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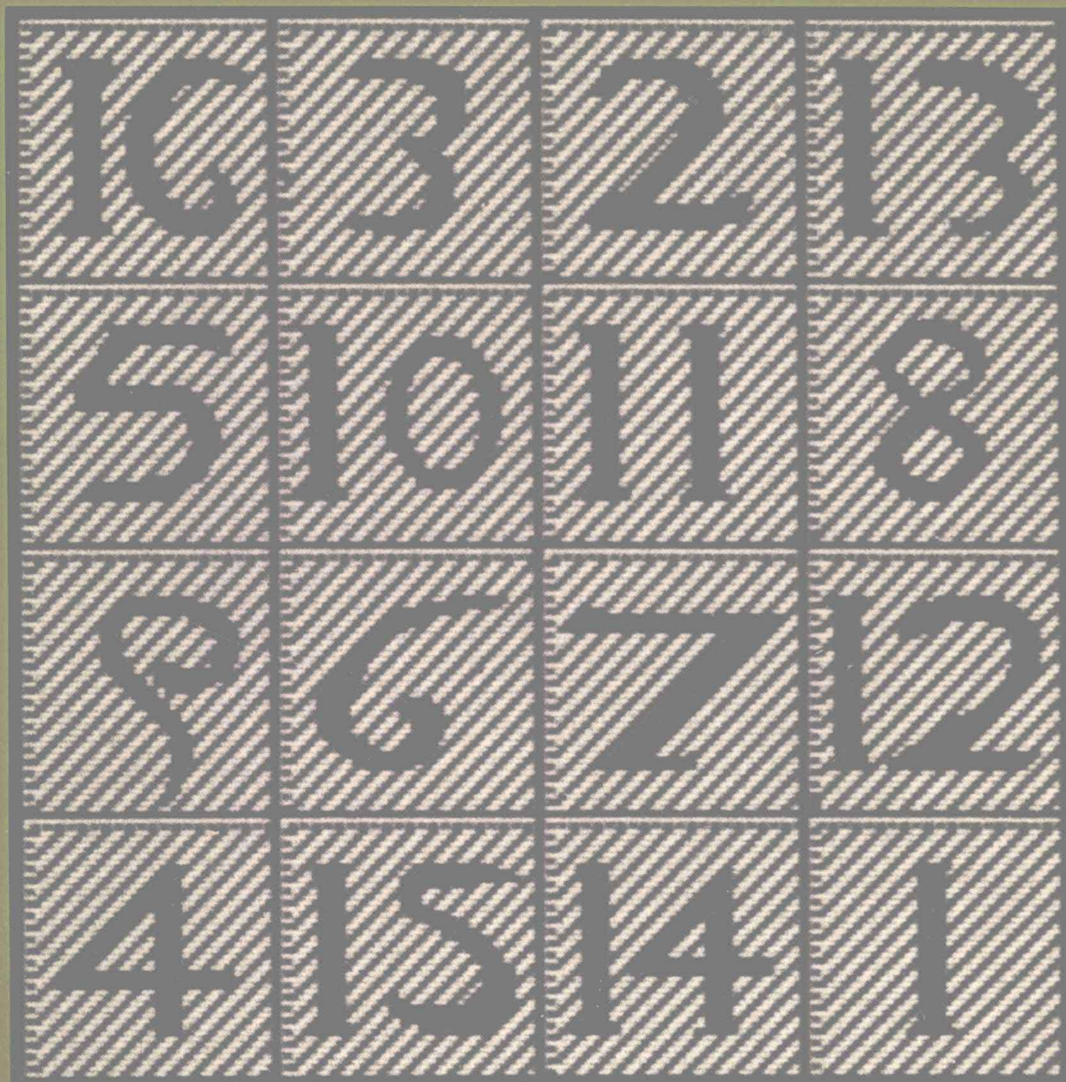

Teaching Secondary School Mathematics

TECHNIQUES AND ENRICHMENT UNITS

2ND EDITION

TEACHING SECONDARY SCHOOL MATHEMATICS

Techniques and Enrichment Units

2nd Edition

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PREFACE

How to teach secondary school mathematics well is what this book is all about. It is specifically written for pre-service mathematics teachers, yet it is designed to be useful for in-service junior and senior high school teachers as well.

The fact is, there is no dearth of books on virtually every aspect of education. The education section may even be one of the largest in a college library. We believe this text offers an alternative to others available with respect to both style and coverage. The style is direct and to the point, with a minimum of theory and a maximum of practical information from experienced teachers.

The book was written in two parts. Part 1 is a clear guide to daily teaching. It is, in effect, a handbook on how to prepare lessons, motivate young people, design homework assignments, prepare tests, evaluate student performance, and organize a classroom. The role of routine and challenging verbal problems and the role of calculators and computers as well as extracurricular activities are also considered. In addition, thoughts on how a teacher's performance may be evaluated and some reflections on the problem of pupil anxiety in a mathematics classroom are offered.

Part 2 is a unique section containing a wealth of enrichment ideas designed to be used by both junior and senior high school mathematics classes. Each enrichment unit is self-contained and includes performance objectives, preassessments, motivation ideas, teaching strategies, and postassessments. A cross-reference table relating the enrichment units to the appropriate topic, grade level, and ability level is provided at the start of Part 2. This table will facilitate the selection of an enrichment activity appropriate for a particular class.

The introductions to Part 1 (page 1) and Part 2 (page 199) develop the presentation of the contents of each part in more detail. It is recommended that you read these introductions at this point so that you may more fully appreciate what the authors are striving to achieve.

In recent years the national shortage of mathematics teachers has been gradually worsening. More new mathematics teachers are entering the classrooms without the traditional preparation formerly provided. At the same time there seems to be a dual thrust in mathematics education aimed at improving the mathematical skills of secondary school students and enriching the instruction of all ability levels of these students. With this Second Edition we have recognized this new audience of mathematics teachers, while at the same time addressing these two current emphases of instruction. In Chapter 1 we added a section on strategies for discovering generalizations in mathematics, while Chapter 8, Remediation, was updated with a discussion of computer assisted instruction. Sections were also added on professional development and responsibilities of the inservice teacher and a topic of concern to many inservice teachers: selecting a textbook for their classes. Also helpful are the newly added reading lists provided at the ends of chapters. Chapter 16 was almost entirely rewritten to reflect current research.

The book may be used as a text for both undergraduate and graduate courses in the teaching of secondary school mathematics as well as for an inservice course on topics in mathematics for secondary school teachers. The inclusion of the enrichment units (Part 2) makes this a valuable source of special topics and ideas for enriching mathematics instruction. The book is thus designed to play an active role in the mathematics teacher's professional library for many years to come.

ACKNOWLEDGMENTS

A book of this scope requires input from a wide variety of professionals. Many secondary school teachers participated in the preparation of the enrichment units, a majority of which were originally published by Croft-NEI Publications (Waterford, Connecticut). We extend our sincere gratitude to these fine teachers: Renee E. Baxter, Peter Catranides, Beatrice F. Cohen, Steven Colello, Joyce A. Dato, James DeMetro, Benito Gomez, Adele Hayda, Cynthia Horvath, Howard Kale, Gladys Kornfield, Arlene Kuperberg, Susan Loeb, David Martinez, Robert Parisi, Patricia Pearson, Steven Pottash, Soraida Rivera, Amelia O. Roges, Howard Sardis, Verna Segarra, Max Sharf, Malcolm Singer, Joseph Skittone, Jon Sontz, Daniel Stolnitz, Richard A. Vitulli, Stanley Weinstein, Barbara Winters, and Betty York.

We appreciate the assistance of two educational psychologists: Professor Robert H. Ellis of the State University of New York, College at Fredonia, made contributions to Chapter 16, "Overcoming Mathematics Anxiety." Professor Alfred Weiss of the City College of the City University of New York contributed "A Psychological View of Problem Solving" in Chapter 9 as well as the section on "Creativity in Problem Solving," also in that chapter.

We also received some expert criticism and input for two specialized chapters. Gerald H. Elgarten, former Director of Mathematics at the New York City Board of Education, and now Assistant Professor of Mathematics Education at The City College of the City University of New York, made some valuable contributions on microcomputers in Chapter 10, "Using Calculators and Computers in the Classroom." Steven Conrad, a well-known problem-solving expert, a leader in national and local mathematics contests, and a mathematics teacher at Benjamin Cardozo High School (New York City), offered some valuable comments on Chapter 9, "Problem Solving." Both have our thanks for their assistance.

Special thanks are offered to Jacob Cohen, assistant principal in charge of mathematics at Theodore Roosevelt High School (New York City), for his careful reading of the entire manuscript and for the very insightful comments he offered throughout. In particular, his work on Chapter 5, "Constructing Tests," and contributions to Part 2 were greatly appreciated. Thanks are also extended to Peter Catranides, teacher of mathematics at George Washington High School (New York City), for drawing some of the more complex geometric figures in Part 2.

Many of the revisions in the Second Edition resulted from comments generously offered by Professor Miriam E. Connellan (Marquette University), Professor William M. Waters, Jr. (North Carolina State University), Professor Claire M. Newman (Queens College, CUNY), and Professor Max Sobel (Montclair State College). We are grateful to these mathematics education experts for reviewing the text and providing their insights.

In the preparation of a manuscript two important services are required, typing and proofreading. For these services we extend our sincere thanks to Norma Levy, Yolanda Lora, Noreen Posamentier, Lisa Posamentier, and Deborah Stepelman.

We are also grateful to our editor, Beverly Kolz, for providing valuable support services and generally allowing us to work in a pressure-free atmosphere.

CONTENTS

PART 1 TEACHING MATHEMATICS

Part I offers "no-nonsense" suggestions covering the entire range of classroom related activities. We begin with one of the most essential skills in teaching: lesson planning. Following the rather detailed overview of the topic, the "nuts and bolts" of this skill are considered. Specifically, Chapters 2–5 discuss classroom questioning, motivation, assigning homework, and test construction in a very practical and easily adaptable manner. For example, the chapter on motivation describes (with lots of illustrative examples) eight techniques for motivating a lesson.

To function effectively in a school setting a teacher must not only be able to evaluate student performance, but also be aware of how his or her performance is evaluated. This is discussed in Chapters 6 and 7.

The first three "Recommendations for School Mathematics of the 1980s" set forth by the National Council of Teachers of Mathematics (*An Agenda for Action,* 1980) are that:

1. Problem solving be the focus of school mathematics in the 1980s.
2. Basic skill in mathematics be defined to encompass more than computational facility.
3. Mathematics programs take full advantage of the power of calculators and computers at all grade levels.

Each of these recommendations is thoroughly treated in a useable manner in Chapters 8–10 of the book. Of particular note is the chapter on problem solving, which contains implications for all levels of students and provides an extensive list of sources of non-routine mathematics problems.

By convention, enrichment and extracurricular activities are generally reserved for gifted students. We try to show in the next two chapters that this need not necessarily be the case. We discuss enrichment for the slow, average, and gifted students separately, with the hope that *all* students will eventually receive enriched instruction in mathematics. Extracurricular activities are often viewed as a burden by mathematics teachers. We address this problem by offering a wealth of ideas. Lots of activities are presented, including some useful notes for a mathematics team, and an extensive bibliography to assist in establishing extracurricular activities.

Chapters 13 and 14 presuppose an acquisition of the skills and concepts discussed in earlier chapters. They are intended to prepare and assist teachers in the organizational aspects of teaching. Chapter 15 is principally provided to buttress and sharpen the process of planning a lesson.

Chapter 16 is designed to make the teacher aware of possible causes of mathematics anxiety, so common among today's students. It offers some suggestions on how to deal with this problem and is specifically relegated to the end of this part of the book, so that it will remain clear in the reader's mind when he or she departs for the classroom.

Throughout Part 1 of the book samples of real, as well as ideal, teaching situations, strategies, lessons, tests, activities, and so on are offered in as practical and useful a manner as possible. This approach has frequently resulted in a diminution of explicit theoretical bases for many suggestions offered. This was done with the intention of not detracting from the usefulness of the book. End-of-chapter exercises are offered to allow readers to check their understanding of the skills and concepts discussed as well as provide an opportunity to expand their consideration of the topic.

CHAPTER 1 PLANNING A LESSON

A successful teacher instinctively possesses certain traits. He is capable of making simple and clear that which is complex and clouded. He enjoys what he is teaching and is an inspiration to those he is teaching. In addition, he should possess skills that may be acquired through training. For example, he should be capable of planning, organizing and presenting a formal lesson to a class of young people.

How does one acquire the skills to plan and organize a lesson? In this chapter we will show you some techniques for preparing several types of lessons. You will study a variety of actual lesson plans, some good, some not so good, taken from teachers' files. Through evaluation of these plans, you will gain insight into the thinking that goes into preparation, organization and execution of a lesson.

Can you imagine an actor without a script? A musician without a score? A speaker without an outline? A teacher without a lesson plan? These are anamolies indeed, for in every case it is as if the heart is removed from the soul. Because of the formal structure of our schools, every teacher is expected always to be well prepared, and entering one's classroom without a planned lesson is not tolerated. In fact, it may become grounds for severe warning, reprimand or dismissal if the pattern becomes a regular one. The teacher who walks into a classroom and begins a lesson with "Where did we end yesterday?" or who begins a lesson by taking out the text and merely reading from it to the class, or who teaches "off the cuff" from memory, performs a gross disservice to the young people in his charge. Such actions show that the teacher has a lack of concern, perhaps even disdain for them, that might easily be construed as unprofessional conduct.

THE UNIT PLAN

While it is most desirable that daily lesson plans be prepared on a day-to-day basis, they can be prepared as much as a week in advance. Of course, the probability of any individual lesson following the script exactly as written is small, so that when long range plans are written, adjustments must be made on a daily basis. Teachers should not be reluctant to make modifications just because it is easier to leave the original plan as written.

No meaningful daily lesson can be prepared in a vacuum, for the whole unit of work is the sum of its specific lessons. Each lesson plan should be prepared with the intent of meeting unit goals. Suppose that you are teaching a unit on "Radicals" in the first year algebra course. After reading the course of study and the appropriate material in texts, and after seeking guidance from experienced colleagues, you must decide on a sequence of logically arranged lessons that will cover all relevant topics. A review lesson and time for an examination should be included. Following is an example of such a unit:

UNIT: RADICALS

1. Powers and roots; square-root table
2. Pythagorean theorem; quadratic equations of the form $x^2 = 9$ and $x^2 = 3$
3. Square root algorithm
4. Simplifying radicals
5. Adding and subtracting radicals
6. Multiplying radicals
7. Dividing radicals
8. Rationalizing denominator
9. Radical equations
10. Review
11. Test

THE DAILY LESSON PLAN

You should plan in advance, in writing, every detail of your classroom activities. The amount of written detail versus the detail kept in the memory of the teacher will vary with the teacher and the number of years of experience as well as the number of times the particular course has been taught. But in any event you must know what you are going to do and what you expect to have your pupils do throughout the period. You must think through the statements you are going to make, the questions you will ask, and the answers you expect to get. You should try to anticipate incorrect answers so that you may prevent a lesson from going astray. An unexpected wrong answer or a sudden question by a slow learner, both of which might require extensive explanation by the teacher, can easily upset a lesson for the entire class. The beginner, in particular, should include the wording of his statements and questions in the written lesson plan, for it is so easy to forget the exact phraseology when you face a class.

A lesson may not turn out as planned for reasons you could not foresee. When this happens once in a while, there is no harm done. It merely supports the contention that every lesson must be carefully thought through.

Just as a successful performer spends lots of time rehearsing a performance, a teacher needs to rehearse a lesson. But the practical way for a teacher to rehearse is "mentally." You can best achieve this by planning your "performance" (lesson) on paper. Writing a good lesson plan will force you to go through a detailed "mental dry run" of the lesson. This activity not only allows you to crystallize your thoughts but also to anticipate possible pitfalls in the actual lesson.

Lessons should be written in a "plan book" so you will have an organized daily record of the work your classes have done every day of the term. This book can be used as a guide for you in the future, or as a record for your supervisors, or the parents, and as an aid to a substitute. Yet it is advisable to write new plans each time a course is taught. This will not only allow you to tailor each lesson to the specific class being taught, but it should go far in keeping your mind continuously stimulated.

The script that an actor uses follows a certain format, with appropriate variations which are determined by the nature of the performance itself, whether comedy, tragedy, musical, and so on. Similarly, the teacher's lesson plan follows a certain basic format. These plans will also vary somewhat, depending on the nature of the lesson being planned.

Following is the basic format of a lesson plan. Not all parts of this suggested arrangement need be included in every lesson. Following the chart are details and explanations of specific items found in this suggested format.

Topic

By stating the topic at the beginning of each lesson plan your focus becomes clear when you are planning the lesson as well as when you are simply filing it or looking for it.

Apperceptive Basis

This section calls for a listing of the previously learned material which is required for the present lesson and for which the students are responsible. Although the experienced teacher may frequently omit this section, it is useful for the starting teacher.

A SUGGESTED FORM FOR A LESSON PLAN

Class: ______________________ Date: ______________________

TOPIC:

APPERCEPTIVE BASIS (Previously learned knowledge):

AIM:

MOTIVATION:

DO-NOW EXERCISE:

DEVELOPMENT AND METHODS:

DRILL:

MEDIAL SUMMARY:

APPLICATIONS AND DRILL:

FINAL SUMMARY AND CONCLUSION:

HOMEWORK ASSIGNMENT:

SPECIAL EQUIPMENT NEEDED:

IF TIME:

Aim of the Lesson

1. Should be stated succinctly and written near the top of the lesson plan.
2. Should be made clear to the class early in the period, preferably elicited from the students as a result of a "need" you have demonstrated for topic (see Chapter 3, "Motivating Students").
3. May be written on the board, depending on teacher preference, school requirement or the particular lesson.

Motivation

This portion of the lesson channels student interest toward the topic to be learned. A common medium to achieve this is the do-now exercise. Further discussion of techniques of motivation is left to Chapter 3.

Do-Now Exercise

1. Is a class exercise, written in the same section of the board every day. It helps pupils settle down at the beginning of the period. It does indeed set the tone for the lesson and is therefore a most important part of the lesson.
2. Might serve the purpose of reviewing and reinforcing a previously learned topic, motivating the lesson, helping to develop a new topic or assessing the class' achievement.
3. Might also serve as a managerial device, allowing the teacher the time needed to perform the multitude of "little" chores needed to be done at the beginning (open or shut windows, clean boards, take attendance, return test papers, have students write homework solutions on the board, etc.). It could take five minutes, or ten or only three.
4. Could consist of examples made up by the teacher, or taken from the text, or it could be a brief selection to be read silently or aloud or a quiz to be collected and graded.
5. Should usually be discussed upon its completion, before the class moves on to the next phase of the lesson. Yet, occasionally its review could become part of the lesson.
6. May be omitted when the lesson does not require it. Beginning teachers, however, are best advised to use the do-now exercise regularly to achieve a smoother transition into the lesson than they might otherwise get. More experienced teachers may have developed techniques that don't rely so heavily on the do-now exercise.

Development and Methods

This is the heart of the lesson. The ability to develop and implant a new idea in students' minds distinguishes the teacher-skilled craftsman from the teacher-unskilled laborer. Just as in formal painting classes one can learn techniques and styles of painting, so in education courses one can learn techniques and styles of teaching. In neither course will you learn to be an "artist," for it is only after the training in these courses that your talent takes over. Thus, your first two or three years as a teacher are critical ones in your development. The hours you spend planning a lesson, the thinking you do about creating ways to introduce new topics, the reading you do to broaden your mathematics background, the enjoyment you derive from working with teenagers—all these will enrich you in the years ahead to make you the kind of teacher you want to be. Your attitude towards your teaching will set the tone for your performances and their results.

1. Major questions and statements should be recorded. These should be written in the order they will be asked, at the appropriate points in the lesson plan. You may refer to the plan book on your desk, or hold it with you, if you wish, as you walk about the room. (While holding the plan with you may indicate some insecurity on your part, if you, as a new teacher, feel that it will provide you with some confidence during a particularly difficult development that you want to do "just right," you should not hesitate to do so.)
2. Boardwork should be planned so there will be no crowding and ample room to write all elements of the lesson. The teacher ought to know such details as where the do-now will be written, where the development will take place, where the previous day's homework will be written, whether solutions to certain algebraic problems should be arranged horizontally or vertically, and finally, which drill exercises students will be asked to write on the board.

   The boardwork serves as a model for students. If you are neat, write legibly, are well organized and use a straightedge and compasses for diagrams, the chances are they will, too. This is an opportunity to help develop work habits that will benefit them (as well as you, when you mark their test papers) for the remainder of their lives.

Drill

The drill work, coming after the developmental portion of the lesson, is where theory yields to practice. The lesson is "clinched" at this point. Model solutions are demonstrated by the teacher and are followed by having students work on their own. At this point in the lesson, drill may be provided which involves *only* the newly developed concept or topic, not yet integrated with other previously learned material.

Medial Summary

This is a rather brief summary of the key points of the topic developed earlier. Among the many functions this medial summary can serve is allowing a student who is not quite clear about the aim of the lesson, or the nature of what was developed and its significance, or who may have simply daydreamed for some key moments at the early part of the lesson and is now lost, to catch up with the rest of the class and benefit from the remainder of the lesson.

Applications and More Drill

Now that everyone, hopefully, is caught up to the appropriate part of the lesson, and everyone has had an opportunity to practice using the new concept or topic in very simple drill exercises, this section provides for applications and drill which integrates the new concept or topic with other previously learned concepts or topics. These applications or drill exercises will be more complicated and more embedded in other topics than the first set of drill exercises. Naturally, the specific nature of the topic under consideration will determine the type of problems included in this or the other drill section; sometimes there may be little difference between the two sections.

Final Summary and Conclusion

The final summary is a brief review of the highlights of the lesson. Here the teacher should tie together all the parts of the lesson. This summary, as is the case with prior summaries as well, may likely be in the form of pointed questions from which students will provide the summary of the lesson, or simply statements of the summary, or combinations of both. Sometimes a simple question to the class such as "Suppose your classmate was absent and called you on the phone to ask what today's lesson was about; what would you tell him about the lesson so that he would get a clear idea of the topic he missed?" will elicit a summary.

Homework Assignment

Chapter 4 is devoted entirely to the nature of homework assignments. Since most mathematics lessons require some follow-up homework, it is necessary to include the assignment in each lesson plan. Any remarks you wish to make about any particular part of the assignment should be noted here.

Special Equipment to be Used

Special equipment to be used may be listed. These include items such as board compasses, graph chart, geometric models, overhead projector, etc. This list serves the purpose of reminding you what you must order from the supply room before the lesson. The special equipment to be used need not always be listed as part of the teacher's actual lesson plan. It might be listed, however, if one desires to present to an outside observer a complete picture of the lesson planned.

If Time

Every lesson plan should end with an "if time" section, which may include either additional drill exercises, or enrichment, or any other useful time filler. Also, you should plan to cover more than can be done in one period rather than find yourself with everything done several minutes before the bell is to ring. Those moments can be horribly long and uncomfortable!

TYPES OF LESSON PLANS

The most common types of daily lessons used by teachers in high school mathematics classes are described as "developmental," "lecture," "drill," "review," "mathematics-through-reading," and "individualized." The term "guided discovery" is also sometimes used in connection with "developmental" or "individualized" lessons. The names of the lesson types describe their purpose, and they will be discussed in the remainder of this chapter.

The Developmental Lesson

The *developmental lesson* technique may be used when you want to develop a new topic. It is probably the most difficult lesson of all the lesson types to present well, for it requires planning and a series of questions through which the new concepts are developed. It usually requires the longest preparation time, achieves the best results when done properly, and offers the greatest personal satisfaction to the teacher when it is completed. New teachers as well as experienced ones are capable of presenting this type of lesson well.

Following are two model developmental lesson plans for use with a high school geometry course. These plans demonstrate two types of strategies to direct the class towards discovering conclusions. Other strategies are, of course, possible.

Model Lesson Plan 1, pages 8–10, illustrates how a set of carefully worded, sequentially written questions can guide pupils to discover a proof of the Pythagorean Theorem.

Model Lesson Plan 2, pages 10–11, shows how, through a series of strategically placed (mostly oral) questions, all major properties of a parallelogram can be elicited and proved. Using the suggested technique of listing students' ideas, this lesson plan affords the opportunity to try to *prove* things that are, in fact, false and has the students discover where the proof breaks down and why, therefore, the propositions are false.

Model Lesson Plan 1 — *Comments*

TOPIC: The first lesson on the Pythagorean Theorem

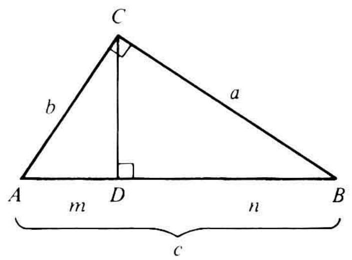
For the high school geometry course.

APPERCEPTIVE BASIS: Similarity, mean proportional theorems for the right triangle.

AIM: To introduce, prove and apply the Pythagorean Theorem.

These are the primary topics which the students need to know to be able to handle this topic.

DO-NOW EXERCISE:
In the figure below, $\overline{CD}$ is an altitude of right $\triangle ABC$ with right angle at C. The lengths of the segments are marked. Referring to the figure, complete each of the following:



This set of exercises will be distributed on spirit duplicator sheets.

1. AC is the mean proportional between (AB) and (AD).
2. Therefore $\frac{c}{(b)} = \frac{b}{(m)}$, or $b^2 =$ (cm). Why?
3. (BC) is the mean proportional between AB and BD.
4. Therefore $\frac{(c)}{a} = \frac{(a)}{n}$; or $a^2 =$ (cn). Why?
5. Adding the results of Exercises 2 and 4, we get $a^2 + b^2 =$ (cm) + (cn) = (c) $(m + n)$.
6. But $m + n =$ (c).
7. Therefore $a^2 +$ (b^2) = (c^2).

N.B. Circles indicate correct answers, which are obviously not included on actual answer sheets.

Notice also that these exercises review the mean proportional theorems while at the same time permit the student actually to prove the Pythagorean Theorem. Although students may not realize this at first, they will be lead to see this during the development of the lesson.

DEVELOPMENT AND METHODS:

1. Ask the class (make up a story) if they can get a 10 ft. diameter circular table top through a door which is only 8 ft. high and 6 ft. wide.

Do not rush through your story in an effort to get to the "meat" of the lesson. This will diminish the effectiveness of this approach.

2. Use the prepared overhead transparency to review to do-now exercise with the class.

This previously prepared transparency should be just like the copy of the do-now exercise given to the class.

3. Indicate to the class the significance of the do-now exercise. That is, *they* have just *proved* the Pythagorean Theorem.

This must be carefully presented so as to reap the full impact intended from these exercises.

4. Ask the class what Euclid and President James A. Garfield had in common. Now give the class some historical notes about the Pythagorean Theorem (such as the Egyptian "rope stretchers," the 360+ proofs, and so on).

Both proved the Pythagorean Theorem.

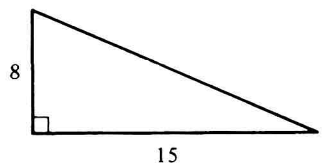
Comments

This brief discussion should generate some extra interest for this topic.

5. Discuss an application of the Pythagorean Theorem with the class (see Item 1 above).

Visual aids would be quite helpful here.

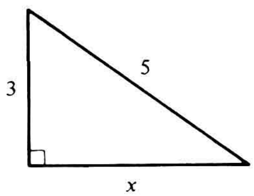
6. Find the length of the hypotenuse of the right triangle shown below:



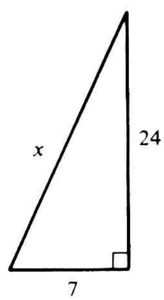
This is a very simple application and should be done together with the class.

DRILL: Find x in each of the following (send students to chalkboard):

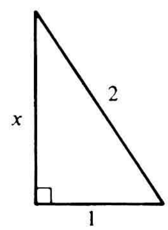
1.



2.



3.



These are simple exercises which only apply the Pythagorean Theorem and require no other prior knowledge.

Students will be asked to place their correct solutions on the chalkboard. These correct solutions will be detected by the teacher during his walk throughout the classroom while the students are working on these exercises.

MEDIAL SUMMARY (ask the following questions):

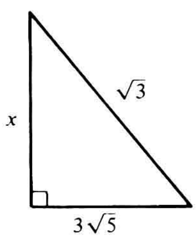
1. State the Pythagorean Theorem.
2. For what can we use the Pythagorean Theorem?
3. Can the Pythagorean Theorem be applied to any triangle where the lengths of two sides are given?

These questions should elicit the key points of the previous part of the lesson and thus serve as a summary to this point in the lesson.

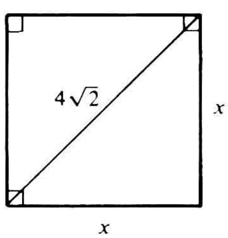
APPLICATIONS AND DRILL:

Find x in each of the following (send students to the chalkboard when assigning the problems to the class):

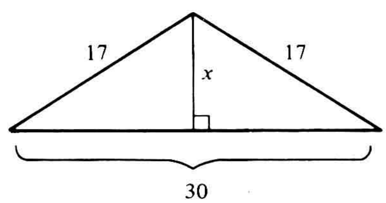
1.



2.



3.



Students should be assigned to do their work on the chalkboard immediately, rather than first to work the problems at their seats and then just to copy them over onto the board. The class can learn from classmate errors as well as from correct solutions.

4. *Given*: $\overline{PS}$ is an altitude of $\triangle PQR$
Prove: $PQ^2 - RP^2 = QS^2 - SR^2$