INTRODUCTION TO
PHYSICAL
GAS DYNAMICS

Walter G. Vincenti and
Charles H. Kruger, Jr.

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Physical Gas Dynamics

Walter G. Vincenti

Department of Aeronautics and Astronautics
Stanford University

Charles H. Kruger, Jr.

Department of Mechanical Engineering
Stanford University

Preface

This book is the outgrowth of a series of courses developed over the past five years in the departments of Aeronautics and Astronautics and of Mechanical Engineering at Stanford University. These courses were introduced to instruct our students in the general features of high-temperature and nonequilibrium gas flows. Requests for the lecture notes that were prepared have encouraged us to believe that a book on these subjects might be useful for similar purposes in other schools of engineering. Since a knowledge of kinetic theory of gases, statistical mechanics, chemical thermodynamics, and chemical kinetics is basic to our subject, material on these disciplines is included. In the book, as in the courses, emphasis is placed on the ideas and structure of the subject and on certain basic results, rather than on technological problems of current concern. The aim is to bring the student to the point where he can understand more advanced treatises in the relevant sciences as well as the pertinent research literature in gas dynamics. In this way we attempt to do for high-temperature and nonequilibrium flows what existing books do for the classical field of the dynamics of perfect gases.

In developing the material we have attempted to maintain a balance, appropriate for engineering purposes, between microscopic physics and macroscopic gas dynamics. The term "physical gas dynamics" is intended to convey this idea. The treatment itself thus alternates between the discussion of physical and chemical processes and the illustration of how these processes influence the behavior of fluid flow. The material of the book falls, in a still different sense, into two major parts. Except for a brief discussion of transport phenomena in Chapter I, the first six chapters deal with questions of equilibrium. Here the physics and chemistry predominate, and gas dynamics appears only in Chapter VI. The second six chapters are concerned with nonequilibrium phenomena—molecular vibration and chemical reactions (which are essentially similar), molecular transport, and radiative transport. These are taken up separately and in order, with a chapter on the process itself followed by a chapter on the related gas dynamics. We do not mean to suggest that the various phenomena arise so neatly separated in practical problems. Our concern is

rather to illustrate the essential effects of each process with a minimum of complexity.

The assumed background for the book is that common to senior or first-year-graduate students in most engineering schools in the United States. For deliberate reasons, there is a considerable difference in the level at which the physics and chemistry and the gas dynamics begin. In the latter we assume that the student has had or is taking concurrently a course in classical gas dynamics based on material such as that found in the books by Liepmann and Roshko (*Elements of Gas Dynamics*, Wiley, 1957) or Shapiro (*The Dynamics and Thermodynamics of Compressible Fluid Flow*, Ronald, 1953). In the molecular physics and physical chemistry, on the other hand, we suppose only that the student has had the usual first undergraduate courses in physics and chemistry for engineers. At Stanford, and perhaps at other schools, these circumstances are in fact the case. This situation is now changing in engineering schools generally and eventually will cease to be true. In the meantime, the elementary material in the early chapters is available for first study, reference, or review. With regard to thermodynamics, we suppose that the student has had a general course in classical (i.e., equilibrium) thermodynamics as taught to most engineering students but has no acquaintance with the special topic of chemical thermodynamics. As to mathematics, Chapters I through VII presuppose a knowledge of elementary calculus plus a few topics from ordinary differential equations and advanced calculus. The later chapters assume also a familiarity with elementary partial differential equations to the extent required in gas dynamics at the level treated by Liepmann and Roshko or Shapiro. In general, the degree of sophistication of the book—and we trust that of the student—increases as the book progresses.

At Stanford, which operates on the quarter system, the material has been used in five ten-week courses, each meeting for three hours per week. A primary series of three courses extends over one academic year at the first-year graduate level and covers Chapters I through X. Only the first of these courses, covering the basic material of Chapters I through IV, is required as prerequisite for the third, which deals with the kinetic theory of Chapters IX and X. Most students, however, follow the complete series. A fourth graduate course, on radiative gas dynamics, is based on Chapters XI and XII. Again, only the first course in the primary series is a prerequisite. Chapters I, II, and IV also form the basis for a senior-level undergraduate course in kinetic theory and statistical mechanics.

A reader wanting to do independent, specialized study may be assisted by the groupings of topics and background material listed below, which can be pursued separately from a study of the remainder of the book.

Here Roman numerals indicate chapters, and Arabic numerals indicate sections. Items in parentheses are not essential for an understanding of the later material.

Equilibrium gas properties: III, IV, V
Equilibrium flow: III, IV, V, VI
Kinetic theory: I, II, (IV-9), IX, X
Transport properties: I, II, IX, X-1 to 8
Chemically reacting flow: III, IV, V-2 and 3, VII, VIII
Flow with vibrational nonequilibrium: IV-1 to 12, VII-1 to 3, 10, and 11, VIII
Radiative gas dynamics: (II-2 and 3), (IV-1 to 5), (IX-2), XI, XII

References throughout the book are listed alphabetically according to author at the end of each chapter. Citing of references in the text is by the author's name and the year of publication. The references have been chosen for the most part to provide fundamental derivations, alternative treatments, more advanced reading, or practical numerical data. Where we have relied heavily on some particular book, we have tried to acknowledge this fact. We have made no attempt in general to always cite original papers or to assess historical priority. This seemed hardly practical in a textbook covering such a diversity of subject matter. We have avoided using industrial or university reports, since such reports are often not available to the student even with considerable effort. A few exceptions have been made where important material was not available otherwise. It may be that our policies of referencing have led us to slight some of our colleagues. If so, we hope they will accept our apologies.

An index of mathematical symbols, akin to the usual subject index, is provided at the rear of the book.

WALTER G. VINCENTI
CHARLES H. KRUGER, JR.

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W. G. V.

C. H. K.

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Chapter I

Introductory Kinetic Theory

1 INTRODUCTION

We begin our study of physical gas dynamics with a discussion of some of the pertinent problems from the kinetic theory of gases. In kinetic theory a gas is considered as made up at the microscopic level of very small, individual molecules in a state of constant motion. If there is no movement of the gas at the macroscopic level, this motion is regarded as purely random and is accompanied by continual collisions of the molecules with each other and with any surfaces that may be present. The movement of a given molecule is thus to be thought of as a kind of random banging about, with its velocity undergoing frequent and more or less discontinuous changes in both magnitude and direction. Indeed, it is this freedom of motion of the molecules, limited only by collisions, that differentiates a gas from the more ordered situation that exists in a liquid or a solid.

When there is a general macroscopic movement of the gas, as is the usual situation in gas dynamics, the motion of the molecules is not completely random. This statement is, in fact, merely two ways of saying the same thing—the general motion is only a macroscopic reflection of the nonrandomness of the molecular motion. In particular, the familiar flow velocity of continuum gas dynamics is, from the molecular point of view, merely the average velocity of the molecules taken over a volume large enough to contain many molecules but small relative to the dimensions of the flow field. Zero flow velocity, that is, an average molecular velocity of zero, corresponds to random absolute motion of the molecules (i.e., molecules of a given speed have no preferred direction). In a flowing gas the molecular motion, although not random from an absolute point of view, will appear almost so to an observer moving at the local flow velocity. These ideas will be made precise in Chapter IX when we consider

the flow of a nonuniform gas in detail. For the time being we are interested primarily in the properties of the gas as such, and for this a consideration of the random motion is for the most part sufficient.

To make the foregoing ideas quantitative, we must introduce a precise assumption regarding the molecular model, that is, regarding the nature of the molecules and of the forces acting between them. Given such a model and the usual laws of mechanics, it would then be possible, in principle, to trace out the path of each molecule with time, assuming that the initial conditions are known (which, of course, they are not). Such a calculation, though of interest theoretically, would be enormously difficult. Fortunately it is also of little necessity. What we are really concerned with in most practical applications is the gross, bulk behavior of the gas as represented by certain observable quantities such as pressure, temperature, and viscosity, which are manifestations of the molecular motions averaged in space or time. It is the primary task of kinetic theory to relate and "explain" these macroscopic properties in terms of the microscopic characteristics of the molecular model.

The approach to this problem in the present chapter will be deliberately rough and nonrigorous—refinement will come later in Chapter II. The aim here is a qualitative understanding of certain ideas and some feeling for the numerical magnitudes that are involved. The results, however, are essentially correct.

The reader who wants to study kinetic theory more extensively than we shall be able to here will probably enjoy the book by Jeans (1940). His mechanical description (pp. 11–15) of the theory in terms of balls on a billiard table is particularly helpful in fixing the basic ideas. Among the other useful books in the field are those of Kennard (1938), Present (1958), and Loeb (1961). Other references will be given later in Chapter IX.

2   MOLECULAR MODEL

Further word on the molecular model is necessary. Aside from the fundamental attribute of mass, a molecule possesses an external force field and an internal structure. The external field is, as a matter of fact, a consequence of the internal structure, that is, of the fact that the molecule consists of one or more atomic nuclei surrounded by orbiting electrons. The somewhat arbitrary concentration of our attention separately on the "outside" and "inside" of the molecules, however, is a useful procedure. (For a discussion of the origin of interatomic and intermolecular forces at a beginning level, see Slater, 1939, Chapter XXII.)

The external force field is generally assumed to be spherically symmetric. This is very nearly true in many cases and to assume otherwise is almost prohibitively difficult. On this basis, the force field can be represented as in Fig. 1, which shows the force F between two molecules as a function of the distance r between them. At large distances the true representation (solid curve) shows a weak attractive force that tends to zero as the distance increases. At short distances there is a strong repulsive force that increases rapidly as the orbiting electrons of the two molecules intermingle. At
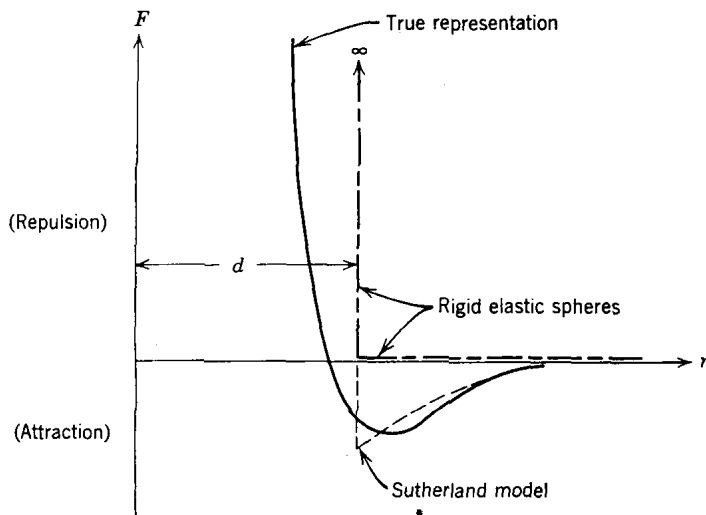

Fig. 1. Intermolecular force F as a function of the distance r between two molecules.

some intermediate distance, F passes through zero. The molecules, if they had no kinetic energy, would remain in equilibrium indefinitely at this distance.

Even with the simplest possible equation for the curve just described, the analysis is complicated. For this reason further approximation is often made. The simplest (Fig. 1) is to regard the molecules as rigid elastic spheres with zero attractive force when apart and an infinite repulsive force at the instant of contact. For like molecules, contact will take place when the distance between the center of the spheres is d, where d is the assumed diameter of the spherical molecule. A considerable amount of surprisingly accurate information can be obtained on the basis of this crude model, provided we chose d to give agreement between the resulting theory and the experimental data for some basic quantity such as viscous stress. Some improvement can be obtained at the expense of further complication by going to the so-called "Sutherland model." This

model supplements the rigid sphere by adding a weak attractive force when the spheres are not in contact. Other models are also possible. One that we shall find useful in Chapter IX dispenses entirely with the idea of solid spheres and assumes a pure repulsive force varying with some inverse power of the intermolecular distance.

The internal structure of the molecules is important primarily for its effect on the energy content of the gas. Being composed of nuclei and electrons that have motion relative to the center of mass of the molecule, the molecules can possess forms of energy (rotation, vibration, etc.) above and beyond those associated with their translational motion. In addition, the internal structure may be expected to affect the collisional interaction of the molecules at short range. Inclusion of the effects of internal structure considerably complicates the kinetic theory.

In the present book the treatment of intermolecular forces in any but the simplest way will be reserved for the discussion of nonequilibrium kinetic theory in Chapters IX and X. The effects of internal structure, at least insofar as the internal energy is concerned, will be taken up in detail in the discussion of equilibrium statistical mechanics in Chapter IV. For the present the molecular model that will be assumed, except for certain digressions, is that of the structureless, perfectly elastic sphere with no attractive forces and with repulsive forces existing only during contact. The characteristics of this "billiard-ball" model are completely specified by giving the mass and diameter of the various types of molecules, the number of each type of molecule per unit volume, and some measure of the speed of the random motion. 'Our task is to relate the macroscopic properties of the gas to these assumedly given microscopic quantities.

3   PRESSURE, TEMPERATURE, AND INTERNAL ENERGY

Consider a gas mixture in a state of equilibrium inside a cubical box. Suppose that the box is at rest so that the molecular motion is purely random. We wish to study the pressure exerted on the walls of the box by this random motion.

For simplicity we assume that the molecules do not collide with each other but only with the walls. We also assume that upon collision with a wall a molecule is reflected specularly (i.e., angle of reflection equals angle of incidence) and with its speed unchanged. It must be emphasized that neither of these assumptions is true; the fate of a given molecule upon collision with the wall is not known, and molecules do in reality collide with each other with great frequency. Intermolecular collisions are, in