HADLEY & WHITIN

ANALYSIS OF INVENTORY SYSTEMS

PRENTICE-HALL INTERNATIONAL
SERIES IN MANAGEMENT - AND

ANALYSIS OF INVENTORY SYSTEMS

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Prentice-Hall, Inc., Englewood Cliffs, N. J. 1963

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY., LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL FRANCE, S.A.R.L., *Paris*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL DE MEXICO, S.A., *Mexico City*

Library of Congress Catalog Card No. 62-7406

Printed in the United States of America

03295-C

PREFACE

During the last fifteen years there has been a rapid growth of interest in what is often referred to as scientific inventory control. Scientific inventory control is generally understood to be the use of mathematical models to obtain rules for operating inventory systems. The subject has attracted such wide interest that today every serious student in the management science or industrial engineering areas is expected to have had some experience working with inventory models. Originally, the development of inventory models had practical application as an immediate objective. To a large extent this is still true, but as the subject becomes older, better developed, and more thoroughly explored, an increasing number of individuals are working with inventory models because they present interesting theoretical problems in mathematics. For such individuals, practical application is not a major objective, although there is the possibility that their theoretical work may be helpful in practice at some future time. Thus, today work is being done with inventory models at many different levels, ranging from a concern only for practical problems to a concern only for the abstract mathematical properties of the model.

The purpose of this text is to introduce the reader to the techniques of constructing and analyzing mathematical models of inventory systems. In doing so, it cuts across many of the levels of work being done with inventory models. Thus, by reading the entire text it should be possible for the reader to gain an understanding of the sort of work that is being done and the kinds of problems that are encountered when studying inventory models at all levels—from the purely practical to the purely theoretical.

It was recognized that in many cases readers would be only interested in one facet of the subject, such as practical applications. An effort has been made to write the text in such a way that it can be read by a broad group of readers with widely varying mathematical backgrounds. To accomplish this, the material presented first in each chapter is of practical interest, and involves the most elementary mathematics. The more detailed and exact developments, which are also more advanced mathematically, come later. Topics of theoretical interest are treated in the discussion of these more advanced models. An exception to this rule is Chapter 8, which is devoted almost exclusively to topics that are more advanced mathematically. Thus someone who was mainly interested in practical applications could read Chapter 1, the first few sections of Chapters 2 through 7, none of Chapter 8, and all of Chapter 9.

Because of its flexibility this text could be used for a variety of courses including a full semester course in inventory theory, part of a course in production, or part of a course in operations research. Both of the authors have used essentially all of the material in this text in an inventory control course. This course was one quarter in length at the University of Chicago (requiring about fifteen hours per week of work) and one semester in length at the University of California (requiring about nine hours per week of work). One of the authors (G.H.) has also used the material for slightly less than one half of a one quarter course in the production area. This course covered all of Chapter 1, Chapter 2 through Sec. 2-9, Chapter 4 through Sec. 4-4, Chapter 5 through Sec. 5-2, Chapter 6 through Sec. 6-4, and all of Chapter 9.

The material treated in this book is concerned almost exclusively with the determination of optimal operating doctrines for systems consisting of a single stocking point and a single source of supply. The reasons for doing this are: 1) Many practical problems fall into this category. 2) Many interesting mathematical problems arise even when attention is restricted to these relatively simple systems. 3) It is extremely difficult to determine optimal operating doctrines for more complex systems. Indeed, little has been done in this area. Frequently, when one desires to examine a complex multi-echelon system as a whole, he is, for a variety of reasons, far more interested in the dynamic response and the stability of the system than in determining an operating doctrine which will minimize some cost expression for a specified stochastic input. The analysis of the dynamic response and stability of a system by analytical methods and simulation techniques will be treated in a separate volume, and hence these topics are not considered in the present text either.

An effort has been made to provide a large number of original and interesting problems. The authors consider the problems to be very important, and any serious reader should at least look them over and attempt to work out a fair number.

The Graduate School of Business, University of Chicago, very generously provided the secretarial services for having the manuscript typed. Jackson E. Morris once again did an excellent job of providing the quotations which appear at the beginning of the chapters. The authors are indebted to Paul Teicholz and B. Lundh of Stanford University, who used their digital computer programs to compute one of the examples for Chapter 4 and

one for Chapter 5. Also the authors are indebted to a number of their students who pointed up errors and misprints in the manuscript. Finally, the authors express their appreciation to the editor of *Operations Research* for permission to reprint a number of the Poisson properties which they first published there.

G.H.
T.M.W.

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1-1 Inventory Problems

The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy. For example, inventories must be maintained in agriculture, industry, retail establishments, and the military. In the United States the total dollar investment in inventories at any one time is immense. The sum runs to more than 50 billion dollars for defense projects alone and more than 95 billion for private enterprise sectors of the economy. There are many reasons why organizations should maintain inventories of goods. The fundamental reason for doing so is that it is either physically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories customers would have to wait until their orders were filled from a source or were manufactured. In general, however, customers will not or cannot be allowed to wait for long periods of time. For this reason alone the carrying of inventories is necessary to almost all organizations that supply physical goods to "customers". There are, nonetheless, other reasons for holding inventories. For example, the price of some raw material used by a manufacturer may exhibit considerable seasonal fluctuation. When the price is low, it is profitable for him to procure a sufficient quantity of it to last through the high priced season and to keep it in inventory to be used as needed in production. Another reason for maintaining inventories, a reason particularly important to retail establishments, is that sales and profits can be increased if one has an inventory of goods to display to customers.

Two fundamental questions that must be answered in controlling the inventory of any physical good are when to replenish the inventory and how much to order for replenishment. In this book we shall attempt to show how these questions can be answered under a variety of circumstances. Essentially every decision which is made in controlling inventories in any organization, regardless of how complicated the inventory supply system may be, is in one way or another associated with the questions of when to order and how much to order. There are certain types of inventory problems, such as those concerned with the storage of water within dams, in which one has no control over the replenishment of the inventory. (Is other words, the resupply of the inventory of water within the dam depends on the rainfall, and the organization operating the dam has no control over this.) We shall not consider this type of problem here. The only problems with which we shall concern ourselves are those in which the organization controlling the inventory has some freedom in determining when, and in what quantity, the inventory should be replaced. On the other hand, we shall assume that, in general, the inventory system has no control over the demands which occur for the item, or items, which it stocks. Again, this is just the opposite of what one encounters in dealing with inventory problems such as storage of water within dams, since the efflux of water through the dam is completely within the control of the organization operating the dam. In short, we are going to consider the type of inventory problem encountered in business, industry, and the military.

We shall concentrate on showing how mathematical analysis can be used to help develop operating rules for controlling inventory systems. When mathematics is applied to the solution of inventory problems, it is necessary to describe mathematically the system to be studied. Such a description is often referred to as a mathematical model. The procedure is to construct a mathematical model of the system of interest and then to study the properties of the model. Because it is never possible to represent the real world with complete accuracy, certain approximations and simplifications must be made when constructing a mathematical model. There are many reasons for this. One is that it is essentially impossible to find out what the real world is really like. Another is that a very accurate model of the real world can become impossibly difficult to work with mathematically. A final reason is that accurate models often cannot be justified on economic grounds. Simple approximate ones will yield results which are good enough so that the additional improvement obtained from a better model is not sufficient to justify its additional cost.

In this book we shall study a variety of mathematical models of inventory systems. Many of these are intended for practical application. Others, however, have no immediate practical application because of the restrictive nature of the assumptions. They are interesting and relevant, however, because they exhibit some theoretical properties which are important in understanding the nature of inventory systems.

1-2 Brief Historical Sketch

Although inventory problems are as old as history itself, it has only been since the turn of the century that any attempts have been made to employ analytical techniques in studying these problems. The initial impetus for the use of mathematical methods in inventory analysis seems to have been supplied by the simultaneous growth of the manufacturing industries and the various branches of engineering—especially industrial engineering. The real need for analysis was first recognized in industries that had a combination of production scheduling problems and inventory problems, i.e., in situations in which items were produced in lots—the cost of set up being fairly high—and then stored at a factory warehouse.

The earliest derivation of what is often called the *simple lot size formula* was obtained by Ford Harris of the Westinghouse Corporation in 1915 [5]. This same formula has been developed, apparently independently, by many individuals since then; it is often referred to as the *Wilson formula* since it was also derived by R. H. Wilson as an integral part of the inventory control scheme which he sold to many organizations. The first full length book to deal with inventory problems was that of F. E. Raymond [6], written while he was at M.I.T. It contains no theory or derivations, and only attempts to explain how various extensions of the simple lot size model can be used in practice.

It was not until after World War II, when the management sciences and operations research emerged, that detailed attention was focussed on the stochastic nature of inventory problems. Prior to that the systems had been treated as if they were deterministic, except for a few isolated cases, such as the work of Wilson, where some attempts were made to include probabilistic considerations. During the war, a useful stochastic model was developed which we shall refer to in Chapter 6 as the Christmas tree model. Shortly thereafter, a stochastic version of the simple lot size model was developed by Whitin, whose book [8], published in 1953, was the first book in English which dealt in any detail with stochastic inventory models.

As has been noted above, the original interest in using analytical techniques to solve inventory problems arose in industry where engineers were seeking solutions to practical problems. It is interesting to observe that economists were not the first to take an active interest in inventory problems even though inventories play a crucial role in the study of dynamic economic behavior. The reason for this lack of interest probably lies in the fact that economists were concentrating their attention mainly on static equilibrium models. Recently, however, some economists and mathematicians have taken an interest in inventory models. They have not been especially concerned with immediate practical applications; instead, they have been interested in the models because of their mathematical properties and economic interpretations. The paper by the economists Arrow, Harris, and Marschak [1] was one of the first to provide a rigorous mathematical analysis of a simple type of inventory model. It was followed by the often quoted and rather abstract papers by the mathematicians Dvoretzky, Kiefer, and Wolfowitz [3, 4]. Since then a number of papers by mathematicians have appeared. A recent full length book devoted to the mathematical properties of inventory systems is that of Arrow, Karlin, and Scarf [2]. At the present time, work on inventory problems is being carried on at many different levels. At one extreme a considerable amount of work is concerned strictly with practical applications, while, at the other extreme, work is being done on the abstract mathematical properties of inventory models without regard to possible practical applications. The material presented in this text will, in similar fashion, cover a fairly broad area. Some material will be directly concerned with practical applications while other material will be concerned with the mathematical structure of inventory systems. In this way, the reader will be introduced to the methods of analysis used and the problems involved in carrying out investigations of inventory systems at these various levels.

1-3 Inventory Systems

There are great differences between existing inventory systems. They differ in size and complexity, in the types of items they carry, in the costs associated with operating the system, in the nature of the stochastic processes associated with the system, and in the nature of the information available to decision makers at any given point in time. All these differences can be considered to reflect variations in the structure of the inventory system. These variations can have an important bearing on the type of operating doctrine that should be used in controlling the system. By an operating doctrine we simply mean the rule which tells us when to order and how much to order.

It is desirable to spell out in somewhat more detail the differences which can exist in inventory systems—either real world systems or mathematical models. In the following sections we shall make explicit some of these differences.

1-4 The Echelon Structure of Inventory Systems

An item may be stocked in an inventory system at only a single physical location, or it may be stocked at many locations. For example, if the organization under consideration is the U.S. Air Force supply system, a spare part for a certain type of aircraft may be stocked at over 100 bases and repair facilities all over the world. If the organization under study is a single privately owned lumber yard, the entire stocks of the organization will be held at this lumber yard.

When there is more than a single stocking point, there exists the possibility for many forms of interaction between the stocking points. One of the simplest forms of interaction involves one stocking point which serves as a warehouse for one or more other stocking points. This leads to what is referred to as a multiechelon inventory system. One possible type of multiechelon system is illustrated in Fig. 1-1. The arrows indicate the normal pattern for the flow of goods through the system. This might be referred to as a four echelon system since there are four levels. Each level is called an echelon. In the system shown, customer demands occur only at the stocking points in level 1. These stocking points have their stocks

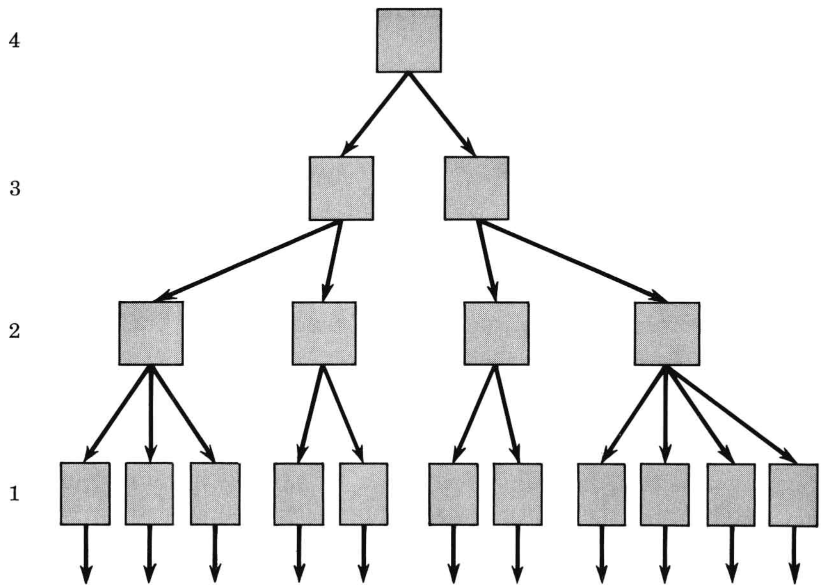

FIGURE 1-1.

replenished by shipments from warehouses at level 2, which in turn receive replenishments for their stock from level 3, etc. Figure 1-1 represents only one type of multiechelon system. In other cases, customer demands might occur at all levels, or stocking points at any level might not only receive shipments from the next highest level but might also get replenishments from any higher level or from the source. Also, it might be allowable, on occasion, to permit redistribution of stocks among various stocking points at a given level.

Most inventory systems encountered in the real world are multiechelon in nature. However, it is often true that one need not or cannot consider the multiechelon system in its entirety. The reason for this is that different organizations operate different parts of the system. For example, Fig. 1-1 might refer to a production distribution system in which the source is a plant where the item is manufactured, level 4 is a factory warehouse, level 3 represents regional warehouses, level 2 represents warehouses in various cities, and level 1 represents the retail establishments which sell the item to the public. In such a system the manufacturer might control only the plant and the factory warehouse, while different organizations operate the regional warehouses and still different organizations operate the city warehouses and the retail establishments. Note that even at a given level many different organizations may be involved. For example, each of the warehouses in different cities may be under different ownership. In such a system, each organization has the freedom to choose the operating doctrine for controlling the inventories under its jurisdiction. One could not, in general, attempt to analyze the system as a whole and dictate what operating doctrine should be used by each stocking point at each level. Instead, one might be concerned with the best way for one of the warehouses at level 2 to control its inventories. In making the analysis, the customers would be the retailers at level 1 and the source from which replenishments are obtained would be the appropriate warehouse at level 3.

Frequently, there will be just a single source from which an inventory system replenishes its stocks when it is desirable to do so. This source may be the plant where the item is made, a factory warehouse, or simply a warehouse at a higher echelon. Sometimes, however, the system has two or more alternative sources of supply available. For example, one of the retailers in Fig. 1-1 might be able to order from several different warehouses at level 2. A special case which can occur is that where the system under study also controls the source of its supplies, i.e., the plant where the item or items are made. In this case the problem is not strictly an inventory control problem but also involves production scheduling. In this book we shall not consider the general problem of combined production scheduling and inventory control except in some cases where production is carried out in lots rather than being continuous.

The basic inventory system that will be studied in this text will be much simpler than the general sort of multiechelon system shown in Fig. 1-1. It will consist of just one stocking point with a single source for resupply. Customer demands arrive at the single stocking point, and at appropriate times orders are placed with the source for replenishing the inventory. The operation of this system is illustrated schematically in Fig. 1-2.

There are good reasons for restricting our attention to the structure illustrated in Fig. 1-2. Perhaps the most important reason is that it is very difficult to study analytically multiechelon systems of the type shown in Fig. 1-1. In fact, very little work has been done in this area. It will be seen

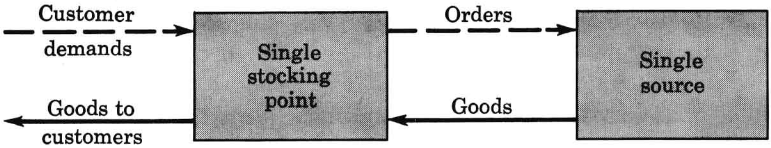

FIGURE 1-2.

that even the relatively simple structure that will be studied can become very complex to analyze. The other reason is that for practical applications the simple structure of Fig. 1-2 is often (although by no means always) adequate. This is true because, as was noted above, even though real world systems are usually of the multiechelon variety, it is often necessary to consider the various stocking points individually because different organizations control them. Even when a single organization controls a number of stocking points, however, the interactions between them are frequently sufficiently small to allow each to be studied independently of the others.

1-5 The Nature of the Items

A large military supply system stocks over 500,000 different items, while a typical department store may carry as many as 150,000 items. Other inventory systems stock only one or two items. The items stocked can differ from each other in many ways. They differ in cost, and in their physical properties, such as weight and volume. Some items are perishable and cannot be stored for long periods of time; others can be stored indefinitely without deterioration; others are subject to rapid obsolescence. Often, items can be stored only under specially controlled conditions of temperature, humidity, etc., and require special types of packaging for storage. When more than a single item is stocked there can be interactions between the items. For example, the items may be substitutes for each other so that if the system is out of stock on one item a customer will accept another. On the other hand, they may be complements so that usually one will not be sold without the other. Frequently, the interactions will take the form of having items compete for limited warehouse floor space or for investment dollars, which are also limited.

Another different form of interaction can exist between the items carried by an inventory system. This type of interaction is represented in what might be called a multistage inventory system associated with a production process. A typical block diagram for such a system is illustrated in Fig. 1-3.