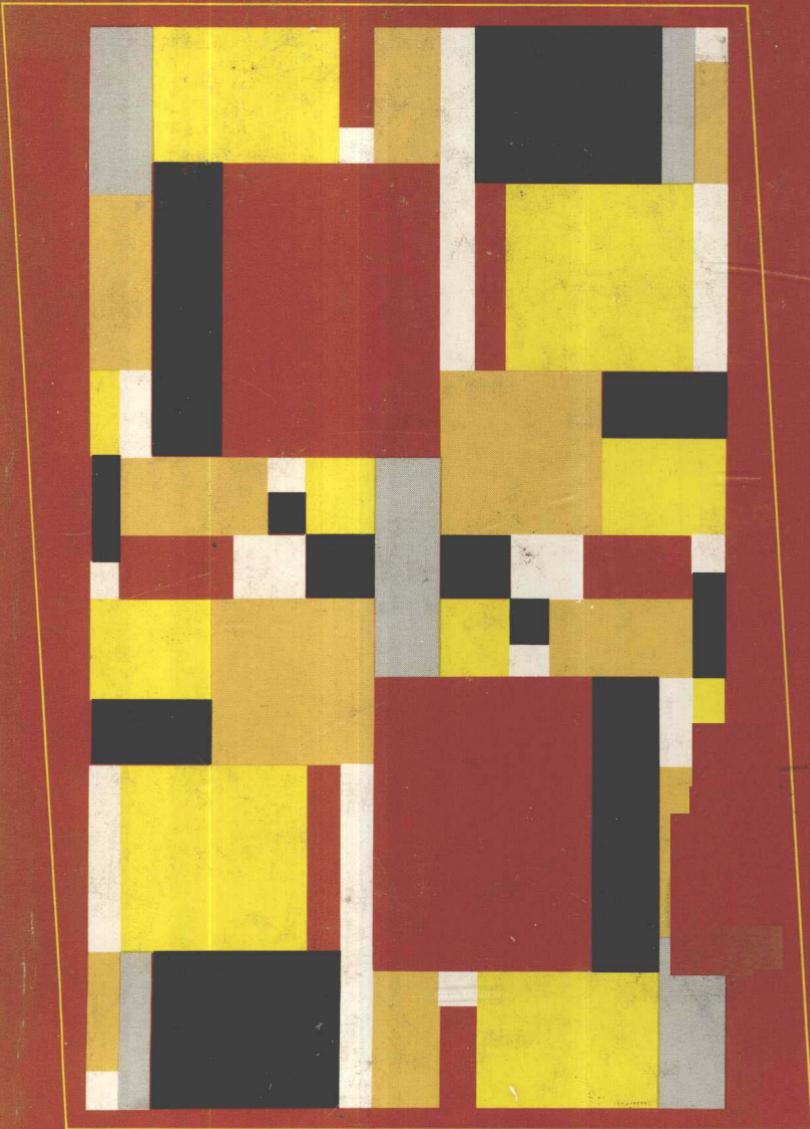


PROFESSIONAL COPY
NOT FOR SALE
PRENTICE-HALL, INC.

Larry J. Goldstein
David C. Lay
David I. Schneider



MODERN
MATHEMATICS
AND ITS APPLICATIONS

MATHEMATICS

LARRY J. GOLDSTEIN

DAVID C. LAY

DAVID I. SCHNEIDER

Department of Mathematics
University of Maryland

Library of Congress Cataloging in Publication Data

GOLDSTEIN, LARRY JOEL.

Modern mathematics and its applications.

Includes indexes.

I. Mathematics—1961– I. Lay, David C., joint author. II. Schneider, David I., joint author.

III. Title.

QA37.2.G64 1980 510 79-20346

ISBN 0-13-595173-9

Modern Mathematics and Its Applications

Larry J. Goldstein, David C. Lay, David I. Schneider

© 1980 by Prentice-Hall, Inc.,

Englewood Cliffs, N.J. 07632.

All rights reserved. No part of this book

may be reproduced in any form

or by any means without permission in writing

from the publisher.

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

Editorial/production supervision by Nancy Milnamow

Interior design and cover design by Walter A. Behnke

Manufacturing buyer: Ed Leone

Photo researcher: Frances L. Orkin

PHOTO CREDITS

Chapter 7 (page 254) Reprinted with the special permission of DUN'S REVIEW, February 1979, copyright 1979, Dun & Bradstreet Publications Corporation.

Chapter 8 (page 304) Courtesy of The Bowery Savings Bank, New York.

Chapter 15 (page 546) © ARCH. PHOT., Paris/S.P.A.D.E.M.

PRENTICE-HALL INTERNATIONAL, INC., *London*

PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*

PRENTICE-HALL OF CANADA, LTD., *Toronto*

PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*

PRENTICE-HALL OF JAPAN, INC., *Tokyo*

PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*

WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

Mathematics and Its Applications

This volume is one of a collection of texts for freshman and sophomore college mathematics courses. Included in this collection are the following.

Calculus and Its Applications, Second Edition by L. Goldstein, D. Lay, and D. Schneider. A text designed for a two-semester course in calculus for students of business and the social and life sciences. Emphasizes an intuitive approach and integrates applications into the development. Much expanded from the highly successful first edition.

Calculus and Its Applications, Brief Edition by L. Goldstein, D. Lay, and D. Schneider. Consists of the first eight chapters of the above book. Suitable for shorter courses.

Finite Mathematics and Its Applications by L. Goldstein and D. Schneider. A traditional finite mathematics text for students of business and the social and life sciences. Allows courses to begin with either linear mathematics (linear programming, matrices) or probability and statistics.

Modern Mathematics and Its Applications by L. Goldstein, D. Lay, and D. Schneider. A text for a two-semester course covering finite mathematics, precalculus, and calculus.

PREFACE

Many colleges and universities offer yearlong sequences devoted to a combined treatment of finite mathematics and calculus. Such “combined” courses are populated by students in a variety of nonphysical science disciplines, but most frequently by business students. The goal is to provide exposure to both finite mathematics and calculus, usually in preparation for a general statistics course. The present work is designed as a text for such a “combined” course.

In designing this book, we have incorporated both a wide range of topics from which an instructor may design a curriculum as well as a high degree of flexibility in the order in which the topics may be presented.

What, in our opinion, distinguishes this book from other texts is its student-oriented approach and its much more thorough treatment of calculus and its prerequisites. Our student-oriented approach manifests itself throughout and, in particular, in the following features:

1. *Applications.* We have, wherever possible, motivated mathematical ideas by applications. The applications are realistic and often are based on real-life data. The reader may survey the variety of applications by turning to the Index of Applications on page xvii.

2. *Intuitive approach.* We develop mathematical ideas by generalizing from concrete examples. Furthermore, formal proofs are often replaced by informal justifications designed to enhance both understanding and intuition.

3. *Examples.* We have provided more worked examples than is customary (572). These examples often illustrate computational points and interesting applications. An instructor may wish to assign some of the examples as student reading. We have included most calculational details to ease the student's way.

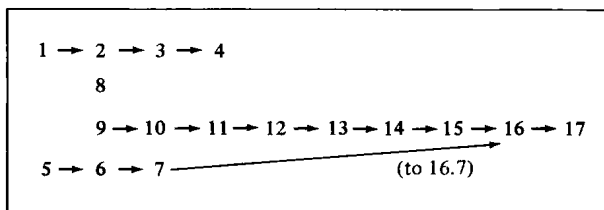
4. *Exercises.* Each section contains an extensive exercise set which illustrates and extends the material in the body of the text. Furthermore, each chapter has a review of new terms and a set of supplementary exercises. A total of 2458 exercises appear in the text.

5. *Practice Problems.* The practice problems are carefully selected exercises that are located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In classroom use over the past three years, several thousand students have found practice problems to be a valuable learning aid.

6. *Minimal Prerequisites.* Because of great variation in student preparation, we have kept the formal prerequisites to a minimum. We assume only a first year of high school algebra. Furthermore, we review as needed those topics which are typical student weak spots.

A traditional shortcoming in existing texts is their sketchy treatment of calculus, as may be seen from the proportion of space devoted to calculus. Furthermore, preparation for the calculus often tends to be weak. In contrast, we have accorded approximately equal amounts of space to calculus and to finite mathematics and have given ample attention to precalculus topics.

This book contains sufficient material to teach a first semester of finite mathematics followed by a second semester of calculus. Actually, the text contains substantially more material than can be comfortably covered in many courses. So most instructors will use only a proper subset of the material we provide. The following chapter dependence chart should be helpful in choosing a curriculum based on this book.



Note that the finite mathematics component of this book primarily divides into two parts. The first consists of “linear mathematics”—linear equations, matrices, and linear programming (Chapters 1–4). The second part consists of probability and statistics (Chapters 5–7). We prefer to begin with linear mathematics since it makes for a smooth transition from high school mathematics and leads rather quickly to interesting applications, especially the linear programming. Our preference notwithstanding, the instructor may begin the book with Chapter 5 (Sets and Counting). The final chapter of the finite mathematics component (Chapter 8) develops a single difference equation to present a unified treatment of interest, mortgages and annuities. This same difference equation can also be used to model various phenomena in sociology, biology, and economics.

It is our belief that the study of calculus should not be unduly delayed with prerequisites. Therefore, we have deliberately kept the precalculus chapter brief and to the point. It reviews exactly those topics which are necessary to begin learning calculus. Other precalculus topics are reviewed as needed.

The chapters on calculus are adapted from our widely-used text *Calculus and Its Applications* which was designed for a one year calculus sequence for business and the biological and social sciences. Our development of calculus is much more detailed and more leisurely than that in most existing books. Furthermore, our development of calculus of several variables, especially the method of Lagrange multipliers is both extensive and, as the experience of the users of *Calculus and Its Applications* has shown, is quite teachable.

Answers to the odd-numbered exercises are included at the back of the book. Answers to the even-numbered exercises are contained in the Instructor’s Manual.

LARRY J. GOLDSTEIN

DAVID C. LAY

DAVID I. SCHNEIDER

College Park, Maryland

ACKNOWLEDGMENTS

While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we should like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm, with us.

Evan G. Houston, University of North Carolina, Charlotte; Richard Bouldin, University of Georgia; David H. Carlson, University of Missouri, Columbia; Marc Konvisser, Wayne State University; Kenneth N. Berk, Illinois State University; Rebecca Klemm, Georgetown University; Jack P. Tull, Ohio State University; James F. Hurley, University of Connecticut.

The staff of Prentice-Hall, Inc. has done a magnificent job of transforming a somewhat dog-eared, often ill-prepared manuscript into a beautiful bound volume. Especially, we should like to thank Susan Formilan for directing the traffic between the authors, reviewers, and publisher, Nancy Milnamow for her expertise as production editor, Robin Bartlett and Robyn Juback for their help in promotion and Walter Behnke for his imaginative and beautiful design work. Finally, we would like to express our gratitude to Harry Gaines, Executive Editor of Prentice-Hall, for favors too numerous to mention. His partnership, friendship, and constant encouragement have had a profound, positive effect on the ultimate form of this work.

INDEX OF APPLICATIONS

- Business and Economics***
- Analyzing business and investment options, 268, 277, 280
 - Annuity, 324–25, 326, 328
 - Auditing of income tax returns, 253
 - Average cost, 496
 - Compound interest, 305–306, 312, 315, 316, 319, 320, 325, 326, 327, 328, 558–63, 565, 600, 604, 605, 629, 671
 - Consumer loans, 312, 316, 326, 328
 - Consumers' surplus, 599–600, 603–604, 629
 - Consumption of food (as a function of income and retail prices), 648
 - Consumption of fuel oil, 439, 568, 572, 577–78, 590–91, 592, 605
 - Continuous stream of income, 605, 611–12, 614
 - Cost of electricity, 488
 - Cost functions, 348, 354, 368, 383, 421, 439, 478, 479–80, 487, 573, 578, 582, 583
 - Counting number of ways of assigning employees to tasks, 195, 197
 - Counting possibilities for management positions, 173, 178
 - Demand for beer (as a function of income, retail prices, and strength of the beer), 647
 - Demand equation, 480–85, 487, 488, 491, 539, 599–600, 603–604, 629
 - Demand functions (several variables), 638, 645–46, 647–48
 - Distribution of revenue between labor and capital, 648
 - Double-declining balance method of depreciation, 321
 - Duration of phone calls, 625
 - Earnings of investment trust, 56
 - Effect of advertising on sales, 418, 424, 477, 554, 671
 - Effect of an excise tax on sales price, 485, 488
 - Effect of a second shift on a factory's cost function, 403
 - Growth rates of U.S. population and national debt per person, 433–34
 - Input-output analysis, 81–89
 - Insurance probabilities, 240, 250, 272, 274
 - Inventory problems, 470–72, 474, 476, 491
 - Investment planning, 40
 - Lifetimes of equipment, 624, 627, 631
 - Linear depreciation, 11, 320
 - Marginal cost, 421–22, 424, 426, 479–80, 486, 487, 496, 573, 575, 578, 582, 583, 591, 630
 - Marginal productivity of labor and capital, 644, 664, 669
 - Marginal productivity of money (as a Lagrange multiplier), 664
 - Marginal profit, 425, 483–86, 575, 583, 592
 - Marginal revenue, 425, 426, 480–83, 486, 573, 630
 - Maximizing investment returns, 106–109, 113
 - Maximizing production, 476, 662–64, 669–70
 - Minimum average cost, 496

Minimizing transportation costs, 110, 113–14
 Monopoly, profit of, 483–85, 488, 491, 652, 658, 669
 Mortgage, 320, 322, 323, 326, 328, 646
 Optimal airline fares, 469–70
 Optimal reorder quantity, 470–72, 474, 476, 491
 Optimizing production schedules, 92, 97, 105, 106, 113, 115, 138, 147
 Organizing economic and business data, 152, 153, 156, 167, 170
 Output, as a function of time, 354, 424, 429, 477, 575, 583, 630
 Present value, 562, 563, 638
 Present value of an income stream, 605, 611–12, 614
 Price discrimination, 652
 Producers' surplus, 603–604
 Production costs, 62
 Production functions, 637, 644, 647, 648, 662–64, 668–69
 Production possibilities curve, 669
 Profitability of franchised restaurants, 626
 Profit functions, 474, 475, 476, 483–86, 487–88, 491, 539, 583
 Profit functions (2 variables), 657, 658, 668–69
 Quality control, 187, 225, 228, 229, 230, 236, 237, 239, 248, 250, 253, 278, 281, 292, 293, 297, 303
 Rate of increase of income, 439
 Ratio of marginal productivities, 664, 669
 Reliability of equipment, 631
 Revenue as matrix product, 52, 57
 Revenue functions, 469–70, 475–76, 480–83, 487, 573
 Sales decay curve, 554
 Sales function (2 variables), 645–46, 667, 671
 Sales performance, 299, 300
 Salvage value, 11
 Savings account transactions, 307, 312, 326
 Shipping, 96, 113
 Simple interest, 314, 317, 320
 Supply and demand curves, 24, 603–604
 Volume of sales, 290, 292
 Wage per unit of labor, capital, 648
 Waiting times, 623, 626, 627
 Wheat production, 626

***Social
Sciences***

Accidental nuclear war, 246
 Census statistics, 251
 Counting options for Senate committee selection, 187
 Counting ways to cast ballots, 193
 Crime rate (as a function of several variables), 671
 Demographic analysis, 70, 73
 Distribution of incomes, 626
 Effects of immigration and emigration, 310, 312, 328
 Evaluating performance in tests, 299, 300
 IQ scores, 292
 Opinion sampling, 179, 185, 229, 239
 Organizing sociological data, 156, 159, 163, 165, 166, 168, 169, 200, 201

Political polling, 241, 251, 299
 Psychology experiment, 245
 Radiocarbon dating, 553–54, 556
 Spread of a rumor, 365–66
 Test for discrimination, 303
 Voter analysis, 62
 Voting model (in political science), 334–35, 499

Biology and Medicine

Age distribution of cells, 621–23, 627
 Allometric equation, 543
 Birth weights of infants, 289
 Calcium metabolism, 421
 Concentration of a drug in the body, 564, 630
 Contraction of the trachea during coughing, 472–73
 Counting possible outcomes of a medical experiment, 171
 Designing an optimum diet, 101, 105, 113, 114, 147
 Drug time-concentration curve, 341–42, 432–33, 534
E. Coli infection, 556
 Elimination time of a drug, 564
 Enzyme kinetics, 348
 Epidemiology, 62, 73, 544
 Exponential decay, 551–54
 Exponential growth, 548–51, 555, 556, 565, 602
 Flu epidemic, 420–21, 425, 575
 Gompertz growth curve, 525
 Growth of bacteria, 548–49, 555, 556, 565
 Growth of a tumor, 424
 Half-life and decay constants, 551–54, 555, 556
 Heights of adult males, 302
 Heights of elephants, 292
 Life expectancy of wild life, 267–68
 Medical diagnosis, 243, 246, 247–48
 Medical screening, 225
 Mitosis in a cell population, 627
 Nutrition planning, 40
 Organizing medical data, 200
 Recovery from disease, 294, 297, 299, 300
 Relief times of arthritic patients, 627
 Response of heart muscle to acetylcholine, 340
 Support of bacteria culture, 30
 Surface area of a human body, 648
 Urinary tract infection, 556
 Weight of an animal, 419–20
 Which group had the fewer cavities?, 273

Ecology

Average world population, 602
 Contamination of hay by radioactive iodine-131, 552–53
 Depletion of natural resources, 575, 630 (*see also* World consumption of oil)
 Food chain, 352
 Net primary production of nutrients, 345

Oxygen content of a lake, 424, 426, 439, 477
 Population growth, 312, 548–51, 555, 556, 604
 Radioactive fallout and waste products, 552, 556, 604
 Water pollution, 344, 424, 439
 Wind velocity, 535
 World consumption of oil, 568, 572, 577–78, 590–91, 592, 605
 World's need for arable land, 556–57

**General
Interest**

Bridge, 196
 Carbon monoxide level in the air, 363
 Carnival games, 263, 264, 273, 274
 Coin tossing, 221, 229, 236, 240, 252, 258, 264, 296, 299
 Comparing sports performances, 272, 280
 Conversion from Celsius to Fahrenheit, 334
 Counting options for everyday tasks, 173, 174, 175, 179, 180, 181, 187, 192, 193, 197, 198, 200, 201
 Counting possible contest outcomes, 197, 198, 200
 Counting possible outcomes of sports events, 175, 179, 181, 186, 187, 193, 200
 Counting travel options, 172, 174, 175, 186
 Design of an athletic field, 475
 Design of a building to minimize heat loss, 636, 652–54, 665–66
 Design of a rain gutter or trough, 475
 Design of a wind shelter, 466, 468, 668
 Dice, 221, 234, 253, 262, 264, 269, 270, 274, 282, 299
 Dimensions of a Norman window, 468
 Flow of water into a tank, 605
 Interarrival times at a toll booth, 627
 Maximum height of a projectile, 423, 462, 467
 Optimal design of containers and mailing cartons, 465–66, 467, 468, 474, 475, 490, 657, 672
 Optimal design of rectangular enclosures, 462–64, 468, 474, 475, 672
 Optimizing exercise routine, 95
 Poker, 181, 187, 201, 229
 Predicting sports performance, 299, 300, 302
 Principle of optimal design in architecture, 666
 Probabilities of everyday occurrences, 221, 226, 229, 230, 231, 233, 236, 238, 239, 251, 253
 Roulette, 221, 271, 273, 300
 R-rating of insulation, 349
 Strength of a beam, 491
 Terminal velocity of a skydiver, 439
 Velocity and acceleration, 422–23, 425, 426, 428, 572–73, 575, 578, 582, 583

CONTENTS

Preface *xiii*

Index of Applications *xvii*

Part I: FINITE MATHEMATICS

Introduction 2

1. Linear Equations and Straight Lines 5

1.1. Coordinate Systems and Graphs 5

1.2. Linear Inequalities 13

1.3. The Intersection Point of a Pair of Lines 22

2. Matrices 29

- 2.1. Solving Systems of Linear Equations, I 29
- 2.2. Solving Systems of Linear Equations, II 41
- 2.3. Arithmetic Operations on Matrices 49
- 2.4. The Inverse of a Matrix 64
- 2.5. The Gauss-Jordan Method for Calculating Inverses 75
- 2.6. Input-Output Analysis 81

3. Linear Programming, A Geometric Approach 91

- 3.1. The Feasible Set of a Linear Programming Problem 91
- 3.2. Linear Programming, I 99
- 3.3. Linear Programming, II 106

4. The Simplex Method 117

- 4.1. Slack Variables and the Simplex Tableau 118
- 4.2. The Simplex Method, I—Maximum Problems 127
- 4.3. The Simplex Method, II—Minimum Problems 139

5. Sets and Counting 151

- 5.1. Sets 151
- 5.2. A Fundamental Principle of Counting 158
- 5.3. Venn Diagrams and Counting 165
- 5.4. The Multiplication Principle 171
- 5.5. Permutations and Combinations 176
- 5.6. Further Counting Problems 182
- 5.7. The Binomial Theorem 188
- 5.8. Multinomial Coefficients and Partitions 194

6. Probability	203
6.1. Introduction	203
6.2. Experiments, Outcomes, and Events	205
6.3. Assignment of Probabilities	214
6.4. Application of Counting Methods	223
6.5. Conditional Probability and Independence	230
6.6. Tree Diagrams	240
6.7. Bayes' Theorem	247
7. Probability and Statistics	255
7.1. Probability Distributions and Random Variables	256
7.2. The Mean of a Probability Distribution	266
7.3. The Variance and Standard Deviation of a Probability Distribution	275
7.4. The Normal Distribution	283
7.5. Binomial Trials	292
8. Difference Equations and the Mathematics of Finance	305
8.1. Introduction to Difference Equations, I	305
8.2. Introduction to Difference Equations, II	313
8.3. The Mathematics of Personal Finance	321

Part II: CALCULUS

Introduction	330
9. Functions	333
9.1. Functions and Their Graphs	333
9.2. Some Important Functions	343
9.3. The Algebra of Functions	349
9.4. Zeros of Functions—The Quadratic Formula and Factoring	355

10. The Derivative 365

- 10.1. The Slope of a Straight Line 366
- 10.2. The Slope of a Curve at a Point 377
- 10.3. The Derivative 383
- 10.4. Limits and the Derivative 392
- 10.5. Differentiability and Continuity 401
- 10.6. Some Rules for Differentiation 408
- 10.7. More About Derivatives 414
- 10.8. The Derivative as a Rate of Change 419

11. Applications of the Derivative 431

- 11.1. Describing Graphs of Functions 431
- 11.2. The First and Second Derivative Rules 440
- 11.3. Curve Sketching (Introduction) 447
- 11.4. Curve Sketching (Conclusion) 454
- 11.5. Optimization Problems 461
- 11.6. Further Optimization Problems 469
- 11.7. Applications of Calculus to Business and Economics 478

12. Techniques of Differentiation 493

- 12.1. The Product and Quotient Rules 494
- 12.2. The Chain Rule 501

13. The Exponential Function 509

- 13.1. Properties of Exponents 510
- 13.2. Graphs of Exponential Functions 513
- 13.3. Differentiation of Exponential Functions 520

14. The Natural Logarithm Function	529
14.1. The Natural Logarithm Function	529
14.2. The Derivative of $\ln x$	535
14.3. Properties of the Natural Logarithm Function	539
15. Applications of the Exponential and Natural Logarithm Functions	547
15.1. Exponential Growth and Decay	548
15.2. Compound Interest	558
16. Integration	567
16.1. Antidifferentiation	567
16.2. Definite Integrals	576
16.3. Areas in the x - y Plane	584
16.4. Riemann Sums	594
16.5. Techniques of Integration	607
16.6. Improper Integrals	615
16.7. Applications of Calculus to Probability	621
17. Functions of Several Variables	633
17.1. Examples of Functions of Several Variables	633
17.2. Partial Derivatives	639
17.3. Maxima and Minima of Functions of Several Variables	649
17.4. Lagrange Multipliers and Constrained Optimization	659