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Theory of Resistive Mixers

Adel A. M. Saleh



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FOREWORD

This is the sixty-fourth volume in the M.I.T. Research Monograph Series published by the M.I.T. Press. The objective of this series is to contribute to the professional literature a number of significant pieces of research, larger in scope than journal articles but normally less ambitious than finished books. We believe that such studies deserve a wider circulation than can be accomplished by informal channels, and we hope that this form of publication will make them readily accessible to research organizations, libraries, and independent workers.

Howard W. Johnson

PUBLISHER'S FOREWORD

The aim of this format is to close the time gap between the preparation of a monographic work and its publication in book form. A large number of significant though specialized manuscripts make the transition to formal publication either after a considerable delay or not at all. The time and expense of detailed text editing and composition in print may act to prevent publication or so to delay it that currency of content is affected.

The text of this book has been photographed directly from the author's typescript. It is edited to a satisfactory level of completeness and comprehensibility though not necessarily to the standard of consistency of minor editorial detail present in typeset books issued under our imprint. A detailed table of contents is included and also an index.

The appendices are included on COSATI-standard microfiche and enclosed in an envelope at the back of the book.

The MIT Press

PREFACE

This research monograph deals with the theoretical aspects of mixers and frequency converters employing purely resistive nonlinear devices. The effects of parasitic reactances, which are bound to exist in practice, are neglected. It is the belief of the author that this is justified because today's semiconductor technology is capable of producing mixer diodes which exhibit almost purely resistive characteristics even at microwave frequencies. The need for new methods which make full use of the capabilities of these diodes are continuously on the rise. This book is an attempt to satisfy this need by finding fundamental limits on the performance of mixers and indicating the conditions necessary for their fulfillment.

It is hoped that the book will be useful to three classes of people; the student, the researcher and the design engineer. For the student, a large part of the available theory and principles of operation of mixers and frequency converters are given. For the researcher, the book adds new concepts to the theory of mixers, provides the answer to some fundamental questions and perhaps more important, it confronts the reader with more unanswered questions than those it answered. Finally, for the design engineer who is not satisfied with the performance of classic mixer circuits, the book shows

xiv

the way to improved circuits and compares them to their classic counterparts.

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TABLE OF CONTENTS

FOREWORD	xi
PUBLISHER'S FOREWORD	xii
PREFACE	xiii
ACKNOWLEDGMENTS	xv
1. INTRODUCTION	1
2. ANALYSIS AND CLASSIFICATIONS OF MIXER CIRCUITS	7
2.1. Notation for Linear Mixers	8
2.1.1. Frequency notation	8
2.1.2. Voltage and current notation	8
2.2. The Z- and the Y-Mixers	11
2.2.1. The basic circuits	13
2.2.2. Circuits with idlers	13
2.3. Separation of the Odd- and Even Order Frequencies	17
2.3.1. Single-ended and balanced mixer circuits	17
2.3.2. Four basic mixer circuits	20
2.4. The H- and G-Mixers	23
2.4.1. Time-domain analysis	23
2.4.2. Frequency-domain analysis	26
2.5. The N-ary Division Schemes	29
2.5.1. Basic concepts	29
2.5.2. The H_N - and G_N -mixers	34
2.6. Conclusions and Topics for Further Research	39
3. THE OPTIMUM RESISTANCE WAVEFORM AND IMBEDDING NETWORK	45
3.1. Three Basic Questions	45
3.2. Question 1: The Optimum Resistance Waveform	48
3.2.1. The Z- and Y-mixers	48

3.2.2.	The H- and G-mixers	54
3.2.3.	The H_N - and G_N -mixers	59
3.3.	Question 2: The Optimum Imbedding Network	64
3.3.1.	General properties of the optimum imbedding network	64
3.3.2.	Sinusoidal resistance and conductance waveforms	65
3.3.3.	Square-wave resistance or conductance	71
3.3.4.	Rectangular-pulse resistance and conductance waveforms	75
3.4.	Question 3: The Optimum Mixer	76
3.4.1.	Fundamental limit on the conversion loss	76
3.4.2.	Effect of junction capacitance and series resistance	78
3.5.	Conclusions and Topics for Further Research	81
4.	MIXERS WITH EXPONENTIAL DIODES	87
4.1.	Introduction	87
4.2.	Single-Diode Mixers	90
4.2.1.	The Y-mixer	91
4.2.2.	The Z-mixer	99
4.2.3.	The G-mixer	109
4.2.4.	The H-mixer	119
4.2.5.	Comparison	129
4.3.	Multi-diode Mixers	144
4.3.1.	Duality in ring mixers with exponential diodes	144
4.3.2.	Sinusoidal voltage pumping	147
4.3.3.	Sinusoidal current pumping	158
4.4.	Conclusions and Topics for Further Research	165

APPENDIX*

A1.	CONVERSION LOSS, PASSIVITY AND STABILITY OF TWO-PORT NETWORKS	173
A1.1.	Generalized Matrix Representation	173
A1.2.	Passivity	177
A1.3.	Stability	179
A1.4.	Conversion Loss	182
A1.5.	Sensitivity to Source Impedance Variations	188
A2.	MIXERS WITH IMAGE FREQUENCY TERMINATIONS	194
A2.1.	Generalized Matrix Representation	194
A2.2.	Mixers Represented by Real Matrices	198
A3.	THE LINEAR PERIODICALLY TIME-VARYING RESISTANCE	203
A3.1.	Time- and Frequency-Domain Representations	203
A3.2.	Power Relations	205
A4.	MATHEMATICAL DERIVATIONS OF OPTIMUM RESISTANCE WAVEFORMS	208
A4.1.	The Z- and Y-Mixers	208
A4.2.	The H- and G-Mixers	210
A4.3.	The H_N - and G_N -Mixers	213
A5.	PROOF OF THEOREMS CONCERNING OPTIMUM IMBEDDING NETWORKS	218
A5.1.	Proof of the Theorem	218
A5.2.	Discussing the Hypothesis	223
A6.	MATHEMATICAL DERIVATIONS FOR OPTIMUM IMBEDDING NETWORKS	228
A6.1.	The Symmetric Tridiagonal Matrix	228
A6.2.	Analysis of a Mixer with a Sinusoidal Resistance Waveform	231
A6.3.	Analysis of Two Modified H-Mixers with Square-Wave Resistances	235
A7.	SOME ASYMPTOTIC APPROXIMATIONS OF INTEGRALS	242
A8.	GENERAL STUDY OF Y-MIXERS WITH PULSED CONDUCTANCE WAVEFORMS	249

* The Appendices for this volume, pp. 173-271, will be found on microfiche at the back of the book.

x

A8.1.	General Approximations of the Mixer Equations	250
A8.2.	General Pulsed Conductance Waveforms	254
A8.3.	Some Specific Pulsed Conductance Waveforms	256
A8.4.	Comparison and Conclusions	259
A9.	STUDY OF AN EXPONENTIAL-DIODE Y-MIXER WITH AN OPEN-CIRCUIT SECOND HARMONIC	261
A9.1.	Purpose of the Study	261
A9.2.	Large Signal Analysis	262
A9.3.	Small Signal Analysis	266
A9.4.	Comparison with a Standard Y-Mixer	270

BIBLIOGRAPHY

INDEX

CHAPTER 1

INTRODUCTION

"Frequency converters" are used in communication receivers and transmitters, radars, radio and radar astronomy, control systems and many other applications. The frequency converter consists of a "local oscillator" or "pump" and a network containing one or more nonlinear devices and means to couple the local oscillator and the input and output signals. This network is called a "mixer". Inside the mixer the input signal, which is simply called the "signal", and the local oscillator output are "mixed" together in the nonlinear device to produce, among other frequencies, the required output frequency which is separated by filtering. The object is to achieve this with the least possible conversion loss, noise and nonlinear distortion. In heterodyne receivers, and many other applications, the output signal is called the "intermediate frequency" (i-f). This frequency is usually the difference between the signal and local oscillator frequencies, and in most applications it is much smaller than all the other frequencies involved in the mixer.

The nonlinear device can be a resistance, a reactance or, usually, a combination of both. Traditionally, if the basic mixing process is accomplished by a

nonlinear resistance, which might include some parasitic reactances, the network is simply referred to as a mixer. On the other hand if the mixing is basically accomplished by a nonlinear reactance, possibly containing resistive parasitics, the network is called a "parametric" frequency changer, although generally speaking it is still a mixer.

It is well known that a parametric up converter can produce a power gain which is equal to the ratio between the output and input frequencies;^{**} moreover it is unconditionally stable. This makes parametric up converters very attractive in practice. On the other hand, a parametric down converter is not desirable since, although capable of producing gain, it is potentially unstable. Other examples of mixers having conversion gain are those employing negative resistance devices, such as tunnel diodes.^{***} However, they too have considerable stability problems. For these reasons, down converters are almost always mixers employing essentially nonlinear positive resistances. This is the type of mixer which will be dealt with throughout this book. However, all our results will be applicable to, or can be slightly modified to include up converters; and some of our results can be applied to negative resistance mixers.

^{**} See for example Manley and Rowe (I) or Penfield and Rafuse (I).

^{***} See for example Chang, Heilmeyer and Prager (I); Kim (I); or Pucel (I).

As suggested from the title, the book will only be dealing with purely resistive mixers having ideal selective filters. The effect of parasitic reactances, which are bound to exist with any resistive device, will be neglected. It is the belief of the author that this theoretical work is needed and that it is justified with the aforementioned idealizations, even for mixers operating in the microwave frequency region, for the following reasons:

- 1) Today's semiconductor technology is capable of producing mixer diodes, such as Schottky-barrier diodes,** which exhibit almost purely resistive characteristics even at microwave frequencies.
- ii) The mixer imbedding networks which are built at present, together with their methods of pumping, and not the diodes and their parasitic reactances, are responsible for the greatest part of deterioration of their performances.
- iii) The treatment of mixers employing purely resistive diodes, although complex enough and contains many unresolved problems as will be seen from this thesis, is considerably simpler than a treatment involving the diode parasitics. This will help us to gain a better understanding of what constitutes

** These are metal-semiconductor junction diodes where current conduction is carried out almost entirely by majority carriers. See for example Sze (I, Chapter 8) for an excellent theoretical treatment.

a good mixer and to establish fundamental limits on its performance.

In Chapter 2, the methods of analysis of different mixers employing linear periodically time-varying resistances are presented. A logical classification system based on the terminations offered to the undesired out-of-band frequencies for the different mixers is given. The system employs names based on the types of network matrices, such as Z-, Y-, H- and G-matrices, which have to be used for the analysis. The analysis given to each mixer is limited to deducing the matrix equation relating the input and output voltages and currents from the resistance waveforms. To find the complete mixer performance one should use Appendix 1, where a complete analysis is given for conversion loss, stability and passivity of two-port networks based on generalized matrix representations. One of the new results in this appendix is the problem of sensitivity of the mixer performance to source impedance variations. Also, Appendix 2 provides the necessary equations for mixers with different image frequency terminations based on the generalized three-port matrices found in Chapter 2.

In Chapter 3, three basic questions concerning fundamental limits on the performance of resistive mixers are given. The questions involve obtaining the optimum

resistance waveforms for the different mixer imbedding networks introduced in Chapter 2, and finding the optimum imbedding networks for different nonnegative resistance waveforms. The parameter to be optimized is the mixer conversion loss. The theme of Chapter 3 can be considered as a search for the world's best mixer.

In both Chapters 2 and 3, the resistance waveform is assumed to be completely arbitrary and independent of the imbedding network. In practice, however, this is not true since the resistance waveform is obtained by pumping a nonlinear device in the imbedding network; and thus it depends on the device used, the method of pumping employed and the imbedding network. This is the purpose of Chapter 4.

In Chapter 4 different mixers employing ideal exponential diodes are analyzed and compared. The analysis given for each mixer consists of two main parts: large-signal and small-signal analyses. The first part includes the calculation of the pumped resistance waveform, the pump power, the resistance offered to the pump and the d-c bias conditions. In the second part of the analysis, the optimum values of the conversion loss and source and output resistance are given for different image frequency terminations. In each case asymptotic approximations are used to obtain simple formulas which are accurate for strongly driven mixers.