Elementary Nuclear Theory

H. A. BETHE

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A SHORT COURSE ON SELECTED TOPICS

By H. A. Bethe

Professor of Physics at
Cornell University

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MELVIN LAX

CONRAD LONGMIRE

ARTHUR S. WIGHTMAN

PREFACE

This book is not meant to be a textbook of the theory of atomic nuclei. It is merely a selection of certain topics in the theory, and even these topics are treated in only an elementary way. Until a more complete textbook is written, the reader who wishes to obtain a thorough knowledge of nuclear theory will have to consult the original literature, or for certain topics the articles of the present author in *Reviews of Modern Physics* (Vol. 8, p. 83, 1936; Vol. 9, pp. 69 and 245, 1937).

The emphasis in this book is placed on the problem of *nuclear forces*. This problem is the central problem of nuclear physics. The problem is treated entirely from the empirical point of view, and I have made an effort to present the evidence available on nuclear forces from the behavior of the simplest nuclear systems. Purely theoretical considerations about nuclear forces, particularly the meson theory of these forces, are treated with the greatest brevity, because they are not yet in a form in which they would permit useful predictions.

As a second field of nuclear physics which is sufficiently well developed and fundamental, I have chosen the theory of beta disintegration.

The theory of the compound nucleus and its consequences for the prediction of the probabilities of nuclear reactions I have treated only very briefly. The reason for this was partly a matter of time: the lecture course on which these chapters are based contained only twenty lectures, and it seemed more profitable to treat part of the theory thoroughly than to treat all of it superficially. Partly, however, the brevity of treatment of the more complicated nuclei was purposeful; in the last ten years the workers in this field have shown an inclination to devote a large proportion of their effort to the study of the complicated nuclei, and the danger exists that the right perspective may be forgotten. The wartime research in the atomic energy project tended further to emphasize the usefulness of the predictions from the theory of the compound nucleus. To correct this tendency, it seemed even more important to put special emphasis on the fundamental theory of nuclear forces and off the theory of the complicated nuclei.

The theory of the fission process has been left out entirely for the same reason: this process is, after all, only a very special phenomenon in nuclear physics.

The theory of alpha radioactivity could be left out with a good conscience because it is given in many elementary textbooks on wave mechanics. With some regrets I also had to leave out the theory of nuclear systems containing from 3 to 60 nuclear particles, especially the successful calculations of binding energies on the basis of group theory by Wigner.

H. A. BETHE

CORNELL UNIVERSITY
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A. DESCRIPTIVE THEORY OF NUCLEI

I. BASIC FACTS ABOUT NUCLEI

Each atomic nucleus has a charge Ze, a mass M, and a mass number A. Ze is an integral multiple of the charge e of the proton. M is very close to an integral multiple of the proton mass. The integer A which gives the multiple closest to M is the mass number.

The nuclear charge Z determines all the chemical properties associated with an element. It has values from $Z = 0$ (neutron) to $Z = 96$ (curium) for observed nuclei. Some of these do not occur in nature: $Z = 0$, 43, 61, 85, 87 (87 occurs in very small abundance as a member of a branch of the radioactive family of Ac), 93, 94, 95, 96.

The mass number A ranges from $A = 1$ (proton or neutron) to $A = 242$ (curium). Nearly every mass number in this range is found in nature. The notable exceptions $A = 5$ and $A = 8$ have good reasons for not being stable long enough to be observed even in the laboratory. The mass numbers of form $4n + 1$ beyond 209 (Bi) are not found in nature but many of them have been produced in the laboratory. These nuclei belong to a radioactive series which does not contain any long-lived members and, therefore, could not have survived on earth.

Isotopes. Nuclei of the same Z but different A are called isotopes. On the average there are about three stable isotopes for each Z. To distinguish isotopes A is usually written as a right superscript and for convenience Z is sometimes written as a left superscript. To illustrate: Si^{28}, Si^{29}, and Si^{30} are the stable isotopes of Si. In addition to the stable isotopes, most elements also possess radioactive isotopes, e.g., Si has the known isotopes Si^{27} and Si^{31}. Of these, Si^{27} is β^+ radioactive (having too little mass for its charge) and decays with a half-life of 4 seconds to Al^{27} and a positron

$$Si^{27} = \beta^+ + Al^{27}$$

Si^{31} (having too little charge for its mass) decays with a half-life of 170 minutes to P^{31} and an electron

$$Si^{31} = \beta^- + P^{31}$$

Isobars. For a given A, there may well be several possible values of Z (isobars). There are many instances of stable isobaric pairs, e.g., $^{22}Ti^{50}$ $^{24}Cr^{50}$, or $^{44}Ru^{104}$ $^{46}Pd^{104}$, and some stable isobaric triples, e.g., $^{40}Zr^{96}$ $^{42}Mo^{96}$ $^{44}Ru^{96}$, as well as numerous radioactive isobars.

Regularities. There are several striking regularities in a table of the stable nuclei. Nuclei of even Z are much more numerous than those of odd Z. Nuclei of even A are more numerous than those of odd A. Nearly all nuclei with even A have even Z; the exceptions are $^{1}H^{2}$, $^{3}Li^{6}$, $^{5}B^{10}$, and $^{7}N^{14}$. (There are also $^{19}K^{40}$ and $^{71}Lu^{176}$ but these are not properly stable, being β-radioactive with very long lifetimes.) The fact that nuclei with odd Z cannot have even A with the listed exceptions is what makes stable nuclei with even Z more numerous than those with odd Z, for a nucleus with even Z may have A either odd or even. Table 1 illustrates

TABLE 1

SAMPLE OF ISOTOPE STATISTICS

Z	Number of Stable Isotopes	Number with Odd A	Number with Even A
48	8	2	6
49	2	2	0
50	10	3	7
51	2	2	0

all three rules. For odd A, there is apparently no preference between even Z and odd Z.

Energy. In considerations involving the energy of nuclei the mass M is important. According to Einstein's relation, the energy equivalent of a change in mass ΔM is

$$\Delta E = \Delta M c^2$$

Such changes in mass occur when protons and neutrons are changed from one configuration to another in which they are bound more or less strongly. There is no evidence at present for the total annihilation of heavy particles (protons or neutrons). Such a thing might happen if an "antiproton" ($Z = -1$, $A = +1$) met a proton ($Z = +1$, $A = +1$) but the antiproton has not as yet been observed. On the other hand, the total annihilation of electrons and positrons with the emission of two light quanta does occur.

Modern mass spectrographic techniques permit the determination of M to better than one part in 10^5 (an improvement by another order of magnitude would just make possible the deter-

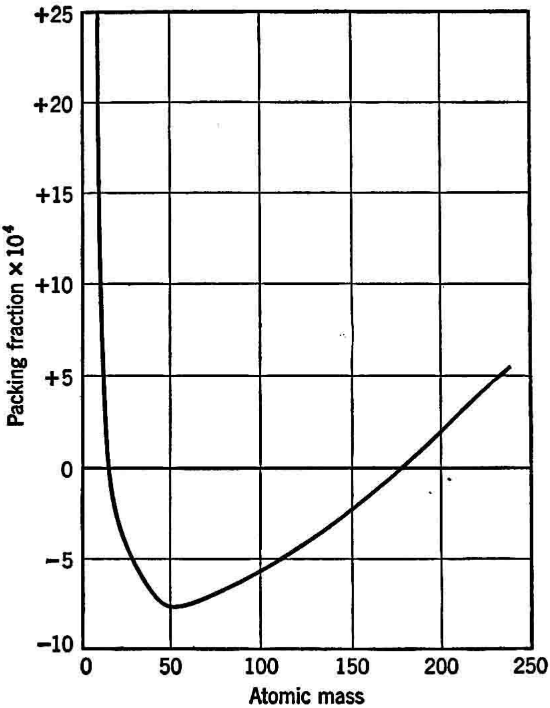

FIG. 1. Packing fractions of atomic nuclei.

mination of the decrease in the atomic weight of a heavy atom due to the binding of the electrons in the field of the nucleus). From such data the binding energies of nuclei can be calculated. For example, using the atomic weight scale based on O^{16}

$$M(O^{16}) = 16.00000$$

there results (using the masses given in the Appendix) $M(^1H^1) = 1.00812$, $M(n) = 1.00893$. Supposing that the O^{16} nucleus is made up of 8 protons and 8 neutrons, the binding energy is

$8M(H^1)+8M(n) - 16.00000 = 0.13640$ mass unit. It is to be noted that the masses of the neutral atoms O^{16} and H^1 are used here (and will be used throughout the book). The justification for this is that the masses of 8 electrons of the O^{16} are canceled in the calculation by the masses of the 8 electrons of the hydrogen. (The change in the mass of the 8 electrons, due to their stronger binding around the O^{16} nucleus, is beyond the experimental error in the mass determinations.)

Two quantities useful in describing the binding energy of nuclei are:

$$\text{Mass excess} \equiv \Delta \equiv M - A$$

$$\text{Packing fraction} \equiv f \equiv \Delta/A$$

The packing fraction is plotted as a function of A in Fig. 1.

Consider now a nuclear reaction

$$^{3}\text{Li}^{7} + {}^{1}\text{H}^{1} \rightarrow {}^{2}\text{He}^{4} + {}^{2}\text{He}^{4}$$

Both the mass number and charge balance. In addition, mass-energy conservation must hold. The balance sheet is as follows:

Initial mass:	
$M(^{3}\text{Li}^{7})$	$= 7.01822$
$M(^{1}\text{H}^{1})$	$= 1.00812$
Total	$= 8.02634$
Final mass:	
$2M(^{2}\text{He}^{4})$	$= 8.00780$
Mass decrease	$= 0.01854$ mass unit

To get the energy equivalent in electron volts $\Delta E = \Delta Mc^2$, the conversion factor

$$1 \text{ milli-mass unit} = 0.931 \text{ Mev}$$

is used (see Nuclear Physics A,* p. 86). This gives 17.26 Mev which is released as kinetic energy. If the $^{3}\text{Li}^{7}$ and $^{1}\text{H}^{1}$ had little

* The three papers by H. A. Bethe in *Reviews of Modern Physics*, namely, Vol. 8, 83, 1936 (with R. F. Bacher); Vol. 9, 69, 1937; and Vol. 9, 245, 1937 (with M. S. Livingston), are hereinafter referred to as Nuclear Physics A, B, and C.

velocity, the α-particles will fly off in nearly opposite directions, each carrying 8.63-Mev kinetic energy. Systematic observations of reactions such as this have verified the Einstein relation very accurately over a great range of nuclear phenomena and are one of the strongest bulwarks of the special theory of relativity. In all nuclear reactions involving heavy particles only, energy has been found to be strictly conserved.

Stability. For a nucleus to be stable it must have a mass which is less than the combined masses of any pair of nuclei made by subdividing it. For example, $^3Li^7$ is stable against the subdivision

$$^3Li^7 \rightarrow {}^2He^4 + {}^1H^3$$

because $M(^3Li^7) = 7.01822$ and $M(^2He^4) + M(^1H^3) = 4.00390 + 3.01702 = 7.02092$. $^2He^5$ is unstable because the decomposition

$$He^5 \rightarrow He^4 + n^0$$

is energetically possible. The mass of He^5 can be found by studying the reaction

$$Li^7 + H^2 \rightarrow He^4 + He^5$$

Knowing the masses $M(Li^7)$, $M(H^2)$, and $M(He^4)$, and measuring the kinetic energy and momentum of Li^7, H^2, and He^4, the mass of He^5 can be determined. It is 5.0137 mass units. This is 0.9 milli-mass unit greater than $M(He^4) + M(n^0)$. (There is the possibility that the measured mass of He^5 might not be for the ground state, but in all known nuclear reactions involving heavy particles, whenever a reaction yields an excited state, it also yields the ground state. Since the experiment gives a unique mass it is presumed to correspond to the ground state.) Li^5 is unstable to the decomposition $Li^5 \rightarrow {}^2He^4 + H^1$, and Be^8 to the decomposition $Be^8 \rightarrow He^4 + He^4$. This explains the absence of nuclei of mass numbers 5 and 8 which was mentioned above.

Fundamental Particles in Nuclei. Present ideas are that a nucleus is composed of protons and neutrons: Z protons and $(A - Z)$ neutrons. This replaces older conceptions which let a nucleus be made up of protons and electrons. Thus the binding energy of any nucleus will be $M - (A - Z)M(n^0) - (Z)M(H^1)$.

II. THE SIZE OF NUCLEI

METHODS OF DETERMINING SIZE

There are four main methods of determining the size of nuclei.

1. *Lifetimes for Alpha Radioactivity.* Nuclei with a mass number A greater than 208 are found to emit helium nuclei (α-particles) spontaneously according to the equation

$$Z^A \rightarrow (Z - 2)^{A-4} + {}^2\text{He}^4$$

The lifetimes of such radioactive nuclei are found to vary over a wide range and to depend strongly on the amount of energy available for the reaction. This is illustrated by the tabulation:

Element	Lifetime	Energy	Radius
Th	2×10^{10} years	4.34 Mev	8.7×10^{-13} cm
RaC′	10^{-3} second	7.83 Mev	9.4×10^{-13} cm

A factor of 2 in energy is thus seen to be equivalent to a factor of the order of 10^{20} in lifetime. This strong energy dependence

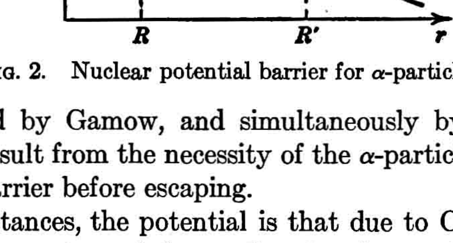

FIG. 2. Nuclear potential barrier for α-particles.

was explained by Gamow, and simultaneously by Gurney and Condon, to result from the necessity of the α-particle to penetrate a potential barrier before escaping.

At large distances, the potential is that due to Coulomb repulsion between a nucleus of charge $Z - 2$ and one of charge 2. At some very short distance, attractive nuclear forces predominate. The potential as a function of separation r between α-particle and residual nucleus is shown in Fig. 2.

The inner radius R at which nuclear forces come into play is defined as the nuclear radius. The probability of an α-particle of energy E penetrating the barrier is given by the Wentzel-Brillouin-Kramers method to be proportional to

$$\exp\left[-\frac{2}{\hbar}\int_R^{R'}\sqrt{2M[V(r)-E]}\,dr\right] \tag{1}$$

This is called the transmission coefficient of the barrier.

Comparison of this formula with experimental lifetimes shows that the enormous variation of lifetime with energy is indeed explained by the theory, using very nearly the same radius for all radioactive nuclei. Moreover, the formula permits a determination of nuclear radii. With three exceptions, all of these lie between 8.4 and 9.8×10^{-13} cm. The success of this first application of quantum mechanics to nuclear phenomena gives us confidence in the general use of quantum mechanics for the description of the motion of heavy particles in nuclei.

2. *Cross Section for Fast Neutrons.* The cross section presented by a nucleus to a fast neutron should approach the geometrical cross section for neutron wave lengths small compared to the nuclear radius: $\lambda/2\pi = \lambda\!\!\!^{-} \ll R$. (This condition is required in order to make a geometrical point of view permissible.) Under this condition every neutron hitting the nucleus strongly interacts with it and should, therefore, cause some reaction.

The geometrical cross section is πR^2, thus permitting a calculation of the nuclear radius from the observed cross sections for fast neutrons. ("Shadow scattering" must be excluded.)

Heavy elements Pb, U, etc., are found to have cross sections of about 3×10^{-24} cm^2 so that their radius is 10^{-12} cm. Middle elements such as Fe are found to have cross sections of about 1×10^{-24} cm^2, corresponding to radii of about 6×10^{-13} cm.

3. *Electrostatic Interaction of Protons in the Nucleus.* If the binding energies of a pair of nuclei which differ only in the interchange of neutrons and protons are compared, a difference in binding energy which increases with the charge of the nuclei is found. Examples of such "mirror" nuclei are:

$$^1H^3\ ^2He^3;\quad ^3Li^7\ ^4Be^7;\quad ^5B^{11}\ ^6C^{11};$$

$$^6C^{13}\ ^7N^{13};\quad ^7N^{15}\ ^8O^{15};\quad ^{14}Si^{29}\ ^{15}P^{29}$$

If neutrons and protons are assumed to be the same as far as nuclear forces alone are concerned, this difference in binding energy is the result of the additional Coulomb repulsion of the extra proton in the field of the original Z protons. To calculate this, all protons are assumed to be uniformly distributed over a sphere of radius R. Then the extra Coulomb repulsion energy due to the replacement of a neutron by a proton is

$$C = \tfrac{6}{5}\, Ze^2/R \tag{2}$$

Using this formula and the observed differences in binding energy to determine nuclear radii leads to the empirical formula

$$R = 1.5 \times 10^{-13} A^{1/3} \text{ cm} \tag{3}$$

This is a reasonable result since it implies that there is roughly a constant volume for each nuclear particle. It further supports the original assumption that neutrons and protons have similar nuclear forces. Furthermore, extrapolation of the result to high atomic weight is in very good agreement with radii given by the α-activity and the neutron-scattering method.

4. *Cross Sections for Nuclear Reactions Involving Charged Particles.* These reactions also involve the penetration of a barrier. The cross sections, in comparison with neutron cross sections, give the transmission of the barrier. Nuclear radii can be computed from these transmissions, thus extending the "α-activity method" down to non-radioactive nuclei. The results are in agreement with the empirical formula 3.

CONCLUSIONS REGARDING THE CONSTITUENTS OF NUCLEI

The size of nuclei is a strong argument for the presence of protons and neutrons in the nucleus rather than protons and electrons. The de Broglie wave length of a neutron or a proton in the nucleus can be estimated to be:

$$\lambdabar = \hbar/p = \hbar/\sqrt{2ME} \sim 1.5 \times 10^{-13} \text{ cm} \tag{4}$$

if we use a kinetic energy E of 8 Mev, in other words of the same order of magnitude as the binding energy per nucleon.

On the other hand, for electrons at this relativistic energy, we would have

$$\lambda\!\!\!^{-} = \hbar/p \approx \hbar c/E \sim 2.5 \times 10^{-12} \text{ cm} \tag{5}$$

Thus the neutron or proton wave length is of the right order of magnitude for the space available in the nucleus, whereas the electron wave length is much too large.

Another argument against the presence of electrons is the long lifetime found for β-emitting nuclei. The long lifetime is not explainable by a potential barrier, because the low electron mass would result in a high transmission coefficient in any barrier the width of which is reasonable considering the nuclear size. Moreover, no barrier at all should be expected for electrons because they are attracted by the Coulomb field of the nucleus. Finally, great difficulties would be encountered in any relativistic theory of the electron if barriers of height greater than $2mc^2$ (m = electron mass) were assumed.