's. We are now witnessing a resurgence of interest in robotics, not only in manufacturing, but in other areas such as medical robotics, search and rescue, entertainment, and service robotics. Recent years have seen robots exploring the surface of Mars, locating sunken ships, searching out land mines, and finding victims in collapsed buildings. Robotics is now seen as part of the larger field of mechatronics, which is defined as the synergistic integration of mechanics, electronics, controls, and computer science. The robot is the ultimate mechatronic system.

The present text began as a second edition of M.W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, Inc., 1989. Early on it became apparent, due to both the length of elapsed time since

the first edition and also to the evolution of the field in the intervening years, that the end product would be an entirely new book. We have retained the philosophy and a good portion of the material from that earlier book but have added much that is new. The material on motion planning, computer vision, and visual servo control is entirely new. All of the control chapters have been rewritten to reflect the maturation of the field of robot control that took place during an intensive period of research at the end of the 1980's and early 1990's. The fundamentals of kinematics and dynamics remain largely the same but has been expanded and improved from a pedagogical standpoint.

Organization of the Text

This text is organized into twelve chapters. The first six chapters are at a relatively elementary level whereas the last six chapters are more advanced. The chapters can be conceptually divided into three groups. After an introductory chapter, Chapters 2 through 5 deal with issues related to the geometry of robot motion. Chapters 6 through 10 deal with dynamics and control. Finally, Chapters 11 and 12 discuss computer vision and how it can be incorporated directly into the robot control loop. A more specific description of the chapters is as follows.

Chapter 1 is an introduction to the terminology and history of robotics and discusses the most common robot design and applications.

Chapter 2 presents the mathematics of rigid motions; rotations, translations, and homogeneous transformations.

Chapter 3 presents solutions to the forward kinematics problem using the Denavit-Hartenberg representation and to the inverse kinematics problem using the geometric approach, which is especially suited for manipulators with spherical wrists.

Chapter 4 is a lengthy chapter on velocity kinematics and the manipulator Jacobian. The geometric Jacobian is derived in the so-called cross product form. We also introduce the so-called analytical Jacobian for later use in task space control. Chapter 4 also discusses the important notion of manipulability.

Chapter 5 is an introduction to the problems of motion planning and trajectory generation. Several of the most popular methods for motion planning and obstacle avoidance are presented, including the method of artificial potential fields, randomized algorithms, and probabilistic roadmap methods. The problem of trajectory generation is presented as essentially a problem of polynomial spline interpolation. Trajectory generation based on cubic and quintic polynomials as well as trapezoidal velocity trajectories are derived

for interpolation in joint space.

Chapter 6 is an introduction to independent joint control. Linear control based on PD, PID, and state space methods is presented for the tracking and disturbance rejection problem for linear actuator and drive-train dynamics. The concept of feedforward control is introduced for tracking time-varying reference trajectories.

Chapter 7 is a detailed account of robot dynamics. The Euler-Lagrange equations are derived from first principles and their structural properties are discussed in detail. The recursive Newton-Euler formulation of robot dynamics is also presented.

Chapter 8 discusses multivariable control. This chapter summarizes much of the research in robot control that took place in the late 1980's and early 1990's. Simple derivations of the most common robust and adaptive control algorithms are presented that prepare the reader for the extensive literature in robot control.

Chapter 9 treats the force control problem. Both impedance control and hybrid control are discussed. We also present the lesser known hybrid impedance control method which allows one to control impedance and regulate motion and force at the same time. To our knowledge this is the first textbook that discusses the hybrid impedance approach to robot force control.

Chapter 10 is an introduction to geometric nonlinear control. This chapter is considerably more advanced than the other chapters and can be reserved for graduate level courses in nonlinear control and robotics. However, the material is presented in a readable style that should be accessible for advanced undergraduates. We derive and prove the necessary and sufficient conditions for local feedback linearization of single-input/single-output systems which we then apply to the flexible joint control problem. We also briefly discuss Chow's Theorem for the problem of control of systems subject to nonholonomic constraints.

Chapter 11 is an introduction to computer vision. We present those aspects of vision that are most useful for robotics applications, such as thresholding, image segmentation, and camera calibration.

Chapter 12 discusses the visual servo control problem, which is the problem of controlling robots using feedback from cameras mounted either on the robot or in the workspace.

This text is suitable for several quarter or semester long courses in robotics, either as a sequence or as stand-alone courses. The first six chapters can be used for a junior/senior level introduction to robotics for students with a minimal background in linear control systems. One of the key

changes we made in this text from the earlier text was to place the independent joint control chapter before the dynamics chapter. The independent joint control problem largely involves the control of actuator and drive train dynamics; hence most of the subject can be taught without prior knowledge of Euler-Lagrange dynamics.

Below we outline two possible courses that can be taught from this book:

Course 1: Introduction to Robotics

Level: Junior/Senior undergraduate

For a one quarter course (10 weeks):

Chapter 1: Introduction

Chapter 2: Rigid Motions and Homogeneous Transformations

Chapter 3: Forward and Inverse Kinematics

Chapter 4: Jacobians

For a one semester course (16 weeks) add:

Chapter 5: Motion Planning and Trajectory Generation

Chapter 6: Independent Joint Control

Chapter 11: Computer Vision

Course 2: Robot Dynamics and Control

Level: Senior undergraduate/graduate

For a one quarter course (10 weeks):

Chapters 1–5: Rapid Review of Kinematics (selected sections)

Chapter 6: Independent Joint Control

Chapter 7: Dynamics

Chapter 8: Multivariable Control

Chapter 9: Force Control

For a one semester course (16 weeks) add:

Chapter 10: Geometric Nonlinear Control

Chapter 11: Computer Vision

Chapter 12: Visual Servo Control

We have taught both of these one semester courses at the University of Illinois. The students who take the first course typically come from Computer Science, Electrical and Computer Engineering, General Engineering and Mechanical Engineering Departments. For this reason, we have tried to make these chapters accessible to a wide variety of engineering students. The second course has typically been taken by upper level students pursuing graduate studies in robotics or control, and therefore these chapters are written at a more advanced level.

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Mark W. Spong
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Chapter 1

INTRODUCTION

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their application requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics and factory automation.

This book is concerned with fundamentals of robotics, including **kine-matics**, **dynamics**, **motion planning**, **computer vision**, and **control**. Our goal is to provide an introduction to the most important concepts in these subjects as applied to industrial robot manipulators and other mechanical systems.

The term **robot** was first introduced by the Czech playwright Karel Capek in his 1920 play *Rossum's Universal Robots*, the word *robota* being the Czech word for work. Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, underwater vehicles, autonomous land rovers, etc. Virtually anything that operates with some degree of autonomy, usually under computer control, has at some point been called a robot. In this text the term robot will mean a computer controlled industrial manipulator of the type shown in Figure 1.1.

This type of robot is essentially a mechanical arm operating under computer control. Such devices, though far from the robots of science fiction, are nevertheless extremely complex electromechanical systems whose analytical description requires advanced methods, presenting many challenging and interesting research problems. An official definition of such a robot comes from the **Robot Institute of America (RIA)**: