

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1370

G. Carlsson R.L. Cohen
H.R. Miller D.C. Ravenel (Eds.)

Algebraic Topology

Proceedings, Arcata 1986



Springer-Verlag

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Algebraic Topology

Proceedings of an International Conference
held in Arcata, California, July 27 – August 2, 1986



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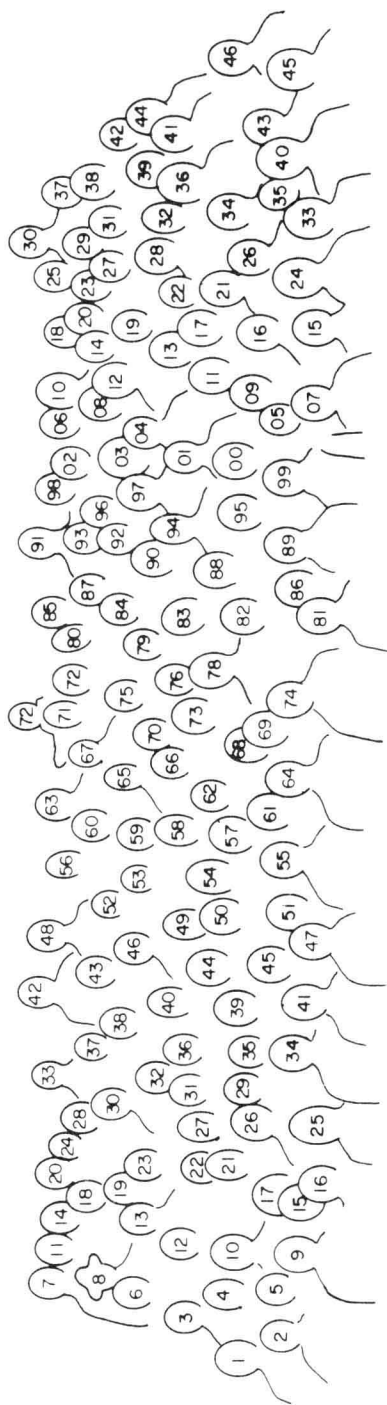
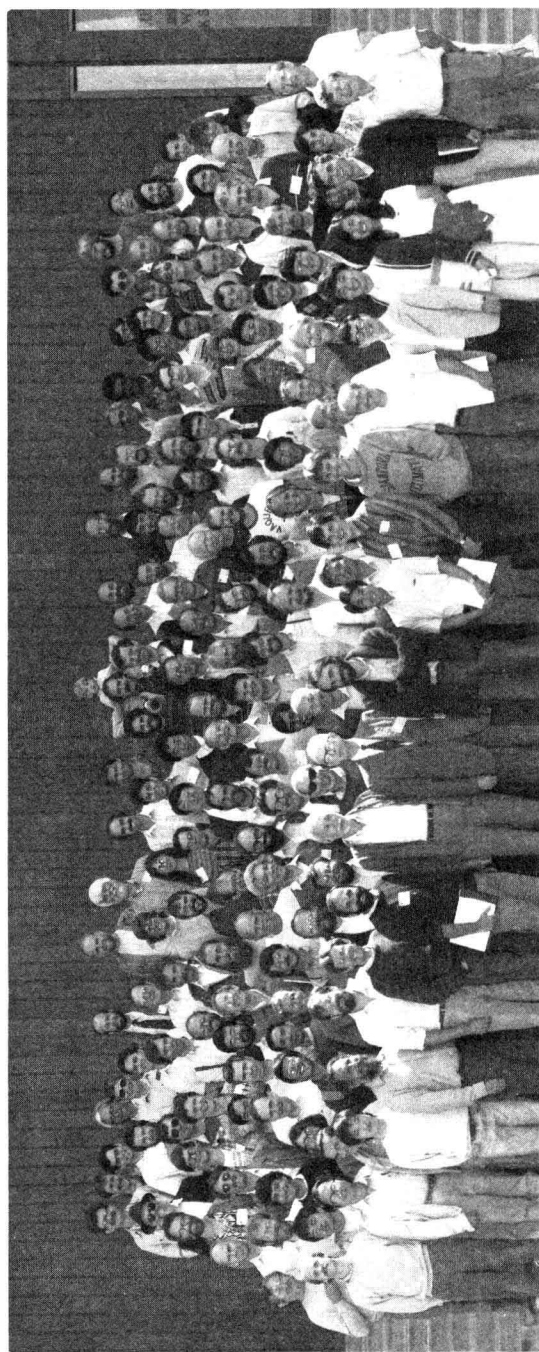
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PREFACE

An international conference on algebraic topology was held July 27 to August 2, 1986, at Humboldt State University in Arcata, California, prior to ICM86 in Berkeley. The conference served in part to celebrate the silver jubilee of the journal Topology, and the sixtieth birthday of Edgar H. Brown. It was supported by a grant from the National Science Foundation and a bequest from Pergammon Press, publishers of Topology.

The Proceedings contain papers submitted by conference participants. All were refereed, and we take this occasion to thank the referees. We would also like to thank the staff of the Jolly Giant Conference Center for its assistance in organizing this conference.

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Topology: past, present and future

I.M. JAMES

Since one of the main purposes of this meeting is to celebrate the Silver Jubilee of our journal Topology, the organizers have invited me, as one who has been an editor of the journal since its inception, to say a few words on the past, present and future of Topology. I hasten to say that the word is to be written with a capital T, not a small t; to discuss the past, present and future of the subject topology would be a mammoth task which might well occupy the whole of the five days we have at our disposal. However if I might have your attention for twenty minutes or so I should like to say something about the journal Topology. In fact to give my remarks an appropriate context I would like to begin with a few words about mathematical journals generally.

Mathematical journals, of course, have much in common with other scientific journals. These originated in the Proceedings of the older Scientific Academies. Pride of place must go to the Philosophical Transactions of the Royal Society of London, which has been appearing regularly since 1665. By comparison the Comptes Rendus Hebdomadaires of the Academy in Paris (1835) and the Proceedings of the Academy in Washington (1863) are newcomers. In this category one might also include the Proceedings of the Cambridge Philosophical Society (1843).

Journals such as these aim to cover the whole of science, although nowadays they are often issued in parts which concentrate on one area or another. They seem to me to belong to an age when science was more of a

unity and it was not unusual for an individual scientist to maintain a serious interest in and indeed to make research contributions to a wide range of different subjects. I would therefore be quite surprised if any more journals of this type were to be founded in future.

The second half of the nineteenth century saw the foundation of learned societies devoted to mathematics alone, rather than science generally. Each of these held regular meetings, at which papers were presented in the form of lectures, and it became usual to publish the proceedings of these meetings, just as the academies did. I believe the earliest example of such a journal is the Proceedings of the London Mathematical Society, which has been published without a break since 1865.

Later these Society journals were expanded to include other papers and gradually that kind of material became the norm so that the formal record of meetings now occupies only a small part of such publications. These days the leading Societies publish several journals of this type, usually including one or more of them in the membership package.

The Learned Society journals cover the whole range of mathematics, on the pure side at any rate. However there are also a number of distinguished independent journals which fulfill much the same role. Of these the oldest, so far as I am aware, are the *Journal für die Reine und Ungewandte Mathematik*, founded in 1826 and known by the name of Crelle who was its editor for so many years, and the similar *Journal de Mathématiques Pures et Appliquées*, founded ten years later, which is likewise generally known by the name of its first editor Liouville. Other independent

journals of high distinction included the Acta, which is closely associated with the Mittag-Leffler Institute, and the Annals, which draws its Editorial Board largely from members of Princeton University and the Institute for Advanced Study.

It was not until comparatively recently that specialist journals started to appear. An early example is the Fundamenta (1920) which as its name suggests was originally intended to specialize in the foundations of mathematics, particularly mathematical logic and general topology but which has developed into a journal of broad coverage like others I have mentioned earlier. This example, and a few others, dates from the period between the wars. However the idea of having specialist journals did not really catch on until the fifties, when several academic publishing houses began to promote specialist journals in various areas of pure and applied science, including mathematics. Not all of these new journals survived for more than a few issues but among those which did are some of very high repute. In this category I feel confident that Topology can be included and at this stage I would like to say a few words about the way in which it originated.

Shortly before his death in 1960, Henry Whitehead had been engaged in discussions with his old friend Robert Maxwell, the Chairman of Pergamon Press, about the foundation of a new International Journal for Mathematics, to be called Topology. He had got as far as selecting the Editorial Board, the Founder Editors being Michael Atiyah, Raoul Bott, Fritz Hirzebruch, René Thom and myself. After Henry Whitehead's untimely death it fell to me to carry out his intentions and continue negotiations with Pergamon Press

about all the things which need to be settled before a new journal can be launched. Robert Maxwell took a very personal interest in the new venture and made a number of valuable suggestions himself - for example at one stage a cover design by Picasso was contemplated and it would be fascinating to have seen what this would have been like.

In fact the first four issues which make up volume one mainly appeared in 1961 - which is why we are celebrating the silver jubilee this year. The call for contributions for the first volume met with an encouraging response, but I must admit that it was quite hard work obtaining material of the right quality for the first few volumes after that. This is often the case with a new journal and it is most important not to compromise standards at this critical stage - if one does so then it can be extremely difficult to recover later. Before long, however, first-class material began to arrive of its own accord and the editors found themselves in the strong position which they have enjoyed ever since.

A perusal of those early volumes leaves an impression of both quality and variety. One finds, for example, René Thom's long article "Topological models in biology", later to form the opening chapter of his well-known book of morphogenesis. On the geometric side of the subject one finds Milnor's paper on Microbundles, the basis for so much subsequent work, and on the algebraic side of the subject the paper in which Brown and Peterson established the existence of the spectrum which bears their name and which may possibly contain the key to some of the more intractable problems of homotopy theory. Right from the outset it was editorial policy to keep the

scope of the journal as broad as possible and one finds in these early volumes, just as one does today, articles on algebraic geometry, differential geometry, dynamical systems and many other subjects.

However, in that first decade the theme which seems to stand out in front of everything else is K-theory: so many of the most important papers in the development of that subject appeared in the pages of *Topology*. One must mention, for example, the classic article by Atiyah, Bott and Shapiro on Clifford modules, with which the Shapiro memorial supplement to volume 3 begins. One must also mention the well-known series of papers on the group $J(X)$ by Adams, which have been the stimulus for so much subsequent work in homotopy theory. In one of these papers Adams formulated the conjecture which bears his name, relating the classification of vector bundles by stable isomorphism to their classification by stable fibre homotopy equivalence of the associated sphere-bundle. And it is particularly gratifying to find that some seven years later Quillen published what I believe to have been the first proof of the Adams conjecture, also in the pages of our journal.

Of the original editors, only Fritz Hirzebruch and I now remain on the Board. The other editors at the present time are Bryan Birch, Simon Donaldson, Blaine Lawson, Larry Siebenmann and Graeme Segal, who has taken over from me as Editor-in-Chief.

The editorial organization has not changed greatly over the years. Normally the individual editors who receive papers arrange refereeing and decide, in the light of reports so obtained, which papers have a fair

chance of acceptance. Those which have are then considered further by a committee which meets at Oxford on a monthly basis and takes the final decision. Although all the editors are members of the committee it is usually the Oxford editors who bear the brunt of this work.

At this point I would like to pay tribute to our contributors - no journal could thrive without their support. Some have been loyal to us from the very earliest days, and we are particularly grateful to them for their contributions over the years. However almost every issue, certainly every volume, contains articles by people who have never written for us before. Such papers are always very welcome and we are proud of our record of outstanding "first papers". In fact, looking back over the first 25 years of our journal, I think we can be generally proud of our contribution to the development of some of the most vital areas of mathematical research and I hope we can continue to contribute in the years to come, wherever the future development of the subject may lead.

As with any leading journal, very much depends on the careful and conscientious work of referees, and I would like to take this opportunity to thank the many people who have helped us in this capacity over the years. Authors do not always appreciate referees' suggestions, however tactfully they may be expressed, and yet many an argument has been improved or mistake avoided by this means. In the case of Topology editors do not, as a rule, also act as referees themselves, but generally each of the more promising papers will have been read by one or more of the editors, as well as by the referee, before a final decision is made.

I would also like to express the thanks of the Editors to the staff of Pergamon Press, particularly Mike Church who has done so much for us over the years. The production of a journal is not such an easy matter as some might believe; in the case of Topology practically every difficulty which could conceivably have occurred has occurred. Although occasionally a party of editors has had to set off for Headington in high dudgeon about something or other, on the whole the partnership with the Press works extremely well and it should also be mentioned that Pergamon has contributed with characteristic generosity to the funding of this Silver Jubilee conference.

Among the hundreds of scientific and technical journals published by Pergamon Press, Topology is one of the most successful. It is taken not only by a large number of institutional libraries but also by a large number of individuals who take advantage of the specially attractive subscription rate which is available to them. The Editors try to ensure that every issue is interesting to the readers - to think only of the authors is a recipe for dullness in a journal. I do hope that those present at this meeting who do not already subscribe on an individual basis will consider doing so.

People sometimes ask about the scope of the journal. It is obviously very broad and yet some types of mathematics are unlikely to be found in our pages. The test the editors generally apply is: is this paper likely to interest our readers? There is no hard and fast rule therefore. When a new research area opens up, which passes that simple test, the editors can