Volker Diekert
Mikhail V. Volkov
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Preface

The 2nd International Symposium on Computer Science in Russia (CSR 2007) was held September 3–7 in Ekaterinburg, Russia, hosted by Ural State University. CSR 2007 was the second event in a series of regular international meetings that started with CSR 2006 in St. Petersburg (see LNCS 3967). The symposium was organized under the auspices of the European Association for Theoretical Computer Science.

The symposium was composed of two tracks: Theory and Applications/Technology. The opening lecture was given by Yuri Gurevich and other invited lectures were given by Scott Aaronson, Rajeev Alur, Peter Druschel, Ziyad Hanna, Bertrand Meyer, Alexei Miasnikov, Geraud Senizergues, and Geoff Sutcliffe. This volume contains the accepted papers of both tracks and the abstracts of seven invited presentations.

The scope of proposed topics for the symposium was quite broad and covered many areas of computer science and its applications. We received 95 submissions, the contributors being from 24 countries. Each submission was reviewed by at least three Program Committee members. The committee decided to accept 34 papers. The reviewing process as well as the preparation of this volume were efficiently supported by the EasyChair conference system.

The following satellite events were collocated with CSR 2007:

- Workshop on Computational Complexity and Decidability in Algebra
- Workshop on Infinite Words, Automata and Dynamics
- Russian Summer School in Information Retrieval

We thank our sponsors: Microsoft Research, Russian Foundation for Basic Research, SKB Kontur, Ural State University, and Yandex.

Yandex, the largest resource on the Russian Internet, established the Yandex the Best Paper Awards and Yandex Best Student Paper Awards for the CSR series. The inauguration of the Yandex Awards formed a part of the Business Meeting of CSR 2007 along with the presentations of the first awarded papers. The following three papers were selected by the Program Committee:

- "Conjunctive grammars over a unary alphabet: undecidability and unbounded growth" by Artur Jez and Alexander Okhotin — Best Paper in the Theory Track
- "Estimation of the click volume by large-scale regression analysis" by Yury Lifshits and Dirk Nowotka – Best Paper in the Applications and Technology Track
- "A fast algorithm for path 2-packing problem" by Maxim Babenko — Best Student Paper

We thank the local Organizing Committee (co-chaired by Vladimir Tretjakov, President of Ural State University, and Vitaly Berdyshev, Director of Mathematics and Mechanics Institute of the Ural Branch of the Russian Academy of Sciences), especially Svetlana Goldberg, Grigoriy Povarov, and Elena Pribavkina.

June 2007

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Table of Contents

Proving Church's Thesis
(Abstract)

Yuri Gurevich

Microsoft Research

The talk reflects recent joint work with Nachum Dershowitz [4].

In 1936, Church suggested that the recursive functions, which had been defined by Gödel earlier that decade, adequately capture the intuitive notion of a computable ("effectively calculable") numerical function[1] [2]. Independently Turing argued that, for strings-to-strings functions, the same goal is achieved by his machines [11].

The modern form of Church's thesis is due to Church's student Kleene. It asserts that every computable numerical partial function is partial recursive. (Originally Church spoke of total functions.)

Kleene thought that the thesis as unprovable: "Since our original notion of effective calculability... is a somewhat vague intuitive one, the thesis cannot be proved" [7]. But he presented evidence in favor of the thesis. By far the strongest argument was Turing's analysis [11] of "the sorts of operations which a human computer could perform, working according to preassigned instructions" [7]. The argument convinced Gödel who thought the idea "that this really is the correct definition of mechanical computability was established beyond any doubt by Turing" [5].

Moreover, Gödel has been reported to have thought "that it might be possible ... to state a set of axioms which would embody the generally accepted properties of [effective calculability], and to do something on that basis" [3]. As explained by Shoenfield [10]:

> It may seem that it is impossible to give a proof of Church's Thesis. However, this is not necessarily the case.... In other words, we can write down some axioms about computable functions which most people would agree are evidently true. It might be possible to prove Church's Thesis from such axioms.... However, despite strenuous efforts, no one has succeeded in doing this (although some interesting partial results have been obtained).

We will demonstrate that, under certain very natural hypotheses regarding algorithmic activity, called the "Sequential Postulates" [6], Church's Thesis is in fact provable. In brief, the postulates say the following.

I. **Sequential Time.** *An algorithm determines a sequence of "computational" states for each valid input.*

[1] For brevity we use the term numerical function to mean a function from natural numbers to natural numbers.

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II. **Abstract State.** *The states of a computational sequence can be arbitrary (first-order) structures.*

III. **Bounded Exploration.** *The transitions from state to state in the sequence are governed by a finite description.*

For precise formulation of the three postulates see the article [6]. With Bounded Exploration, an algorithm computes in "steps of limited complexity", as demanded by Kolmogorov [8]. This postulate thereby answers Kolmogorov's implicit question: What does it mean to bound the complexity of each individual step?

The postulates are justified in the article [6]. On this ground, a (sequential) algorithm is defined there as any object satisfying the three postulates. One may worry that this definition is too liberal. To this end, the article proves that (sequential) abstract state machines, introduced earlier by the author, satisfy the three postulates, and that, for every algorithm, there is an abstract state machine that emulates the algorithm.

To focus on numerical algorithms, we add the following postulate:

IV. Only basic arithmetic operations are available initially.

Algorithms satisfying postulate IV will be called numerical.

We will show that Church's Thesis provably follows from these four postulates.

Theorem 1. *Any numerical partial function is computed by a numerical algorithm if and only if it is partial recursive.*

Thus, to the extent that one might entertain the notion that there exist non-recursive effective functions, one must reject one or more of these postulates. In a similar way, we can prove Turing's thesis from postulates I–III and a postulate.

V. Only basic string operations are available initially.

Theorem 1 generalizes to the case when oracles are present. If only oracle functions are available initially, postulates I–III suffice. No additional postulates are needed.

Our goal in this work has been to remedy the situation described thus by Montague [9]: "Discussion of Church's thesis has suffered for lack of a precise general framework within which it could be conducted." We show how the Sequential ASM Postulates provide just such a framework. As we mentioned, Gödel surmised that Church's Thesis may follow from appropriate axioms of computability. But, as far as we can ascertain, no complete axiomatization has previously been presented in the literature. In fact, the challenge of proving Church's Thesis is first in Shore's list of "pie-in-the-sky problems" for the twenty-first century [1].

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