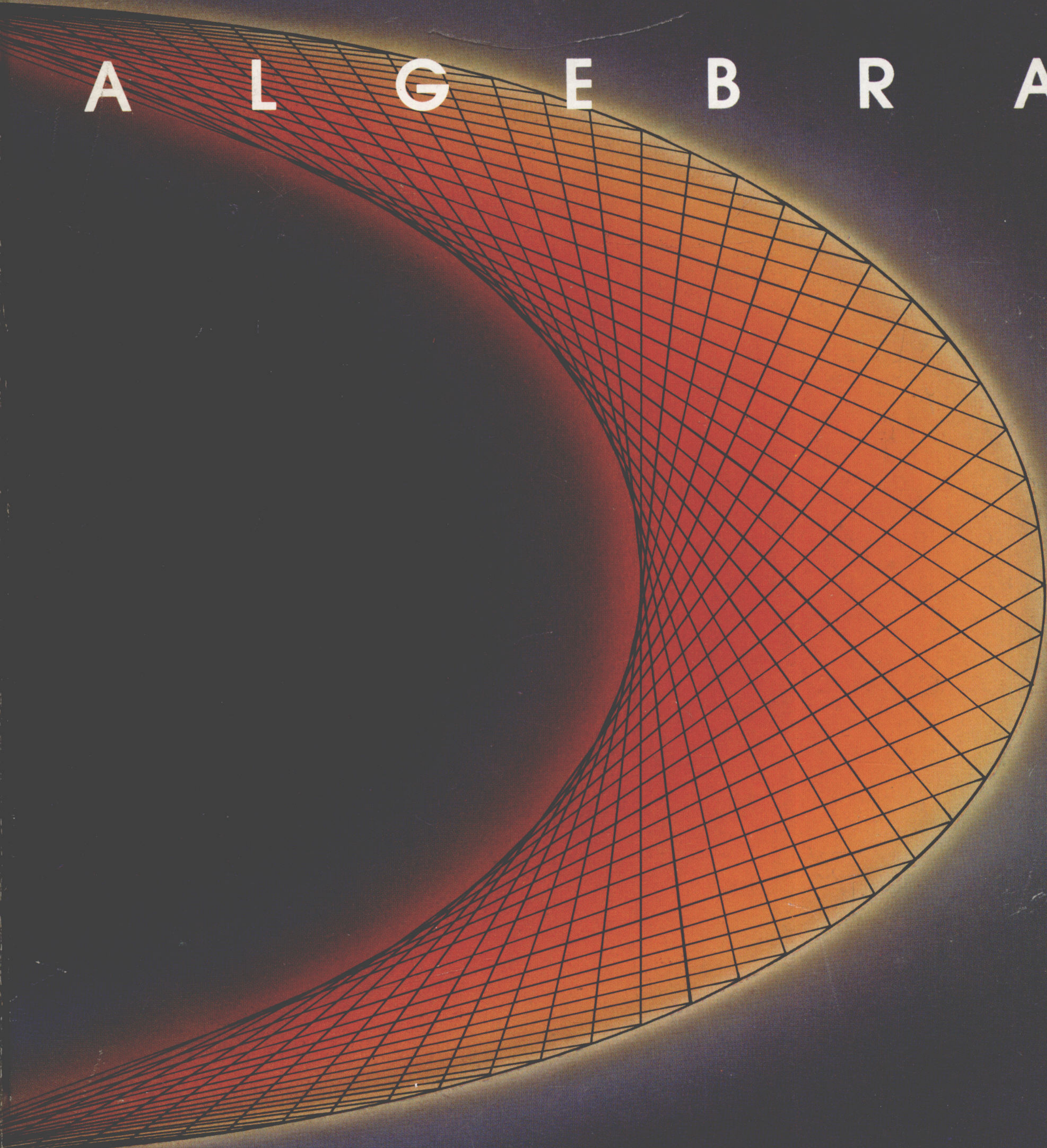
INTERMEDIATE ALGEBRA



R. DAVID GUSTAFSON · PETER D. FRISK

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BROOKS/COLE PUBLISHING CO.
MONTEREY, CALIFORNIA

To

Harold and Monie,
Harder and Evelyn,
with love and affection

Brooks/Cole Publishing Company
A Division of Wadsworth, Inc.

Printed in the United States of America

10 9 8 7

Library of Congress Cataloging in Publication Data

Gustafson, R. David (Roy David), [date]
Intermediate algebra.

Includes index.
1. Algebra. I. Frisk, Peter D., [date]
II. Title.
QA154.2.G874 1983 512.9 83-7585

ISBN 0-534-02906-X

Subject Editor: *Craig F. Barth*
Production Editor: *Suzanne Ewing*
Manuscript Editor: *Dex Ott*
Problem Checker: *Gloria Langer*
Interior and Cover Design: *Vernon T. Boes*
Cover Illustration: *David Aguero*
Art Coordinator: *Judith Macdonald*
Interior Illustration: *Lori Heckelman*
Typesetting: *Graphic Typesetting Service, Los Angeles, California*

Much of the discussion on CAREERS AND MATHEMATICS has been adapted from the *Occupational Outlook Handbook,* 1982–1983 edition, Bulletin 2200, published by
U.S. Department of Labor
Bureau of Labor Statistics
For more information on these careers and others, please consult that publication.

1.1 SETS OF NUMBERS

natural numbers: 1, 2, 3, 4, 5, . . .
prime numbers: 2, 3, 5, 7, 11, . . .
composite numbers: 4, 6, 8, 9, 10, . . .
whole numbers: 0, 1, 2, 3, 4, 5, . . .
integers: . . ., −3, −2, −1, 0, 1, 2, 3, . . .
rational numbers: all numbers $\frac{a}{b}$ where a and b are integers and $b \neq 0$
irrational numbers: coordinates of points on the number line that are not rational numbers
real numbers: any rational or irrational number

$$-(-x) = x$$

$$\begin{cases} |x| = x, \text{ if } x \text{ is positive or zero} \\ |x| = -x, \text{ if } x \text{ is negative} \end{cases}$$

1.2 PROPERTIES OF REAL NUMBERS

If a, b, and c are real numbers, then

$a = a$ the reflexive property
If $a = b$, then $b = a$ the symmetric property
If $a = b$ and $b = c$, then $a = c$ the transitive property

$a + b$ is a real number
$a - b$ is a real number
$a \cdot b$ is a real number
$\frac{a}{b}$ is a real number provided $b \neq 0$
} the closure properties

$(a + b) + c = a + (b + c)$
$(ab)c = a(bc)$
} the associative properties

$a + b = b + a$
$ab = ba$
} the commutative properties

$a(b + c) = ab + ac$ the distributive property
0 and 1 the identity elements
$-a$ is the negative of a
$\frac{1}{a}$ is the reciprocal of a $(a \neq 0)$

1.4 A REVIEW OF FRACTIONS

If no denominators are 0, then

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{a}{b} = \frac{ak}{bk}$$

$$\frac{a}{b} = \frac{-a}{-b} = -\frac{a}{-b} = -\frac{-a}{b}$$

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\frac{-a}{-b}$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

2.1 PROPERTIES OF INTEGRAL EXPONENTS

If n is a natural number, then

$$x^n = \overbrace{x \cdot x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x}$$

If m and n are integers, then

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$\frac{x^m}{x^n} = x^{m-n} \quad (x \neq 0)$$

$$x^0 = 1 \quad (x \neq 0)$$

$$x^{-n} = \frac{1}{x^n} \quad (x \neq 0)$$

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m} \quad (y \neq 0)$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n \quad (x, y \neq 0)$$

2.5 MULTIPLYING POLYNOMINALS

$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$
$(x + y)(x - y) = x^2 - y^2$

3.2 THE DIFFERENCE OF TWO SQUARES: THE SUM AND DIFFERENCE OF TWO CUBES

$x^2 - y^2 = (x + y)(x - y)$
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

3.3 FACTORING THE GENERAL TRINOMIAL

$x^2 + 2xy + y^2 = (x + y)^2$
$x^2 - 2xy + y^2 = (x - y)^2$

5.4 VARIATION

$y = kx$	y varies directly as x
$y = \frac{k}{x}$	y varies inversely as x
$y = kxz$	y varies jointly with x and z
$y = \frac{kx}{z}$	y varies directly as x, but inversely as z

5.5 ABSOLUTE VALUE EQUATIONS

If $k > 0$, then
$|x| = k$ is equivalent to $x = k$ or $x = -k$

5.6 LINEAR INEQUALITIES

If a, b, c, and d are real numbers, then
$a < b$, $a = b$, or $a > b$ (trichotomy property)
If $a < b$ and $b < c$, then $a < c$ (transitive property)
If $a < b$, then
$a + c < b + c$
$ac < bc \quad (c > 0)$
$ac > bc \quad (c < 0)$
$c < x < d$ is equivalent to $c < x$ and $x < d$

5.7 INEQUALITIES CONTAINING ABSOLUTE VALUES

If $k > 0$, then
$|x| < k$ is equivalent to $-k < x < k$
$|x| > k$ is equivalent to $x < -k$ or $x > k$

6.1 RATIONAL EXPONENTS

If all expressions represent real numbers, then
$(a^{1/n})^n = a$
$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$
$a^{-m/n} = \frac{1}{a^{m/n}}$

6.2 RADICALS

If all expressions represent real numbers, then
$a^{1/n} = \sqrt[n]{a}$
$(\sqrt[n]{a})^n = a$
$\sqrt{a^2} = |a|$
$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

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COLLEGE ALGEBRA AND TRIGONOMETRY
INTERMEDIATE ALGEBRA
PLANE TRIGONOMETRY

PREFACE

TO THE INSTRUCTOR

In most schools intermediate algebra students have a mathematical background that is quite varied. Some have recently completed a beginning algebra course; many others have not studied algebra for years. Of this mixture, many students are quite talented, and others find algebra very difficult. To make the problem even worse, most intermediate algebra textbooks contain much more material than can be covered in a typical three-hour course. With this type of book it is extremely easy to get mired down in the review material, which leaves little time for advanced topics. Consequently, the weaker students are placed in jeopardy if you omit this review material.

We have minimized the problems presented by this heterogeneous group of students by writing a book that is flexible, comprehensive, but yet manageable. Because this book contains an abundant supply of worked examples and exercises with the odd answers provided, the student can learn a great amount on his or her own time. This will permit you occasionally to cover more than one section per day. In a typical three-hour course, you should have little trouble covering the required sections in the first ten chapters. In an expanded four-hour course you should be able to cover the entire book at a leisurely pace.

We think that you will be pleased with this book because it has the following features:

Review. It includes a thorough review of basic concepts.

Many Exercises. It includes over 3500 exercises.

Comprehensive. It covers all of the topics that are essential in intermediate algebra to provide a strong background for work in college algebra or finite mathematics. Many optional sections are provided for an extended course.

Mathematically Honest. The developments preserve the integrity of the mathematics, but they are not so rigorous as to confuse students.

Relevant. It includes many applications and discusses occupations requiring a strong background in mathematics.

Teacher Support. Teacher's manuals are available that contain the answers to the even-numbered exercises and sample chapter tests.

We think your students will be pleased with this book because it has the following features:

Informal Writing. It is written for the student to read and understand. The writing is informal rather than technical.
Worked Examples. There are over 350 worked examples. Many ideas are presented by examples.
Functional Use of Color. It makes use of color—not just to highlight important definitions and theorems, but to "point" to terms and expressions that you would point to in a classroom discussion.
Review Exercises. Each chapter concludes with a chapter summary and review exercises with *all* answers provided. Interspersed throughout the book are cumulative review exercises with *all* answers provided.
Applications. Careers in mathematics are discussed at the end of each chapter. Applications appear throughout the book.
Summary of Information. Key formulas and ideas are listed inside the front and back covers for quick reference.

ORGANIZATION AND COVERAGE

The 53 required sections in the first 10 chapters form a complete course in intermediate algebra. The last chapter plus the optional sections provide extra material for good students, or an extended course. The first 10 chapters are sequential and should be taught in order.

CALCULATORS

The use of calculators is encouraged throughout the book. We believe that students should learn calculator skills in the mathematics classroom. They will then be prepared to use a calculator in science and business classes, and for nonacademic purposes.

ACCURACY

Dozens of mathematics teachers have reviewed either all or a portion of this book. We are grateful for their many constructive criticisms and helpful suggestions. Both authors have independently worked all of the exercises.

TO THE STUDENT

Because we believe that many students who read this book do not intend to major in mathematics, we have written it in an informal rather than a technical way. We have provided an extensive number of worked examples, and have tried to present them in a way that will make sense to you. This book has been written for *you* to read, and we think that you will find the explanations helpful. If you do not bother to follow the explanations carefully, much of the value of the book will be lost. What you learn here will be of great value both in other course work and in your chosen occupation.

We suggest that you consider keeping your book after completing this course. It is the *one* piece of reference material that will keep at your fingertips the material that you learned here.

We wish you well.

ACKNOWLEDGMENTS

We are grateful to the following people who reviewed the book at various stages: Dale Boye, Schoolcraft College; Lee R. Clancy, Golden West College; Robert B. Eicken, Illinois Central College; George Grisham, Illinois Central College; David W. Hansen, Monterey Peninsula College; Steven Hatfield, Marshall University; John Hooker, Southern Illinois University; Herbert Kasube, Bradley University; and Ray Tebbetts, San Antonio College.

Our special thanks to Ron Braukhoff for arranging for the plotting of the design for the cover.

We also thank Craig Barth, Vernon T. Boes, Sue Ewing, Lori Heckelman, David Hoyt, Gloria Langer, and Laura Zurawski for valuable assistance in creating this book.

R. David Gustafson
Peter Frisk

CONTENTS

CHAPTER ONE
THE REAL NUMBER SYSTEM

The concept of number is fundamental in mathematics. For this reason, we begin by discussing the various kinds (or sets) of numbers, and how they relate to each other.

1.1 SETS OF NUMBERS

If a person were asked to name a number, the likely response would be a number that is used to count with: a number such as 1, 2, or 7. Because the counting numbers come to mind most naturally, they are called the **natural numbers.**

(1.1) **Definition.** The **natural numbers** are the counting numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, . . .

The three dots used in the previous definition, called the **ellipsis,** indicate that the list continues endlessly.

Certain natural numbers can be divided (without a remainder) by other natural numbers. For example, 12 can be divided by 1, 2, 3, 4, 6, and 12. If a natural number is greater than 1 and if it can be divided without remainder *only* by itself and 1, that number is called a **prime number.** A natural number is called a **composite number** if it is greater than 1 and if it can be divided without remainder by a natural number other than itself and 1.

(1.2) **Definition.** A **prime number** is any natural number, greater than 1, that is divisible without remainder only by itself and 1.

(1.3) **Definition.** A **composite number** is a natural number, greater than 1, that is not a prime number.

The prime numbers less than 20 are

2, 3, 5, 7, 11, 13, 17, and 19

The composite numbers less than 20 are

$$4, 6, 8, 9, 10, 12, 14, 15, 16, \text{ and } 18$$

Because the natural numbers begin with 1, zero is not a natural number. When zero is included with the list of natural numbers, however, we obtain a different set of numbers called the **whole numbers.**

(1.4) **Definition.** The **whole numbers** are the numbers

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots$$

It is often necessary to use numbers to indicate direction as well as quantity—for example, profit or loss, temperatures above or below zero, and gains or losses in football. To do so, we must extend the set of whole numbers to include the negatives of the natural numbers. The negatives of the natural numbers are indicated with "$-$" signs. For example, the negative of 2 is written as -2, and the negative of 7 is written as -7. The whole numbers together with the negatives of the natural numbers form the set of integers.

(1.5) **Definition.** The **integers** are the numbers

$$\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$$

All of the previous sets of numbers can be graphed on a number line. To graph the set of integers on the number line, we draw a line as in Figure 1-1, pick a point (called the **origin**), and give that point a number name (a **coordinate**) of 0. We locate points that are equal distances to the right and to the left of 0, place dots at these points, and label them as indicated in the figure. The points to the right of 0 have coordinates that are positive numbers, and the points to the left of 0 have coordinates that are negative numbers. Note that 0 is neither positive nor negative. Both the line and the integers marked on it continue forever in both directions. If a nonzero integer does not have a sign preceding it, that integer is considered to be positive: $5 = +5$ and $7 = +7$.

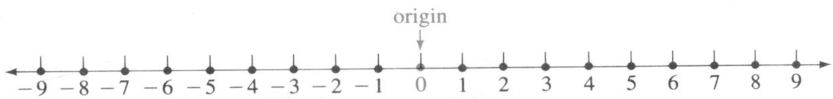

Figure 1-1

Note that 2 represents a point that is two units to the right of 0, and that -2 represents a point that is two units to the left of 0. The coordinates 2 and -2

represent points that are the same distance from 0, but in opposite directions. Two numbers that are the same distance from 0, but in opposite directions, are called **negatives** of each other. For example,

1. the negative of 7 (denoted by -7) is -7, and
2. the negative of -7 (denoted by $-(-7)$) is 7.

Likewise,

3. the negative of 12 (denoted by -12) is -12, and
4. the negative of -12 (denoted by $-(-12)$) is 12.

The results of Parts 2 and 4 generalize as the following rule.

(1.6) **The Double Negative Rule.** If x represents any number, then

$$-(-x) = x$$

The distance on a number line (without regard to direction) that a number is from 0 is called the **absolute value** of that number. For example, the absolute value of 7 (denoted by $|7|$) and the absolute value of -7 (denoted by $|-7|$) are both 7. Likewise, $|12|$ and $|-12|$ are both 12.

More formally, the absolute value of a number is defined as follows.

(1.7) **Definition.** If x is positive or 0, then $|x| = x$.
If x is negative, then $|x| = -x$.

If a number is positive or 0, then it is its own absolute value. If a number is negative, then its negative (which is positive) is its absolute value. Note that $|x|$ is never negative.

Example 1

a. $|6| = 6$

b. $|-5| = 5$

c. $|0| = 0$

d. $-|7| = -(7) = -7$

e. $-|-7| = -(7) = -7$ ■

Some integers can be written as twice another integer. For example, 6 is twice the integer 3, and 16 is twice the integer 8. Such integers are called **even integers.** The integer 9 is not an even integer because it is not twice another integer. Such integers are called **odd integers.** The even integers from -10 to 10 are

$$-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10$$

The odd integers between -10 and 10 are

$$-9, -7, -5, -3, -1, 1, 3, 5, 7, 9$$

If two integers are added, subtracted, or multiplied, the result is always another integer. It is also possible to divide integers, but the result is not always another integer. For example, 8 divided by 4 is the integer 2, but 8 divided by 5 is not an integer. To permit divisions such as $8 \div 5$, mathematicians invented the **rational numbers.**

(1.8) **Definition.** A **rational number** is any number that can be written as the quotient $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.

Note that $\frac{8}{4} = 2$. This is true because $4(2) = 8$. Similarly,

$$\frac{24}{8} = 3, \text{ because } 8(3) = 24$$

and

$$\frac{0}{9} = 0, \text{ because } 9(0) = 0$$

However, the quotient $\frac{5}{0}$ is undefined; there is no number that when multiplied by zero, gives 5. The quotient $\frac{0}{0}$ is also undefined because *all* numbers when multiplied by 0 give 0. Thus, it is understood that the denominator of a fraction can never be 0.

Example 2

a. The fraction $\frac{5}{3}$ is a rational number because it is the quotient of two integers, and the denominator is not 0.

b. The integer 7 is a rational number because it can be written as $\frac{7}{1}$, or as $\frac{14}{2}$, and so on. In fact, all integers are rational numbers.

c. The integer 0 is a rational number because it can be written as $\frac{0}{1}$, the quotient of two integers. Note that although division by zero is *never* allowed, zero can be divided by any nonzero number.

d. The decimal number 0.125 is a rational number because it can be expressed as the quotient of two integers: $0.125 = \frac{1}{8}$.

e. The decimal number 0.6666. . . is a rational number because it can be expressed as the quotient of two integers: $0.6666\ldots = \frac{2}{3}$. ■

The rational numbers also may be represented as points on a number line. In addition to the integers, which are also rational numbers, three other rational numbers are indicated in Figure 1-2. Halfway between the points labeled 0 and 1 is the point associated with the rational number $\frac{1}{2}$. The point marked $-\frac{3}{2}$ is halfway between points -1 and -2. The point marked $\frac{7}{3}$ is one-third of the way from point 2 to point 3.