

Applications of Perturbative QCD

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Editor's Foreword

The problem of communicating in a coherent fashion recent developments in the most exciting and active fields of physics continues to be with us. The enormous growth in the number of physicists has tended to make the familiar channels of communication considerably less effective. It has become increasingly difficult for experts in a given field to keep up with the current literature; the novice can only be confused. What is needed is both a consistent account of a field and the presentation of a definite "point of view" concerning it. Formal monographs cannot meet such a need in a rapidly developing field, while the review article seems to have fallen into disfavor. Indeed, it would seem that the people most actively engaged in developing a given field are the people least likely to write at length about it.

FRONTIERS IN PHYSICS was conceived in 1961 in an effort to improve the situation in several ways. Leading physicists frequently give a series of lectures, a graduate seminar, or a graduate course in their special fields of interest. Such lectures serve to summarize the present status of a rapidly developing field and may well constitute the only coherent account available at the time. Often, notes on lectures exist (prepared by the lecturer himself, by graduate students, or by post-doctoral fellows) and are distributed in mimeographed form on a limited basis. One of the principal purposes of the FRONTIERS IN PHYSICS Series is to make such notes available to a wider audience of physicists.

It should be emphasized that lecture notes are necessarily rough and informal, both in style and content; and those in the series will prove no exception. This is as it should be. One point of the series is to offer new, rapid, more informal, and, it is hoped, more effective ways for physicists to teach one another. The point is lost if only elegant notes qualify.

As FRONTIERS IN PHYSICS has evolved, a third category of book, the informal text/monograph, an intermediate step between lecture notes and formal texts or monographs, has played an increasingly important role in the series. In an informal text or monograph an author has reworked his or her lecture notes to the point at which the manuscript represents a coherent summation of a newly-developed field, complete with references and problems, suitable for either classroom teaching or individual study.

During the past decade, perturbative quantum chromodynamics (QCD) has emerged as a major field of study for physicists working in elementary particle physics; it has significant applications in other sub-fields of physics, including heavy ion collisions, cosmology, and relativistic astrophysics. An introductory account of QCD is thus especially timely and Rick Field is unusually well-qualified to write such an account. Together with Richard Feynman at Caltech he developed many of the basic ideas of QCD, while during the past decade he has lectured on these and subsequent developments in summer schools and in one- and two-semester courses at both Caltech and the University of Florida. I share his hope that his book will benefit theorists and experimentalists at every level of experience, from the graduate student with a beginning understanding of Feynman diagrams to the experimentalist who wishes to carry out his own QCD calculations. As a sometime tennis pal, it is a special pleasure to welcome him to the ranks of FIP authors.

David Pines

Urbana, Illinois
March, 1989

To Jimmie, Jason, Aimee, and Amanda

Preface

I have attempted to provide a book at the level of the first volume of Bjorken and Drell but on perturbative quantum chromodynamics (QCD) rather than quantum electrodynamics. The book contains very little formal field theory. I start with the Feynman diagrams of QCD and attempt to teach QCD by doing calculations. In doing the calculations in this book the reader will not only develop calculational skills and learn mathematical techniques, but I hope gain an appreciation and understanding of the theory that might be missed in a formal field theory course. I believe that by working through this book a student with a rudimentary understanding of Feynman diagrams will develop the skills and understanding necessary to perform individual research and will be able to contribute to the field of perturbative QCD. Furthermore, I feel the book should be beneficial to both theorists and experimenters. Hopefully with this book students interested in experimental physics can learn to perform their own QCD calculations.

QCD is a precise and complete theory of quarks and gluons which purports to be an ultimate explanation of all strong interaction experiments at all energies, high and low. There are many reasons to hope and expect it to be right. The question is, is it indeed right? Mathematical complexity has, so far, prevented quantitatively testing its correctness. The primary obstruction is the fact that the fundamental quarks and gluons of QCD apparently cannot be isolated as free particles, but are always confined within hadrons by strong forces not amenable to treatment by perturbative methods. Nevertheless, because QCD is an asymptotically free theory, interaction forces become weak at small distances (large energies) and calculations using perturbation theory and Feynman diagrams are possible. Unfortunately, most processes involve both low and high energy aspects, and one must separate the low energy pieces, which are not calculable by perturbative methods, from the high energy perturbative parts. The nonperturbative (low energy) pieces are parameterized, taken from data, or a model is built to describe the regime.

It would take only one *precise* comparison with data to "prove" QCD correct. However, "true tests of perturbative QCD" often turn out merely as tests of the authors' cleverness in parameterizing the nonperturbative uncalculable part of the problem and not as actual tests of QCD. Great care must be taken in examining the sensitivity of predictions to the uncalculable parts of the problem. The belief that perturbative QCD is correct comes from the fact that the theory correctly predicts the approximate behavior of a wide class of experiments. It provides an understanding of why the "naive" parton model works so well (weak interaction forces at short distances) and predicts deviations from the parton model that are seen experimentally.

In Chapter 2 and 3 of this book the QCD perturbative predictions for electron-positron annihilations are examined. One can learn a great deal about perturbative QCD by studying electron-positron annihilations. Since this process involves no color in the initial state, it provides an excellent "theoretical laboratory" in which to develop the tools of perturbative QCD. Calculations will be performed using two different regularization schemes; the "massive gluon scheme" and dimensional regularization. There are lessons to be learned in both schemes and by comparing the results one can see clearly those quantities that are regularization scheme dependent and those that are not. Throughout the book we will always compare the "QCD" result with the "naive" parton model.

Deep inelastic lepton nucleon scattering will be covered in Chapter 4 and Chapter 5 covers the large-mass muon pair production in hadron-hadron collisions. Here we will use the techniques learned in Chapters 2 and 3 and again all calculations will be performed using the two regularization schemes. Chapter 6 is a bit more formal and will cover in more detail renormalization and the running coupling constant of QCD which is only briefly discussed in the introduction in Chapter 1.

Chapter 7 covers applications of perturbative QCD to the production of particles and "jets" in hadron-hadron collisions. Much has been learned about QCD by studying hadron-hadron collisions that involve large transverse momentum or result in the production of a large mass object.

In Chapter 8 I present several other applications of perturbative QCD. Here I do not go into quite as much detail as in the previous chapters. However, after working through Chapters 2-7 the reader should be able to perform the calculations presented in Chapter 8. At the end of each chapter I have provided a modest reading list and references. My list is not as extensive as, for example, the book by Quigg and I refer the reader to his book for a more complete reading list.

I have provided a set of appendices containing information that should be useful in working through this book. They contain formulas that I have collected through the years starting when I was a Ph.D. student of Professor J.D. Jackson at Berkeley in 1971.

This book is an outgrowth of the summer school lectures I gave at La Jolla (1978), Boulder (1979), and SLAC (1986) and a graduate course I gave at CALTECH (1980) and at the University of Florida (1984). The CALTECH course was two semesters in length and I was able to cover all the material in Chapters 1 through 6 of this book. The Florida course was one semester in length and I covered only selected topics throughout the book. The book is suitable for a one or two semester special topics course in high-energy physics or it can be used selectively to supplement a course on relativistic quantum mechanics. In addition the book can be used for reference and self-study by both theorists and experimenters.

I could not have written this book without the six years of collaborative work I did with Professor Richard Feynman at CALTECH. Many of the discussions I have presented here are an outgrowth of our work or are an outgrowth of things I learned from Professor Feynman. I am very thankful for the privilege and opportunity of working with Richard Feynman. I am also grateful to Geoffrey Fox and Stephan Wolfram both of whom taught me a lot while I was at CALTECH and both have made contributions to this book. I would like to thank the teaching assistants of the course I taught at CALTECH (1980), Rajan Gupta and Steve Otto. They provided solutions for all the homework problems I assigned throughout the course and thereby contributed greatly to this book.

I am grateful to the Aspen Center for Physics for the warm hospitality shown to me during the summer of 1987 where I wrote the first three chapters of this book. I also appreciate very much the hospitality shown to me by R. Schrieffer, S. Ellis, and A. Mueller at the Institute for Theoretical Physics at Santa Barbara where I attended a six months workshop on "QCD and its Applications" in Spring of 1988. I learned a lot at the workshop that helped me to write this book.

I would like to thank my tennis pals (Aimee, Amanda, Dee Dee, George, Howie, Jason, Jeff, Jimmie, Joyce, Lorna, Mike, Mike, Pierre, Rick, and Ron) for helping me keep my priorities straight.

Finally, I am grateful to my wife Jimmie for taking good care of me and our three children, Jason, Aimee, and Amanda, while I worked on this book.

*R.D. Field
Florida, Fall 1988*

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