

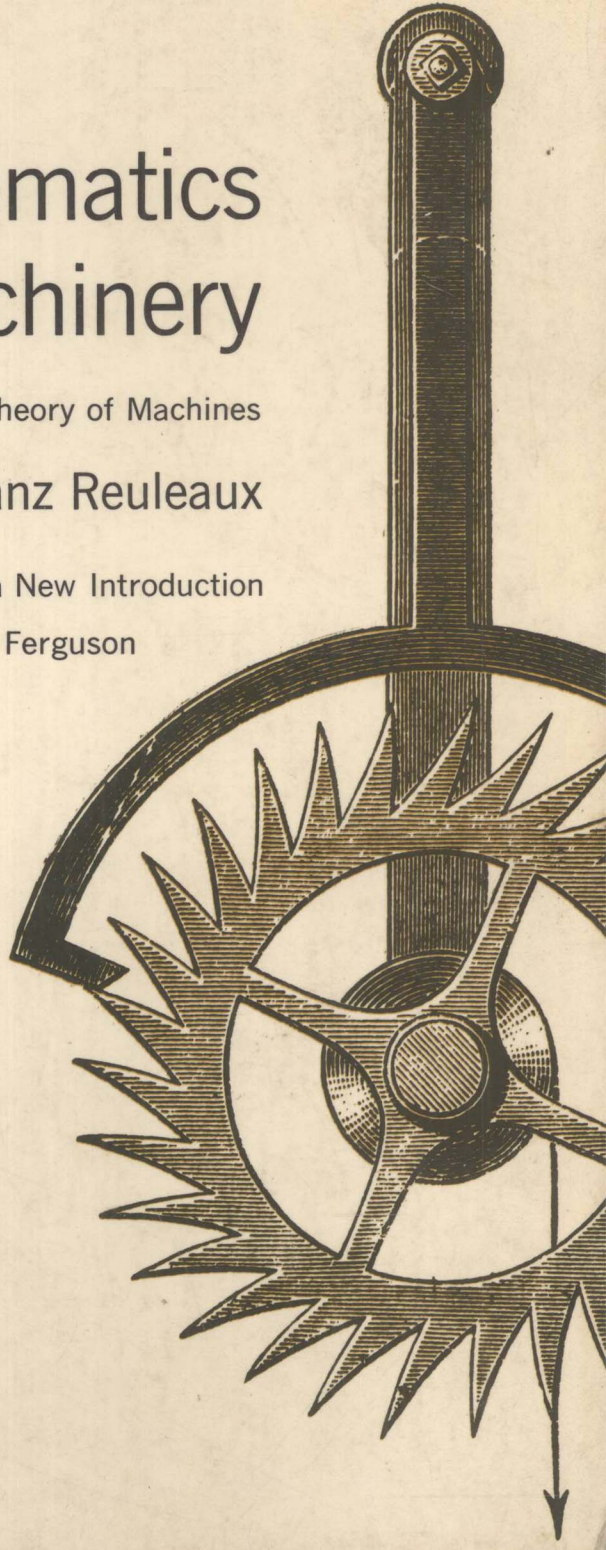
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# The Kinematics of Machinery

Outlines of a Theory of Machines

by Franz Reuleaux

With a New Introduction  
by Eugene S. Ferguson



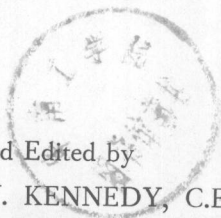
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# THE KINEMATICS OF MACHINERY

*Outlines of a Theory of Machines*

by Franz Reuleaux



Translated and Edited by  
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## INTRODUCTION TO DOVER EDITION

### *The Book*

OUR modern way of thinking about kinematics of mechanisms was established by this book. The study of kinematics, which is concerned with the discovery and application of principles underlying motions occurring in mechanisms, is based upon the assumptions and propositions set forth in these pages. The author's work has been elaborated upon, added to, and superseded in many details, but his conviction that this is the way we must look at mechanisms if they are to be understood has withstood the test of nearly a century of time.

Written by Franz Reuleaux (1829–1905) while he was professor of kinematics and director of the Royal Industrial Academy in Berlin, the treatise was published serially in the *Transactions* of the Society for the Advancement of Industry in Prussia, from 1871 through 1874. In 1875 the entire work was published in book form under the title *Theoretische Kinematik: Grundzüge einer Theorie des Maschinenwesens*; and in 1876 the present English translation by Alexander Blackie William Kennedy (1847–1928) was published in London.

Many of the ideas and concepts introduced in this book have become so familiar to us that we are likely to underestimate Reuleaux's originality and consider him merely a recorder of the obvious. We may feel that there is indeed no other way of approaching kinematics. But that is perhaps the hallmark of genius: to state a new idea in such convincing and uncompromising terms that it becomes immediately obvious and soon a truism.

Many men, before and after Reuleaux, have contributed to the development of kinematics of mechanisms. By any standards, however, the original and incisive ideas of Reuleaux, contained in this book, overshadow the contributions of others in the field.

Reuleaux's *Kinematics of Machinery* is not merely of interest historically, however. In it the reader has the opportunity of observing at work a brilliant mind and of sharing with the author some of the penetrating insights that enabled him to discover and identify the permanent truths of his subject. Reuleaux's sections on the process of innovation, for example (Chapter VI and parts of the Introduction and Chapter I) will be of interest to anybody who has ever seriously wondered how we go about doing anything that has not been done before.

The fundamental and original ideas that were first stated clearly by Reuleaux can be summarized briefly. While the concepts are few and simple, it is instructive to note that they establish the point of view from which we contemplate mechanisms today.

The author observed, for one thing, that machine elements are never employed singly, but always in pairs. He was thus able to identify the "kinematic pair." The idea of expansion of pairs led to the recognition of kinematic similarities between mechanisms that physically are quite dissimilar in form and arrangement.

Reuleaux was first to recognize, moreover, that the fixed link of a mechanism is kinematically the same as any of the moving links. From this discovery followed the powerful concept of inversion of linkages, in which different links are fixed successively to change the function of a mechanism.

His chapter on rotary engines and pumps (Chapter IX) demonstrates clearly the extent to which designers may be confused and deluded in their ignorance of the notions of expansion and inversion of linkages.

Reuleaux's list, furthermore, of the six classes of mechanical components from which all mechanisms are built, which appears near the end of the book (p. 480), has provided chapter or section headings for many if not most subsequent books on kinematics and machine design.

Finally, the possibilities of kinematic synthesis—that is, a systematic approach to the design of a mechanism to perform a given function—were first explored in this book. The author's chapter on



synthesis (Chapter XIII), which draws upon all the rest of the book, stresses a point that modern students of kinematics will heed with profit: that kinematic synthesis will be successful in direct proportion to the designer's understanding and appreciation of analysis. Reuleaux's objective was to show the designer "the essential simplicity of the means with which we have to work" and to remind him of the analytical principles that must be observed in any successful new design.

Reuleaux contributed little to velocity or acceleration analysis. Kennedy, on the other hand, in his *Mechanics of Machinery*, published in 1886, made extensive use of instant centers in velocity analysis. Kennedy receives proper credit for his law of three centers, which he discovered independently. It should be noted, however, that the law had been stated in 1872 by Siegfried Aronhold in an article that appeared alongside one written by Reuleaux in the journal that Reuleaux edited. The fact that Reuleaux did not include Aronhold's law in his book can be taken as evidence of his failure to appreciate its significance.

When we meet a subject at the undergraduate level we seldom have the background or the maturity to learn (or even to be concerned about) the underlying concepts upon which the subject is based. It is true that the technical axioms of the present book can be, and have been, boiled down to a relatively few pages in current textbooks. It is also true that subsequent additions by Reuleaux's successors have provided materials for extensive manipulation and problem solving. We can successfully use the techniques of kinematics without troubling ourselves to inquire into their origins. Nevertheless, unless we know about such things and can appreciate the organic nature of the subject's growth and development, it is unlikely that we shall have any clear or useful vision as to the directions in which we may guide the future development of kinematics.

### *The Author*

Franz Reuleaux was recognized within his own lifetime as a chief authority in mechanical engineering subjects associated with machine design. He wrote in this field many dozens of articles and three important pioneering books, of which the present book was

second to appear. He was also for forty years a gifted teacher, influencing a long generation of engineers directly as well as through his writings.

Reuleaux was born in Eschweiler, near Aachen (Aix-la-Chapelle), on September 30, 1829, the fourth son of Johann Josef Reuleaux, who had established one of the early machine shops in the Rhineland. At the age of twenty-one, having acquired in addition to traditional schooling a background of practical experience in machine building, Franz entered the polytechnic school in Karlsruhe, drawn thither by the reputation of Ferdinand Redtenbacher (1809-1863), professor of mechanical engineering. Redtenbacher was an able and perceptive teacher who conveyed to others his deep impatience with the existing traditional order of machine design and construction, and who was in his writings groping toward true principles of machine design. If the young Reuleaux had not yet settled upon a career when he arrived in Karlsruhe, it is clear that he had, after his two years with Redtenbacher, a sense of direction and dedication that stayed with him throughout his life. He went on to universities in Berlin and Bonn, where he pursued studies covering a wide variety of interests in the physical sciences and in philosophy. In 1856, at the age of twenty-seven, he was appointed professor of machine design in the polytechnic school in Zurich, where he remained for ten years and where his revolutionary ideas on the kinematics of mechanisms took shape.

Reuleaux, in collaboration with a classmate named Moll, had published two books before he was twenty-five years old. The first was a brief work on the strength of materials; the second was a long (983 pp.) practical treatise called *Principles of Machine Design*. Three years later he published a short treatise on springs, which had not before been treated analytically. The first of his three major books was on the theory of machine design. Gaining wide acceptance after its publication in 1861, *Der Konstrukteur* reached its fourth and final edition in 1889. This book was translated into English by the American engineer Henry H. Suplee. It was also translated into French and Russian.

In 1864, while he was yet in Zurich, Reuleaux had begun to lecture on the new kinematics. In 1865, he accepted a call to the Royal Industrial Academy in Berlin, of which he was made director in 1868. In 1879 he was a moving spirit in the founding of the Royal

Institute of Technology in Berlin, where he served as professor until he retired in 1896.

*Theoretische Kinematik*, Reuleaux's second major work, was, as noted above, published in 1875. His third major work, an extension of and enlargement of the author's ideas of kinematics, appeared in 1900 as *Die Praktischen Beziehungen der Kinematik zur Geometrie und Mechanik*. This final book, although not translated into English, has also made a lasting impression upon scholars of kinematics.

In addition to the books that he wrote from time to time, Reuleaux produced a constant stream of technical papers. His writing was loose and often prolix, showing little evidence of rewriting or revision, but it had a practical flavor and clarity of explanation that moved the editor of *Engineering* (London) to write in his review of the present book: "The subject is treated theoretically, but with a recognition of the claims of practice such as Englishmen do not generally associate with the writings of a German scientific professor." The reviewer found in Reuleaux's introductory comments on theory and practice (p. 54) the "promise that the theoretical treatment will not be too abstract to be useful"; he went on to conclude that "this promise is amply fulfilled."

Another side of Reuleaux's career was concerned with the promotion and advancement of German industry. He made significant contributions through his work in connection with international exhibitions. He was a judge of awards for the London Exhibition of 1862 and the Paris Exhibition of 1867; he was a commissioner to the Vienna Exhibition in 1873 and the Centennial Exhibition of 1876 in Philadelphia. He managed the German departments of the Sydney and Melbourne Exhibitions of 1879 and 1881, respectively, and he visited and wrote about the Columbian Exhibition of 1893 in Chicago. He wrote also a short history of the development of exhibitions.

It was after the summer of 1876, when he had served as a German commissioner to the Centennial Exhibition in Philadelphia, that Reuleaux became known throughout Germany as the author of the severely critical "Letters from Philadelphia," in which he characterized the industrial products of Germany as *billig und schlecht* (cheap and ugly). Publication of these letters helped steer technical effort toward the manufacturing excellence that has long been associated with German industry.



Reuleaux found time also to concern himself with the relationships between technology and the culture in which it exists. He wrote a number of papers on this subject, one of which, "Kultur und Technik," was translated into English and published in the Smithsonian *Annual Report* for 1890. While he had little that was original to say about culture, he undoubtedly made a positive contribution by showing other engineers that he considered such questions worth investigating.

In addition to his other projects, Reuleaux found diversion in such activities as translating into German Longfellow's *Hiawatha*, buying paintings and sculpture for the Royal Museum of Art, and encouraging the German Language Association to arrest the deterioration of the German language. According to one of his translators, who had known him personally, he was an accomplished linguist, speaking and writing freely in French, English, and Italian. However, he wrote for publication only in German.

A man of commanding personality, Reuleaux was one of those rare individuals who combine a brilliance of penetration and a clarity of original thought with methodical and untiring industry to produce a significant effect upon the direction in which his contemporaries and successors will go.

He died at his home in Berlin on August 20, 1905, at the age of 75.

EUGENE S. FERGUSON

Ames, Iowa  
April, 1963

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## PREFACE TO FIRST ENGLISH EDITION

THE greater part of the *Theoretische Kinematik* of Prof. Reuleaux, which I have now the pleasure of presenting to English and American readers, was originally published in chapters in the *Berliner Verhandlungen*, under the title of *Kinematische Mittheilungen*. These papers, revised and enlarged, and with the addition of a chapter on Kinematic Synthesis, were published collectively in 1874-5 in the work of which the present is a translation. They have attracted considerable attention in Germany, and the principles laid down in them have already made their way into Polytechnic School instruction, not only in that country but also in Russia and Italy.

The book addresses itself to somewhat different classes of readers, or rather to readers who have had very different training, on the Continent and here. Its readers there are to a great extent the past or present students of the Polytechnic Schools, or at least those who are acquainted with polytechnic teaching. They are familiar with a regularly systematised system of machine instruction and its somewhat extended literature. Here, on the other hand, neither systematised instruction nor extended literature exists. The book addresses itself greatly to practical engineers and mechanics, men who have often enough worked out their knowledge of the subject for themselves to a far greater extent than they have acquired it from books or lectures. To these readers some sections of the book may appear unnecessary, as referring to opinions or combating conclusions of which they have scarcely heard, and the erroneousness of which they are

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