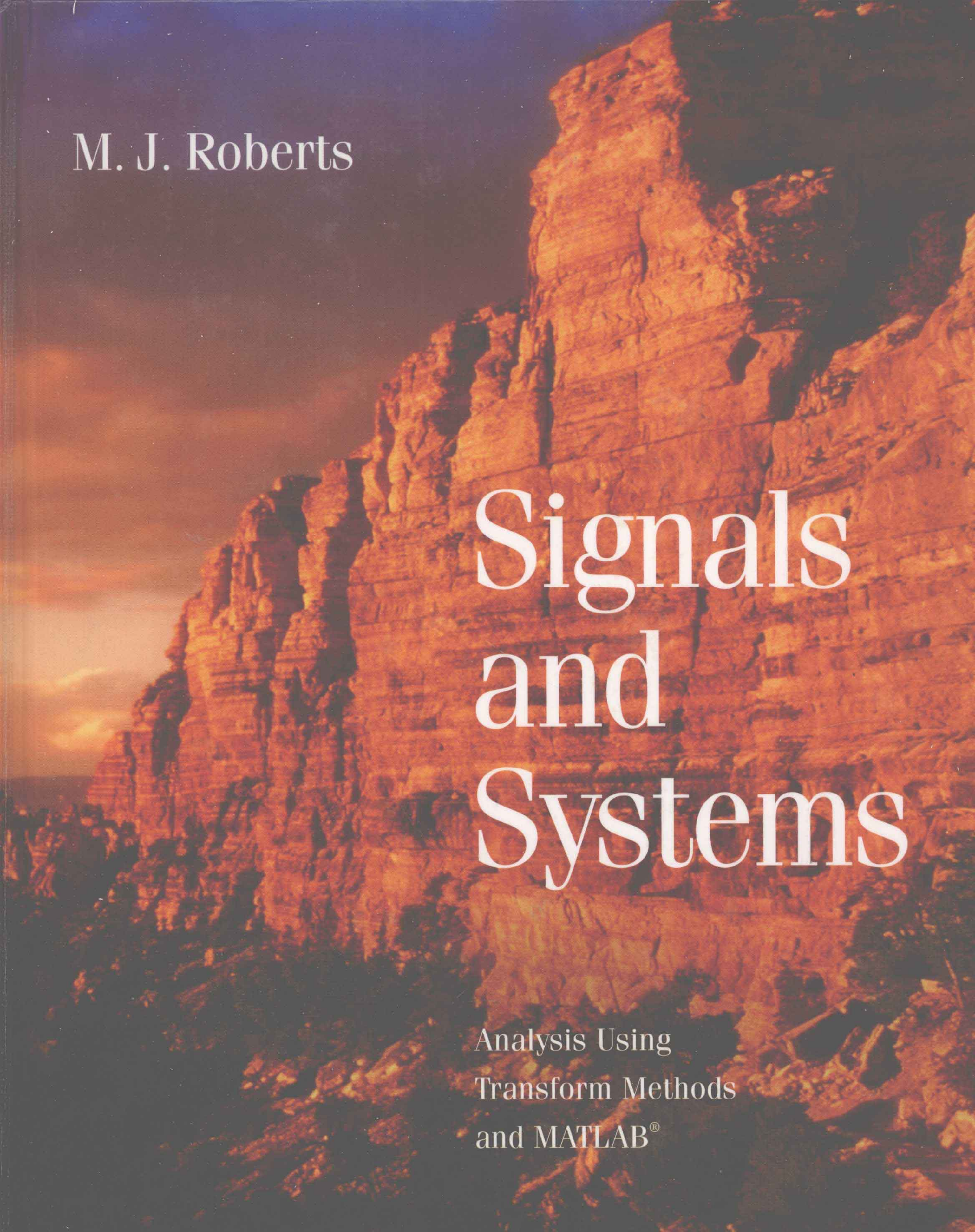
M. J. Roberts

Signals and Systems

Analysis Using Transform Methods and MATLAB®



Signals and Systems

Analysis Using Transform Methods and MATLAB®

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To my wife Barbara for giving me the time and space to complete this effort
and to the memory of my parents, Bertie Ellen Pinkerton and Jesse Watts Roberts,
for their early emphasis on the importance of education.

PREFACE

MOTIVATION

I wrote this book because I love the mathematical beauty of signals and systems analysis. As in most areas of science and engineering the most important and useful theories are the ones, like Newton's laws, Maxwell's equations, and Einstein's theory of relativity, that capture the essence, and therefore the beauty, of physical phenomena. I don't know how many hours I have spent writing this material, although it must be at least several thousand, but I think it would be difficult, if not impossible, for anyone to do this much work without a real passionate commitment to it.

AUDIENCE

I have written class notes for my junior-level classes in this area for many years and in 2000 decided that these notes had reached a level of maturity such that I could contemplate publishing them more widely. This book, which grew out of those class notes, is intended to cover a two-semester course sequence in the basics of signal and system analysis during the junior year. It could also be used in a senior-level course in these topics, although in most engineering curricula this material is covered in the junior year. It can also be used (as I have used it) as a book for a quick one-semester master's-level review of transform methods as applied to linear systems.

OVERVIEW

The book begins with mathematical methods for describing signals and systems, in both continuous and discrete time. I introduce the idea of a transform with the Fourier series, and from that base move to the Fourier transform as an extension of the Fourier series to aperiodic signals. There is a chapter on applications of Fourier analysis, including filters and communication systems. After covering Fourier methods, I use them in explaining the implications of sampling and in the analysis of the correlation between two signals and the energy and power spectral density of signals. I introduce the Laplace transform both as a generalization of the continuous-time Fourier transform for unbounded signals and unstable systems and as a powerful tool in system analysis because of its very close association with the eigenvalues and eigenfunctions of continuous-time linear systems. Then I present applications of the Laplace transform in circuit analysis, feedback systems, and multiple-input, multiple-output systems. I take a similar path for discrete-time systems using the z transform. In the last chapter I spend significant time on approximating continuous-time systems with discrete-time systems with extensive coverage of digital filter design methods. Throughout the book I present examples and introduce MATLAB functions and operations to implement the methods presented. A chapter-by-chapter summary follows.

CHAPTER SUMMARIES

Chapter 1 is an introduction to the general concepts involved in signal and system analysis without any mathematical rigor. It is intended to motivate the student by demonstrating the ubiquity of signals and systems in everyday life and the importance of understanding them.

Chapter 2 is an exploration of methods of mathematically describing signals of various kinds. It begins with familiar functions, continuous-time (CT) sinusoids and exponentials, and then extends the range of signal-describing functions to include CT singularity functions (switching functions) and other functions that are related to them through convolution and/or Fourier transformation. Like most, if not all, signals and systems textbooks, I define the unit step, the signum, the unit impulse, the unit ramp, and the unit sinc function. In addition to these I define the unit rectangle, the unit triangle, and the unit comb function (a periodic sequence of unit impulses). I find them very convenient and useful because of the compact notation that results. The unit comb function, along with convolution, provides an especially compact way of mathematically describing arbitrary periodic signals.

After introducing the new CT signal functions, I cover the common types of signal transformations (amplitude scaling, time shifting, time scaling, differentiation, and integration) and apply them to the signal functions. Then I cover some characteristics of signals that make them invariant to certain transformations (evenness, oddness, and periodicity) and some of the implications of these signal characteristics in signal analysis.

The next major section of Chapter 2 is coverage of discrete-time (DT) signals following a path analogous to that followed in CT signals. I introduce the DT sinusoid and exponential and comment on the problems of determining the period of a DT sinusoid. This is the student's first exposure to some of the implications of sampling. I define some DT signal functions analogous to CT singularity functions. Then I explore amplitude scaling, time shifting, time scaling, differencing, and accumulation for DT signal functions pointing out the unique implications and problems that occur, especially when time-scaling DT functions.

The last section of Chapter 2 is on signal energy and power. I define both for CT and DT signals and comment on the need for both by defining and discussing energy signals and power signals.

Chapter 3 is an introduction to the mathematical description of systems. First I cover the most common forms of classification of systems (homogeneity, additivity, linearity, time invariance, causality, memory, static nonlinearity, and invertibility). By example I present various types of systems which have, or do not have, these properties and how to prove various properties from the mathematical description of the system.

The next major section of Chapter 3 is the introduction of impulse response and convolution as components in the systematic analysis of the response of linear, time-invariant DT systems. I present the mathematical properties of convolution and a graphical method of understanding what the convolution-sum formula says. I also show how the properties of convolution can be used to combine subsystems which are connected in cascade or parallel into one system and what the impulse response of the overall system must be. This section is followed by an analogous coverage of CT convolution. This order of coverage seems best because the students are better able to understand how to find the impulse response of a DT system than the impulse response of a CT system. Also DT convolution is easier to conceive because there are no limit concepts involved. The last section of Chapter 3 is on the relations between block diagrams of systems and the system equations.

Chapter 4, on the Fourier series, is the beginning of the student's exposure to transform methods. I begin by graphically introducing the concept that any CT signal with engineering usefulness can be expressed over a finite time by a linear combination of CT sinusoids, real or complex. Then I show that periodic signals can be expressed for all time as a linear combination of sinusoids. Then I formally derive the Fourier series using the concept of orthogonality (without the name at this point) to show where the signal description as a function of discrete harmonic number (the harmonic function) comes from. I mention the Dirichlet conditions to let the student know that the CT Fourier series applies to all practical CT signals, but not to all imaginable CT signals.

There is a major section on simply following the mathematical process of finding the harmonic function for a time function, with many graphical illustrations, beginning with a single sinusoid and progressing to multiple sinusoids and nonsinusoidal functions. Along the way the concepts of orthogonality and correlation arise naturally and are briefly discussed.

The next few sections in Chapter 4 are an exploration of the properties of the Fourier series. I have tried to make the Fourier series notation and properties as similar as possible and analogous to the Fourier transform which comes later. That is, the harmonic function forms a Fourier series pair with the time function. As is conventional in most signals and systems textbooks, I have used a notation for all the transform methods in which lowercase letters are used for time-domain quantities and uppercase letters are used for their transforms (in this case their harmonic functions). This supports the understanding of the interrelationship among the Fourier methods. I have taken an ecumenical approach to two different notational conventions that are commonly seen in books on signals and systems, control systems, digital signal processing, communication systems, and other applications of Fourier methods such as image processing and Fourier optics; the use of either cyclic frequency f or radian frequency ω. I use both and emphasize that the two are simply related through a change of variable. I think this better prepares students for seeing both forms in other books in their college and professional careers. Also I emphasize some aspects of the Fourier series, especially with regard to using different representation periods, because this is an important idea which later appears in Chapter 7 on sampling and the discrete Fourier transform (DFT). I encourage students to use tables and properties to find harmonic functions, and this practice prepares them for a similar process in finding Fourier transforms and later Laplace and z transforms. I also have a section on the convergence of the Fourier series illustrating the Gibbs phenomenon at function discontinuities.

The next major section of Chapter 4 covers the same basic concepts as the first section, but as applied to DT signals. I emphasize the important differences caused by the differences between continuous- and discrete-time signals, especially the finite summation range of the DT Fourier series as opposed to the (generally) infinite summation range in the CT Fourier series. I also point out the importance of the fact that the DT Fourier series relates a finite set of numbers to another finite set of numbers, making it amenable to direct numerical machine computation. Later, in Chapter 7, I show the strong similarity between the DT Fourier series and the discrete Fourier transform (DFT).

Chapter 5 extends the concepts of the Fourier series to aperiodic signals and introduces the Fourier transform. I introduce the concept by examining what happens to a CT Fourier series as the period of the signal approaches infinity and then define and derive the CT Fourier transform as a generalization of the CT Fourier series. Following that I derive all the important properties of the CT Fourier transform. The next major section covers the DT Fourier transform, introducing and deriving it in an analogous way. There are numerous examples of the properties of both the CT and DT Fourier transform.

The last major section is a comparison of the four Fourier methods. This section is important because it reemphasizes many of the concepts of (1) continuous time and discrete time, and (2) sampling in time and sampling in frequency (which will be important in Chapter 7 which covers sampling and the discrete Fourier transform). I emphasize particularly the duality between sampling in one domain and periodic repetition in the other domain and the information equivalence of a sampled and an impulse-sampled signal.

Chapter 6 is devoted to the application of Fourier methods to two kinds of system analysis for which it is particularly well suited, filters and communication systems. I define the ideal filter and return to the concept of causality to show that the ideal filter cannot be realized as a physical system. This is an example of a design in the frequency domain which cannot be achieved, but can be approached, in the time domain. Then I discuss and analyze some simple practical passive and active filters and demonstrate that they are causal systems. Bode diagrams are introduced as a method of quick analysis of cascaded systems. Then I introduce the simplest forms of modulation and show how Fourier analysis greatly simplifies understanding them. I also explore the concepts of phase and group delay and demonstrate them with a modulated signal. In the next major section I apply the same modulation principles to DT signals and systems in an analogous way. In the last sections I briefly discuss the use of filters to reduce noise in communication systems and the operation of a spectrum analyzer.

Chapter 7 is the first exploration of the correspondence between a CT signal and a DT signal formed by sampling it. The first section covers how sampling is usually done in real systems using a sample-and-hold and an analog-to-digital converter. The second section starts by asking the question of how many samples are enough to describe a CT signal. Then the question is answered by deriving Shannon's sampling theorem, first using the DT Fourier transform to describe a DT signal formed by sampling a CT signal. Then I impulse-sample a signal to show the correspondence between a sampled signal and an impulse-sampled signal and use the CT Fourier transform to show the same result. Then I discuss interpolation methods, theoretical and practical; the special properties of band-limited periodic signals; and finally the discrete Fourier transform, relating it to the DT Fourier series. I do a complete development of the DFT starting with a CT signal and then time-sampling, windowing, and frequency-sampling it to form two signals, each completely described by a finite set of numbers and exactly related by the DFT. Then I show how the DFT can be used to approximate the CT Fourier transform of an energy signal or a periodic signal. The next major section is a sequence of examples of the use of and properties of the DFT, and the last major section discusses the fast Fourier transform and shows why it is a very efficient algorithm for computing the DFT.

Chapter 8 is on correlation, energy spectral density, and power spectral density. These topics are not often covered in a signals and systems textbook. They are traditionally introduced in (or after) a course on random processes. I introduce the ideas here from the point of view of analyzing the similarity of two signals. Correlation concepts are important in system identification and matched filtering. I demonstrate correlation, and then autocorrelation, using both random and deterministic signals, but in the exercises I only ask the students to analyze the correlation or autocorrelation of deterministic signals. Since energy spectral density and power spectral density are the Fourier transforms of the autocorrelation of energy and power signals, I include those topics also.

Chapter 9 introduces the Laplace transform. I approach the Laplace transform from two points of view, as a generalization of the Fourier transform to a larger class of signals and as a result which naturally follows from the excitation of a linear, time-invariant system by a complex exponential signal. I begin by defining the bilateral Laplace transform and discussing the significance of the region of convergence. Then I define the unilateral Laplace transform and use it for most of the rest of the chapter before returning to the bilateral form at the end. I derive all the important properties of the Laplace transform and fully explore the method of partial-fraction expansion for finding inverse transforms. Then examples of solving differential equations with initial conditions are presented. Lastly I return to the bilateral form and show that bilateral transforms can be found by using unilateral transform tables.

Chapter 10 covers various applications of the Laplace transform, including block-diagram representation of systems in the complex-frequency domain, system stability, system interconnections, feedback systems including root-locus and gain and phase margin, block-diagram reduction, system responses to standard signals, frequency response, Butterworth filters, and lastly standard realizations and state-space methods for CT systems.

Chapter 11 introduces the z transform. The development parallels the development of the Laplace transform except it is applied to DT signals and systems. I initially define a bilateral transform and discuss the region of convergence and then define a unilateral transform. I derive all the important properties and demonstrate the inverse transform using partial-fraction expansion and the solution of difference equations with initial conditions. Then I return to the bilateral transform showing that they can be found using unilateral tables. I also show the relationship between the Laplace and z transforms, an important idea in the approximation of CT systems by DT systems in Chapter 12.

Chapter 12 is the last chapter and deals with applications of the z transform. The main topics are approximating CT systems with DT systems, especially digital filter design as an approximation to optimal analog filters, responses to standard signals, system interconnections, standard system realizations, and state-space methods.

There are several appendices ranging from one page on how to find least common multiples to many pages describing the major commands and operations available in MATLAB. Appendices E, F, and G are tables of the Fourier methods, the Laplace transform, and the z transform, respectively. Appendices H, I, and J are on complex numbers and variables, differential and difference equations, and vectors and matrices, respectively, topics which are generally considered as background for a signals and systems course. These appendices are written like book chapters with exercises at the end and can be used for purposes of review if the students in a particular class need it.

CONTINUITY

The book is fairly integrated in its approach, and each chapter builds on earlier chapters. However, in a two-semester sequence spanning the entire book, the following topics could be omitted without loss of continuity.

System characteristics (except for linearity and time invariance).

The response of linear, time-invariant systems to periodic excitation using the Fourier series.

Some applications of Fourier methods including discrete-time filters, some of the modulation techniques, phase and group delay, and spectral analysis.

Sampling methods (as opposed to sampling theory).

Sampling discrete-time signals.

The fast Fourier transform algorithm.

All the discussions of correlation, energy spectral density, and power spectral density in Chapter 8.

The return to the bilateral Laplace transform at the end of Chapter 9.

Certain topics on Laplace transform applications like specific stability analysis methods, block-diagram reduction, and Butterworth filters.

Standard realizations of systems and state-space analysis, continuous-time and/or discrete-time.

The return to the bilateral z transform at the end of Chapter 11.

Digital filter design with MATLAB.

REVIEWS AND EDITING

I often tell my students that if they really want to learn a subject well, they should agree to teach a course in that subject. The process of standing up in front of a group of very intelligent people and presenting material is a strong discipline for learning the material (if the presenter is at all disturbed by public humiliation). After writing this book, I can amend that statement to say that if one wants to learn a subject *very* well, he or she should agree to write a textbook on it. The process of review is a somewhat similar discipline although not quite as public. The public part comes after the book is published. This book owes a lot to the reviewers, especially those who really took time and criticized and suggested improvements. I am indebted to them.

I am also indebted to the many students who have endured my classes over the years. I believe that our relationship is more symbiotic than they realize. That is, they learn signals and systems analysis from me and I learn how to teach signals and systems analysis from them. I cannot count the number of times I have been asked a very perceptive question by a student that revealed not only that the students were not understanding a concept but that I did not understand it as well as I had previously thought.

WRITING STYLE

Every author thinks he or she has found a better way to present material so that students can grasp it, and I am no different. I have taught this material for many years and through the experience of grading tests have found what students generally do and do not grasp. I have spent countless hours in my office one on one with students explaining these concepts to them, and, through that experience, I have found out what needs to be said. In my writing I have tried to simply speak directly to the reader in a straightforward conversational way, trying to avoid off-putting formality and, to the extent possible, anticipating the usual misconceptions and revealing the fallacies in them. Transform methods are not an obvious idea, and, at first exposure, students can easily get bogged down in a bewildering morass of abstractions and lose sight of the goal which is to analyze a system's response to signals. I have tried (as every author does) to find the magic combination of accessibility and mathematical rigor because both are important. I think my writing is clear and direct, but you, the reader, will be the final judge of whether or not that is true.

EXERCISES

The book contains over 500 exercises. Each chapter has a group of exercises with answers provided and a second group of exercises without answers. The first group is intended more or less as a set of drill exercises, and the second group as a set of more challenging exercises.

CONCLUDING REMARKS

Although I have tried hard to make this book a good one, no book in its first edition is perfect, and mine will not disprove that assertion. I have discovered what all authors must discover, that even though they feel they really understand the concepts and how to do all the exercises, with this much text and this many exercises mistakes are inevitable. Therefore, I welcome any and all criticism, corrections, and suggestions. All comments, including ones I disagree with and ones which disagree with others, will have a constructive impact on the next edition because they will point out a problem. If something does not seem right to you, it probably will bother others also, and it is my task, as an author, to find a way to solve that problem. So I encourage you to be direct and clear in any remarks about what you believe should be changed and not to hesitate to mention any errors you may find, from the most trivial to the most significant.

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