

THE THEORY OF HEAT ENGINES

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WITH 246 DIAGRAMS AND NUMEROUS EXAMPLES

NEW IMPRESSION

LONGMANS, GREEN AND CO.

39 PATERNOSTER ROW, LONDON, E.C.4

NEW YORK, TORONTO

BOMBAY, CALCUTTA AND MADRAS

1923

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PREFACE TO NEW EDITION

THE Author, who held a Commission in the Reserve, was called up on the outbreak of war in August, 1914, and was killed in December, 1915. A new edition of his book is required, and as a tribute to the memory of a former colleague I have gone through his book and corrected a number of errors of various kinds which found their way into the first edition.

Thanks are given to Mr. Telford Petrie, to Mr. T. Bevan and others who have kindly taken the trouble to point out slips which they have come across in using the book.

A. MORLEY

January, 1920.

PREFACE

THIS book has been written mainly for engineering students, and covers the ground required for University and similar examinations in the theory of heat engines. It will also be found suitable for students reading for the examinations of the Institution of Civil Engineers, the Institution of Mechanical Engineers, the City and Guilds of London Institute, and the Board of Education, and should prove useful to the engineer who desires a thorough knowledge of the theory of the subject.

The Author feels that no apology is necessary for adding yet another book on this subject because, although many excellent books exist which deal solely with one or two special branches of the subject, there are very few which deal with the subject as a whole. An attempt has here been made to give in a complete and concise form the thermodynamical and mechanical *principles* of the subject; to that end all purely descriptive matter has been designedly omitted.

Many numerical examples are fully worked out in the text, and the student is urged to read all of these, and to work out for himself the examples at the ends of the chapters in order to obtain a thorough knowledge of the subject. Those marked (L.U.) are taken from various London University papers for the B.Sc. (Engineering) examination.

Many important researches in the subject have been noticed, references to which are freely given. In particular, the *Proceedings* of the Institution of Mechanical Engineers, the Institution of Civil Engineers, and the Reports of the Gaseous Explosions Committee of the British Association have, with permission, been freely drawn upon for this purpose.

The Author's thanks are due to many friends for hints and suggestions, particularly to Professor A. Morley, M.Sc., and Professor W. Robinson, M.E., and also to Mr. R. H. King, B.Sc., Mr. J. S. Robinson, B.Sc., and Mr. T. P. G. Stone, who have so kindly checked many of the numerical examples. It is too much to hope that, with so many numerical examples, this edition will be free from errors; any intimation of these or suggestions for future consideration will be cordially appreciated.

W. INCHLEY.

UNIVERSITY COLLEGE,
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September, 1913.

INTRODUCTION

UNITS

SOME of the information given in this introduction will be known to all readers, while the whole of it may be already known to others; it is placed here for convenient reference, and on beginning the study of the Theory of Heat Engines, the student will do well to be thoroughly conversant with the various units used in both the British and the Metric systems.

Units of Work.—The British engineer's unit of work is the foot-pound, being the amount of work done when a force of one pound weight acts through a distance of one foot in its own direction. The unit of work in the metric system is the work done when a force of one kilogramme acts through a distance of one metre; it is called the kilogramme-metre.

Units of Heat.—The *British thermal unit* (B.Th.U.) is the quantity of heat required to raise the temperature of one pound of water 1° F. It is numerically equal to about 778 foot-pounds. Another thermal unit which finds increasing favour with British engineers is the *Centigrade heat unit* (C.H.U.), being the quantity of heat required to raise the temperature of one pound of water 1° C.; it is equal to 778×1.8 or 1400 foot-pounds.

Since the specific heat of water is not quite constant and equal to unity, it follows that the quantity of heat required to raise the temperature of one pound of water 1° F. or 1° C. is not the same at high as at low temperatures; but the difference is so small that in practical calculations it may safely be neglected.

Calorie.—The *gramme-calorie* is the quantity of heat required to raise the temperature of one gramme of water 1° C. The *kilo-calorie* is the quantity of heat required to raise the temperature of one kilogramme of water 1° C.

252 gramme-calories are equivalent to one B.Th.U., and one kilo-calorie is equivalent to 3.96 B.Th.U.

Power.—The British engineer's unit of power is the *horse-power* (usually written H.P.), being the power expended when working at the rate of 550 foot-pounds per second, or 33,000 foot-pounds per minute.

The *French horse-power* (*force de cheval*) is the power expended when working at the rate of 75 kilogramme-metres per second. The electrical engineer's unit of power is the *watt*, being the rate of working when one *ampère* flows under a pressure of one *volt*, *i.e.* when work is being done at the rate of one *joule* per second. One British horse-power is equal to 746 watts.

Kilowatt.—The watt is an inconveniently small unit for practical purposes, hence the electrical engineer usually estimates power in *kilowatts*, one kilowatt being equal to 1000 watts.

Board of Trade Unit.—The Board of Trade unit of electric supply is *one kilowatt-hour*, being the quantity of work done in one hour when working at the rate of one kilowatt, or one thousand joules per second.

Specific Heat.—The usual definition of specific heat may be expressed as the ratio—

$$\frac{\text{Quantity of heat required to raise the temp. of unit mass of a substance } 1^{\circ}}{\text{Quantity of heat required to raise the temp. of unit mass of water } 1^{\circ}}$$

As defined above, the specific heat of a substance is a pure number, and it is immaterial in what units the quantity of heat is expressed. The specific heat of a substance is also frequently expressed as the number of B.Th.U., or foot-pounds, required to raise the temperature of one pound of the substance 1° F.

Specific Heat of Gases.—The specific heat of a gas is not a constant quantity (see Chap. XIV.); it varies with the temperature. The results of most experiments on the energy of gases have been expressed in the form of tables of formulæ giving the specific heat (referred to unit mass of the gas as above) in terms of the temperature. It would appear preferable for most purposes to exhibit them in terms of the internal energy per unit of volume. This is the form most convenient for purposes of thermodynamic calculations, and it has the further advantage that it expresses the actual quantity measured; and in most cases the lower limit of temperature is near that of the room. The rate of change with temperature of the energy so determined is sometimes called the “true” or “instantaneous” specific heat, and sometimes the “thermal capacity” of the gas. The “Gaseous Explosions” Committee of the British Association¹ suggest that this quantity should be called the “volumetric heat,” which, if adopted, should include in its significance that the measurement to which it relates should be made at constant volume and refer to unit volume of the gas. The term “specific heat” could then be restricted to its usual meaning, which refers to unit mass of the substance. They further recommend that the *volumetric heat* be referred to the gramme-molecule under standard conditions, which is nearly the same for all gases, namely 22.25 litres, and that the zero of temperature from which energy is reckoned be 100° C. in order that steam may be included on the same basis as other gases.

The volumetric heat of a gas may also be expressed in foot-pounds per cubic foot by multiplying calories per gramme-molecule by 3.96, since

One calorie per gramme-molecule = 3.96 foot-pounds per cubic foot.

¹ See the First Report of this Committee, Section G, Dublin, 1908.

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THE THEORY OF HEAT ENGINES

CHAPTER I

THERMODYNAMICS AND PROPERTIES OF GASES

1. The First Law of Thermodynamics: Heat and mechanical energy are mutually convertible, and Joule's equivalent is the rate of exchange.—The value of this equivalent (usually denoted by the letter J) is 778 foot-pounds, which are equivalent to one British thermal unit, or 1,400 foot-pounds are equal to one Centigrade heat unit.¹ The function of a heat engine is to convert *heat energy* into *mechanical energy*, which operation is much more difficult to perform than to convert mechanical energy into heat energy. If a heat engine converted all the heat energy supplied to it into mechanical energy, it would convert H units of heat into JH units of mechanical energy, where J represents Joule's equivalent. Hence if W represents the number of units of work done we may write

$$W = JH$$

No engine, however, can convert all the heat it receives into work, as will be seen later, the maximum efficiency (Art. 51) ever reached being certainly not much greater than 45 per cent., the actual efficiency of the commercial engine being considerably less (see p. 346).

The Working Fluid.—In all heat engines the working fluid is either a gas or a vapour; when a liquid is converted into the gaseous state it becomes a vapour, and, as such, possesses properties similar to those of a gas. The higher the temperature to which a vapour is heated the more closely does it approximate to a gas; in fact, all gases are merely vapours at temperatures far above the boiling-point of the corresponding liquid.

All vapours at ordinary temperatures can be condensed or liquefied by the application of pressure at constant temperature, but experiment shows that *if a gas is above a certain temperature, it cannot be liquefied by pressure alone.* This temperature, for any particular gas, is called the critical temperature of that gas. Hence, unless a gas is first cooled *below its critical temperature it cannot be liquefied by pressure alone.* The critical temperatures of oxygen, hydrogen, nitrogen, air, etc., are so very low, however, that at the working temperatures used in heat engines these gases may be considered as permanent gases.

2. The First and Second Laws of Permanent Gases.—The first law (*Boyle's Law*) states that in a perfect gas the absolute pressure is

¹ See Robinson's "Gas and Petroleum Engines," or any text-book on physics.

inversely proportional to the volume when the temperature remains constant. If p denotes the pressure and v the volume, we may write

$$p \propto \frac{1}{v}$$

or

$$pv = \text{a constant.}$$

The second law (*Law of Charles*) says that under constant pressure, equal volumes of different gases expand equally for the same increment in temperature; also, if a gas be heated under constant volume, equal increments of its pressure correspond to equal increments of temperature.

For example—492 cubic feet of gas at 32° F. become 491 cubic feet at 31° F., 460 cubic feet at 0° F., and finally, if it follows this law, the gas will have no volume at -460° F. As a matter of fact, any actual gas would change its physical state before reaching so low a temperature.

3. Absolute Temperature.—The absolute temperature of a substance is its temperature reckoned from absolute zero.

If t = the temperature on the ordinary thermometer scale
and T = the absolute temperature, then—

$$T = t + 460 \text{ on the Fahrenheit scale,}$$

and

$$T = t + 273 \text{ on the Centigrade scale.}$$

The absolute zero of temperature as obtained by the above reasoning (-460° F. or -273° C.) corresponds very nearly to the absolute zero obtained from the purely thermodynamic considerations discussed in Art. 26, and the above values will be taken in making calculations which involve the absolute temperature.

4. Connection between the Pressure, Volume, and Temperature of a Gas.—Boyle's Law states that $p \propto \frac{1}{v}$ when the temperature remains constant; Charles' Law states that $p \propto T$ when the volume remains constant. Combining the two laws, we have for a given weight of gas

$$pv \propto T \text{ or } pv = RT \dots \dots (1)$$

where p = absolute pressure in pounds per square foot.

v = volume in cubic feet.

T = absolute temperature (Fahrenheit or Centigrade).

R = a constant depending on whether T is expressed on the Fahrenheit or on the Centigrade scale.

For dry air the numerical value of the constant R is 53.18 when T is measured on the Fahrenheit scale; it may be obtained as follows:—

Consider one pound of air at normal temperature and pressure (N.T.P.): under these conditions the weight of one cubic foot of air is known to be 0.0807 pound, hence the volume of one pound is $\frac{1}{0.0807}$ or 12.391 cubic feet.

Standard atmospheric pressure is 14.7×144 or 2116 pounds per square foot, and the normal temperature is 32° F. or 492° absolute, hence

$$\begin{aligned} R &= \frac{pv}{T} \\ &= \frac{2116 \times 12.391}{492} \\ &= 53.18 \text{ foot-pounds per pound of air.} \end{aligned}$$

5. **The Third Law of Permanent Gases:** The specific heat at constant pressure is constant for any gas.—Let C_p = specific heat at constant pressure, and C_v the specific heat at constant volume.

Now at *constant volume* the gas does no external work when heated, hence, all the heat supplied is utilised in increasing its stock of internal energy; when heated at *constant pressure* the gas expands and does external work equal to the pressure multiplied by the change in volume.

Suppose 1 pound of gas to be heated at *constant pressure* p from absolute temperature T_1 to absolute temperature T_2 , and let v_1 be the volume of the gas at temperature T_1 , and v_2 the volume at temperature T_2 , then—

$$\begin{aligned} \text{Heat taken in by the gas} &= C_p(T_2 - T_1) \text{ and} \\ \text{Work done by the gas} &= p(v_2 - v_1) \\ &= R(T_2 - T_1) \text{ from (1) Art. 4} \end{aligned}$$

Also

$$\begin{aligned} \text{Increase in internal energy} &= \text{heat taken in} - \text{work done} \\ &= C_p(T_2 - T_1) - R(T_2 - T_1) \\ &= (C_p - R)(T_2 - T_1) \dots (1) \end{aligned}$$

6. **The Fourth Law of Permanent Gases:** When a perfect gas expands without doing external work, and without taking in or giving out heat (and therefore without changing its stock of internal energy), its temperature does not change.—The actual gases met with in practice are not perfect, and for all real gases this law is not perfectly true; for instance, Dr. Joule and Lord Kelvin found that air in expanding freely through a porous plug without doing work became cooled $\frac{1}{4}^\circ$ C. for each atmosphere fall in pressure. From this law we see that whatever may be the change in the pressure and volume of a *perfect* gas when it expands under the above conditions, its store of internal energy remains unaltered, and hence the internal energy of a perfect gas depends only on its temperature.

Suppose one pound of a perfect gas to be heated at *constant volume* from absolute temperature T_1 to absolute temperature T_2 . Then—

$$\begin{aligned} \text{Heat taken in} &= \text{work done} + \text{increase in internal energy} \\ C_v(T_2 - T_1) &= 0 + \text{increase in internal energy} \\ \text{i.e. increase in internal energy} &= C_v(T_2 - T_1) \dots (1) \end{aligned}$$

Now in Art. 5 we saw that the increase in internal energy was equal to $(C_p - R)(T_2 - T_1)$, hence we may say that the expression $C_v(T_2 - T_1)$ represents the increase in internal energy, no matter how the pressure and volume may change during the process.

7. **Relation between the Specific Heats C_p and C_v .**—From Art. 5, the increase in internal energy of one pound of perfect gas when heated at constant pressure from T_1 to T_2 is

$$(C_p - R)(T_2 - T_1)$$

From Art. 6, the increase in internal energy of one pound of perfect gas when heated at constant volume is

$$C_v(T_2 - T_1)$$

Equating these quantities we have—

$$(C_p - R)(T_2 - T_1) = C_v(T_2 - T_1)$$

$$C_p - R = C_v$$

which may be written

$$C_p - C_v = R \dots \dots \dots (1)$$

or the difference between the specific heats is constant, and for dry air is equal to 53.18 foot-pounds per pound of air.

Equation (1) may also be written

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

The ratio $\frac{C_p}{C_v}$ is very important, and is usually denoted by γ , hence

$$\gamma - 1 = \frac{R}{C_v}$$

or

$$C_v = \frac{R}{\gamma - 1} \dots \dots \dots (2)$$

For dry air Regnault found $C_p = 0.2375$ B.Th.U. per pound, which is equal to 0.2375×778 or 184.8 foot-pounds per pound. The same authority found $C_v = 0.1691$ B.Th.U. per pound, which is equivalent to 0.1691×778 or 131.6 foot-pounds per pound, hence

$$\gamma \text{ or } \frac{C_p}{C_v} = \frac{0.2375}{0.1691} \quad \text{or} \quad \frac{184.8}{131.6} = 1.404$$

the value of γ is not a constant quantity, however, because the specific heat at constant volume (C_v) varies with the temperature (see Chap. XIV.).

8. Work done by a Gas expanding according to Boyle's Law. Isothermal or Hyperbolic Expansion.—Suppose the expansion takes place from the initial state p_1, v_1 , and T_1 , to the final state p_2, v_2 , and T_2 .

The law of the expansion curve is $p v = a \text{ constant} = k$, say; for a small change in volume δv during which the average pressure is \bar{p} (Fig. 1) the work done

$$\delta W = \bar{p} \times \delta v$$

Let W denote the work done during the expansion, then

$$W = \int_{v_1}^{v_2} p \, dv.$$

Since

$$p v = k = RT$$

$$p = \frac{k}{v}.$$

$$\therefore W = \int_{v_1}^{v_2} \frac{k}{v} \, dv = k [\log_e v]_{v_1}^{v_2} = k \log_e \frac{v_2}{v_1}$$

or

$$W = p_1 v_1 \log_e \frac{v_2}{v_1} \quad \text{or} \quad RT \log_e \frac{v_2}{v_1} \dots \dots (1)$$

which may be written,

$$W = p_1 v_1 \log_e r \dots \dots \dots (2)$$

where r is the ratio of expansion, *i.e.* $\frac{\text{final volume}}{\text{initial volume}}$.

9. **Work done by a Gas expanding according to the Law $p v^n = \text{a Constant}$.**—As in Art. 8, let the expansion take place from the state $p_1, v_1,$ and $T_1,$ to the state $p_2, v_2,$ and $T_2,$ then

$$W = \int_{v_1}^{v_2} p dv$$

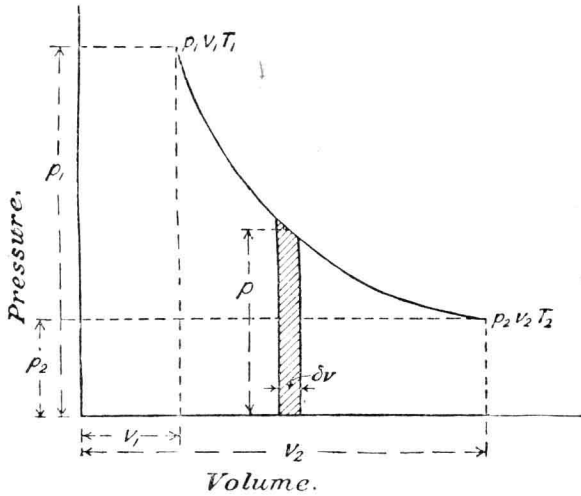


FIG. 1.

The law of the expansion curve is $p v^n = \text{a constant} = k,$ say,

$$\therefore p = \frac{k}{v^n}$$

$$\begin{aligned} \therefore W &= \int_{v_1}^{v_2} \frac{k}{v^n} dv \\ &= k \left[\frac{1}{1-n} v^{1-n} \right]_{v_1}^{v_2} \\ &= \frac{k}{1-n} \{ v_2^{1-n} - v_1^{1-n} \} \\ &= \frac{p_1 v_1^n}{1-n} \{ v_2^{1-n} - v_1^{1-n} \} \dots \dots \dots (1) \end{aligned}$$

and since $p_1 v_1^n = p_2 v_2^n,$ equation (1) becomes

$$W = \frac{p_1 v_1 - p_2 v_2}{n - 1} \dots \dots \dots (2)$$

Also for a perfect gas $p v = RT,$ and (2) may be written

$$W = \frac{RT_1 - RT_2}{n - 1} \quad \text{or} \quad \frac{R(T_1 - T_2)}{n - 1} \dots \dots \dots (3)$$

This expression for work done may be put in several forms, as follows :—

Since $W = \frac{p_1 v_1^n}{1-n} \left\{ v_2^{1-n} - v_1^{1-n} \right\}$ we may write

$$\begin{aligned} W &= \frac{p_1}{1-n} \left\{ v_1^n \times v_2^{1-n} - v_1 \right\} \\ &= \frac{p_1 v_1}{1-n} \left\{ v_1^{n-1} \times v_2^{1-n} - 1 \right\} \\ &= \frac{p_1 v_1}{1-n} \left\{ \left(\frac{v_1}{v_2} \right)^{n-1} - 1 \right\} \end{aligned}$$

or $W = \frac{p_1 v_1}{n-1} \left\{ 1 - \left(\frac{v_1}{v_2} \right)^{n-1} \right\} \dots \dots \dots (4)$

Again, $p_1 v_1^n = p_2 v_2^n$
 $\left(\frac{v_1}{v_2} \right)^n = \frac{p_2}{p_1} \dots \dots \dots (5)$

Substituting (5) in (4), we have—

$$W = \frac{p_1 v_1}{n-1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right\} \dots \dots \dots (6)$$

If all pressures are measured in pounds per square foot, and all volumes in cubic feet, the work done as calculated from either (2), (4), or (6), will be in foot-pounds.

If all pressures are measured in kilograms per square metre, and all volumes in cubic metres, the work done as calculated from the above formulæ will be in kilogram-metres.

For most purposes, particularly when under examination, the student is advised to remember and use the simple form $\frac{p_1 v_1 - p_2 v_2}{n-1}$.

10. Adiabatic Expansion and Compression.—When a gas expands without gaining or losing heat, and does an amount of work equal to the difference between its initial and final internal energy, the expansion is said to be adiabatic; conversely, if a gas is compressed without either heat being supplied to it or taken away from it, its change of internal energy being equal to the work done on it, the compression is said to be adiabatic. The value of “n” in the general $p v^n = \text{a constant}$ may be found as follows :—

When heat is supplied to a gas we have the fundamental relation discovered by Dr. Joule

$$\text{heat supplied} = \text{work done} + \text{increase in internal energy}$$

Let δH denote a small quantity of heat supplied, and δT and δv the resulting small increments of temperature and volume, then

$$\delta H = p \delta v + C_v \delta T$$

If the expansion takes place without gain or loss of heat as in adiabatic operations, $\delta H = 0$, hence

$$C_v \delta T + p \delta v = 0$$

or $\frac{\delta T}{\delta v} = - \frac{p}{C_v}$

hence in the limit :

$$\frac{dT}{dv} = -\frac{p}{C_v} \dots \dots \dots (1)$$

Now for a perfect gas $pv = RT = (C_p - C_v)T$ (Art. 7).

Differentiating we have—

$$\frac{dT}{dv}(C_p - C_v) = p + v \frac{dp}{dv} \dots \dots \dots (2)$$

Substituting the value of $\frac{dT}{dv}$ from (1) in (2) gives

$$-\frac{p}{C_v}(C_p - C_v) = p + v \frac{dp}{dv}$$

$$-p \frac{C_p}{C_v} + p = p + v \frac{dp}{dv}$$

$$-p\gamma = v \frac{dp}{dv}$$

or
$$\frac{dp}{p} = -\gamma \cdot \frac{dv}{v}$$

Integrating we have

$$\log p = -\gamma \log v + \text{a constant}$$

or
$$\log p + \gamma \log v = \text{constant}$$

or
$$pv^\gamma = \text{constant.}$$

Hence the law of an adiabatic expansion or compression curve is $pv^\gamma = \text{constant}$, where $\gamma = \frac{C_p}{C_v}$.

Alternative Method.—This result may also be found as follows :—

From (3) Art. 9,

$$W = \frac{R(T_1 - T_2)}{n - 1}$$

Now if the gas changes in temperature from T_1 to T_2 its internal energy is diminished by the amount

$$C_v(T_1 - T_2) \text{ from Art. 6,}$$

or
$$\frac{R}{\gamma - 1}(T_1 - T_2) \text{ since } C_v = \frac{R}{\gamma - 1} \text{ (Art. 7, equation (2))}$$

Hence equating the loss of internal energy to the work done, the condition for adiabatic expansion is secured when

$$\frac{R}{\gamma - 1}(T_1 - T_2) = \frac{R}{n - 1}(T_1 - T_2)$$

that is when $n = \gamma$.

Hence the expansion or compression will be adiabatic when

$$pv^\gamma = \text{constant.}$$