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# Introduction to Optimization of Structures

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# Author's Introduction

Recently, substantial advances have been made toward the solution of certain important problems of mechanics related to volumetric reduction and the improvement of the mechanical properties of structures.

The concepts of the "best" structures, in one sense or another, and of criteria that are assigned to their performance have been generalized. Numerical optimization techniques have also been improved, thus permitting us to formulate effective estimates of sensitivity of some of the important structural properties to changes in design parameters, and thus to analyze techniques for obtaining optimal solutions. Such results permit, at least in part, one to make extensive use of optimization techniques for the eventual establishment of automated (i.e., computer-assisted) design of systems. However, there are many unsolved problems in optimal design theory that are now being intensively studied.

This volume contains an exposition of some of the fundamental concepts, as well as an attempt to present the "state of the art" in the theory of optimal design. It consists of two parts. The purpose of the first part is to introduce the reader to the theory and techniques of optimal design. Here we offer an exposition of problems of optimal design and techniques for transforming them, necessary conditions for optimality, analytic and numerical methods for optimization of structural systems with distributed parameters, and optimization techniques for discrete systems. We consider problems of designs with multiple objectives, designs with incomplete information, and also fundamental concepts for designs with multiple optimality criteria.

The second part of this book is primarily devoted to applying the individual criteria of strength, rigidity, stability, and weight to optimization processes. Here we examine optimal design solutions for beams, curved rods, trusses, plates, and shells, and for large (heavy) bodies that may be obtained by the use of such specific criteria.

This book also utilizes results of research from the Laboratory for Structural Optimization in the Institute for Problems in Mechanics in the USSR Academy of Sciences.

A substantial part of the material presented in this book originated with lectures given by the author to the students of the Physico-Technical Institute in Moscow.

The author offers his sincere thanks to A.Iu. Ishlinskii, who helped in formulating the overall theme of the present book and also supported subsequent work.

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# Translator's Notes

A number of monographs dealing with the distributed parameter aspects of mechanical and structural optimization has appeared in the 1984 to 1988 time period. These include the Haug–Choi–Komkov monograph [55],[1] a translation of Banichuk's book [8], the Pironeau monograph [67], the Troitskii–Petukhov study [48], the Bogomolov–Simpson monograph [11], and Volume II of Komkov's monograph [57]. Expository works and long articles related to these topics include [53], [63], [65], [66], and [71]. They reinforce some older surveys, such as the excellent survey of Haug given in [23], the Olhoff survey of 1986 [63], the survey of Komkov [26] given at the meeting of the American Mathematical Society in 1983, the Niordson–Pedersen review [30] of 1973, Prager's work [37] of 1972, a shorter review by Venkayya [51], and the Brandt–Wasiutyński review [52] of 1963. A comparison of the recent and older monographs and surveys reveals an accelerated pace of ideas, and the steady incorporation of increasingly more sophisticated mathematical techniques and of new relevant physical data. The mathematical research of R.V. Kohn, Kohn and Vogelius, and Kohn and Strang established theoretical foundations for the "homogenization" of structural designs, an important aspect of shape optimization. Cea, Rousselet, Haug, Choi, Olhoff, and the author of the present study contributed significantly to a better understanding of the optimization of the stability of structural systems, which is absolutely essential in designs of slender aerospace-type structures and in earthquake-resistant designs.

The present monograph provides another and more recent presentation of the "state of the art" by a leading expert in this field. It concentrates on "difficult" aspects of systems with distributed parameters and on continuum mechanics, in general, as opposed to a discretized approach to structural design.

In some respects the distributed parameter theory is essential as a proper modeling background for discrete representation of many physical and engi-

[1] The references cited here are incorporated into the general bibliography given at the end of this monograph.

neering systems. Much of the theory offered in the present monograph is essential to efficient discretization schemes for structural designs and to the construction of numerical algorithms. Although this theory has outpaced the adaptation of relevant computing algorithms, it is the opinion of the translator that recent developments in rapid computational technology, particularly in the parallel computing field, have created a new environment for imaginative uses of this theory in computer-assisted design of very large and complex structural systems.

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# Contents

# Part 1
# The Theory and Techniques of Structural Optimization

# 1 Fundamental Concepts and Problems of Optimal Design

## 1.1 The Choice of a Computational Scheme in the Theory of Optimal Design

The focus of the theory of optimal design is on the best possible choice of a system of forces, the shape and properties of materials, and the working environment for a structure. The study of general laws governing extremal solutions and the development of effective optimization techniques are also part of the theory of optimal design.

Research in design has made it possible to describe the maximal potential for of structural improvement, to estimate the quality of traditional (non-optimal) reinforcements, and to discover the most effective ways of improving these reinforcements. There is a great variety of problem statements in optimal design (cf. [5], [14], [24], and [44]), perhaps because the equations for the loads and deflections of a structure, and the constraints imposed on its expected mechanical properties substantially differ from one type of structure to the next (beams, columns, curved beams, plates, or shells), between the different rheological properties (elasticity, plasticity, creep), the different external loads (such as surface loads, body forces, static, or dynamic loads, "dead loads," loads that depend on the behavior of the structure itself, or the thermal loads), the different types of control variables (such as variables controlling the shape of a structure), different assumptions for measuring the completeness of information available on the working environment (i.e., problems with incomplete information concerning interaction between the structure and the external environment, or concerning the manner of supporting the structure). The accuracy of the model and of the relevant data also influences the formulation of such problems.

The choice of a computational scheme (i.e., model) is essential to both the analysis and the optimization of a structure. Therefore, optimization of design is impossible without first conducting a preliminary study of the assumptions made about both the real and imagined aspects of the structural response, and before preparing a scheme describing the working conditions, and before making various simplifications that would still preserve the adequacy of the computational scheme for a real-life structure. A mathematical model of the real behavior of an object stripped of all unrealistic properties or assumptions shall be called in this presentation "a computational scheme."

Such a description of computational schemes routinely takes place in typical strength of materials courses (e.g., [39]). But, in fact, the choice of computational scheme is not unique.

In some cases several differing schemes may be offered for the study of the same object. On the other hand, a single computational scheme may model several different real-life objects.

In design optimization one tries to apply computational schemes that would uniquely determine the relevant quantities of stress-deformed states, as well as specific values of the design variables.

However, this is not always possible to achieve, either because of the absence of exact data concerning the external loads, or incompleteness of the data concerning the manufacture of the product, the distribution of parameters that describe the materials of the structure, or other factors contributing to the incompleteness of information. To produce an adequate scheme in such a situation it makes sense to relax the demands of an "exact" description of a real-life object and to adopt either a computational scheme for the structure that assumes the worst possible scenario or else to adopt a scheme of stochastic description. These are the so-called "guaranteed" and "probabilistic" approaches to the adoption of a computational scheme.

With regard to the geometric aspects affecting the choice of a computational scheme, we shall discuss only the most commonly used schemes in the theory of optimal structural design. One may model a solid three-dimensional body whose characteristic dimensions are of the same order of magnitude in each spatial direction. Bodies having one of these dimensions "substantially larger" than the other two (such as rods, arches, or systems of beams) and bodies having one dimension "substantially smaller" that the other two dimensions (such as shells and plates) comprise the remaining two cases.

The general setting may vary in optimal design theory. It does depend substantially on the type of the designed structure; that is, is this a traditional structure, or is it an entirely new design? In the first case much useful information is available concerning the prototypes and much of the accumulated experience may be utilized in the form of a "support" solution, or perhaps as the initial approximation for the optimal design process. In that case only a few design parameters are regarded as unknown and optimization of the structure is accomplished with a relatively small number of basic design variations. In the second case the design is determined by a large number of design variables and the optimization process has to be carried out by considering a large number of admissible variations (i.e., of separate design projects). A degree to which the parameters are regarded as known, or conversely, the absence of any knowledge concerning the design parameters that would determine either the exterior shape or the internal properties of a structure is an important factor in determining the computational scheme.

The stage in the computational scheme in which the important unknown variables appear for the first time is critical, and in general, determines the effects of optimization procedures. The introduction of the design parameters frequently amounts to a stipulation of some additional requirements for the computational scheme. Let us clarify the last statement by offering an example of an optimal design of a thin shell with variable thickness. In the optimization we will use the classical theory of elastic thin shells. If, as an outcome of the optimization process, we obtain a distribution of the thicknesses having large gradients or having some singularities (such as discontinuities, a "zero," or an "infinite" thickness), the classical computational scheme is not valid and therefore one must introduce appropriate corrections.

Here the possible corrections are of two principal types:

1. An open introduction into the computational scheme of new constraints that are necessitated by the accepted theory, and the subsequent use of these constraints during the design process. This operation amounts to a "tightening" of the set of admissible design projects.
2. A "broadening" and generalizing of the classical computational schemes by taking into account trends that tend to alter the design projects, thus permitting an analysis of a larger class of admissible structures.

In choosing a computational scheme for optimal design problems an important role is played by the a priori knowledge of properties of the unknown solution. Information about the model, knowledge of some basic properties of the solution, and also a reexamination of the initial hypotheses all permit us to state some essential constraints in the formulation of the optimization problems and to exclude "second layer" considerations, making it possible to state the problem in proper form to apply some of the well-known numerical or even analytical techniques.

Therefore, a large body of results in the theory of optimal design is directly connected with some well-known computational schemes. However, it is frequently quite difficult to "guess" in advance the properties of the unknown optimal solution, and the optimization problem may turn out to be formulated so that the derived solutions violate the fundamental hypotheses essential to the model itself. For example, certain solutions of problems of plate design may have large gradients of thickness. This contradicts some of the assumptions that are fundamental to Kirchhoff's plate theory. Other well-known singularities arising in the use of classical models of plates and shells are caused by the presence of either zero or infinite thickness in the optimal solution. Thus, if one discovers a deviation from the model or a violation of hypotheses that are essential to the computational scheme, it becomes necessary to modify the system of relations used in a design procedure by the introduction of additional constraints, for example, on thickness in a problem concerning the bending of plates. Another method of escaping from such a situation may involve a generalization of the model, consisting in "softening" the hypotheses and constructing a computational scheme for a broader class of structures—one that includes some previously inadmissible designs. Thus, the choice of a model and its evolution constitute important aspects of the design process.

Let us summarize what has been said above. Generally, it is useful in any optimal design process to permit the possibility of making corrections in the computational schemes (that is in the mathematical model) thus sharpening or changing some of the initially assigned conditions.

## 1.2 Formulation of Problems in Optimal Structural Design

The optimal design problems considered here consist in determining the shapes, internal properties, and working conditions of a structure that obey assigned constraints, and produce an extremum (either a maximum, or a minimum) for a chosen quantity characterizing the design. In a rigorous postulation of a problem of optimal structural design one must include the statement of basic constitutive relations (the choice of a mathematical model) and define the functional to be optimized, asign all the necessary constraints to the state functions, and select the unknown control (design) variables. From a purely mathematical point of view, such problems may be classified by identifying the types of systems of differential equations and boundary conditions, the types of optimized functionals and initially given constraints, or, alternatively, by the dimension of the problem, the manner in which the unknown design variables enter into the fundamental relations (e.g., should we control the coefficients or the boundary of the region?), the completeness of information concerning the given initial data (i.e., are we given a problem with full information or one with incomplete information?), the nature of the extremum (e.g., are we given a single extremum or multiple extrema?) and the manner in which the extremum is defined (e.g., are we given a single criterion or multiple criteria?), and perhaps by some other properties of the system.

In this section we shall examine a classical formulation of optimal design problems. An exposition of some possible generalizations will be given in sections 1.5 to 1.8. As we have previously remarked, an essential feature in the formulation of such problems is the choice of a mechanical model. To begin with, we choose the state variables **u** and system of equations

$$L(\mathbf{x}, \mathbf{u}, \mathbf{h}, \mathbf{q}) = 0 \qquad (1.1)$$

relating these variables to the design parameters and also to the external loads. Here the state variable $\mathbf{u} = \{u_1(x), \ldots, u_m(x)\}$ is a vector function that determines the state of the structure. The independent variable $\mathbf{x} = \{x_1, \ldots, x_l\}$ assumes all possible values in the region $\Omega$. The symbol $L$ in the eq. (1.1) denotes a differential operator acting on functions of the spatial coordinates $x_i$ only.

Equation (1.1) can be regarded as a system of generally nonlinear differential equations. In this monograph we will be concerned with theories that assume the validity of both geometric and physical hypotheses of linearity. In accordance with such assumptions, the behavior of structures is described by operators that are linear with respect to the state variables.

The operator $L$ in eq. (1.1) depends on the design vector function $\mathbf{h}$, where $\mathbf{h} = \{h_1(x), \ldots, h_n(x)\}$, and on the vector function of the external loads $\mathbf{q}$. The natural numbers $m$, $n$, and $l$ are given. Here it is assumed that the boundary conditions determining the type of support and the loads applied to the structure are all specified in the definition of $L$.

Given the loads and the structural parameters, the system of differential equations must be "closed" (i.e., is well-posed), and thus should uniquely determine the state variables, which in turn characterize the stresses and deformations of the structure. Determining the state variables from the given design functions shall be referred to as the direct problem.

If the state equations reflect the laws of physics, the choice of the design variables appearing in the functionals considered in the design process, including the optimized functional (i.e., the quality, or cost functional) and the system of constraints, are all governed by the designated purpose of the structure, the working environment, and the technology available for its construction.

The functions $h_i(x)$ determine the shape and also the physical and mechanical properties of the construction materials. As our $h_i(x)$, we can choose the distribution of thickness and the cross-sectional area of the body, functions that determine the location of the median surface for curved beams or shells, the distribution of the density of the reinforcing material inside the structure, or perhaps the angles that define the orientation of the axes of anisotropy at each point inside the region occupied by an elastic body.

In an optimal design problem, besides the state functions and the design variables we also need to know the functionals that characterize the design. These functionals depend on the vectors $\mathbf{u}$, $\mathbf{h}$, and $\mathbf{q}$:

$$J_1 = J_1(\mathbf{u}, \mathbf{h}, \mathbf{q}), \ldots, J_r = J_r(\mathbf{u}, \mathbf{h}, \mathbf{q}).$$

Two types of such functionals are considered in optimal design problems: functionals of the first type are called integral functionals:

$$J_i = \int_\Omega f_i(\mathbf{x}, \mathbf{u}, \mathbf{h}, \mathbf{q})\,d\Omega, \qquad i = 1, \ldots, r_1, \tag{1.2}$$

and of the second type, local functionals, for example,

$$J_j = \max_x f_j(x, \mathbf{u}(x), \mathbf{h}(x), \mathbf{q}(x)), \tag{1.3}$$

with $j = r_1 + 1, \ldots, r_1 + r_2$.

Here $f_i$ denotes given differential forms while $r_1$ and $r_2$, are assigned natural numbers with $r_1 + r_2 = r$.

Integral functionals or combinations of integrals of the type given in (1.2) may represent characteristic properties of the structure, such as the weight, the energy of the elastic deformation (compliance), natural frequencies of vibration, or a critical load that would cause the structure to lose its stability (cf. [1.11], [1.27], [1.39], and [1.40]). A local property could be the magnitude of the maximal deflection or the stress intensity (cf. [7], [8], and [28]).

If other demands are made concerning the properties of the structure, then appropriate constraints must be applied to the design and state variables. The given constraints may form a system of inequalities, which may be displayed in vector form:

$$\boldsymbol{\psi}(\mathbf{x}, \mathbf{u}, \mathbf{h}, \mathbf{q}, J_1, \ldots, J_r) \leq 0. \tag{1.4}$$

The components $\psi_i$ of the vector $\{\boldsymbol{\psi}\} = \{\psi_1, \ldots, \psi_k\}$ are regarded as a priori known functions of their arguments. Various notations used for the constraints (1.4) are discussed in section 2.2. In specific cases inequalities (1.4) represent different types of bounds on the stresses, deformation, or displacements, the integrands in rigidity or compliance functionals, and also the natural frequencies of vibration and values of critical parameters that determine the loss of stability. One of these functionals, or perhaps a function of several functionals, $F(J_1, \ldots, J_r)$, is then chosen as the functional that is to be optimized.

The optimization problem now consists in finding a specific (vector) function that assigns a minimum (or a maximum) to the functional

$$J = F(J_1, \ldots, J_r) \tag{1.5}$$

and also satisfies (1.1) to (1.4).

Note that, generally speaking, there may be arbitrarily many functionals and assigned constraints, as long as they are not contradictory. However, there can be only a single optimized functional (or quality criterion) for the structure in each specific problem of the form of (1.1) to (1.5). Thus, in the bending of a beam with variable thickness it is possible to formulate a problem of weight minimization with a constraint on the maximal deflection, or the problem of minimizing the maximal deflection for a given total weight. However, simultaneous optimization of both these functionals does not make sense within the context of the classical concepts of optimality.

A correct formulation of optimization problems with a vector of assigned quality criteria becomes possible if one makes use of the concept of optimality in the sense of Pareto, or in some other sense appropriate to a multiple criteria optimization. A large number of publications (e.g., [46], [1.25], [1.37], and [1.38]) contain an exposition of the basic ideas of multi-criterion optimization theory. Nevertheless, approaches to structural design based on non-classical concepts of extrema are presently only in the initial stages of development.

It has been made clear in several recent research articles on optimal design theory that in many instances the formulation of optimization problems in the manner of (1.1) to (1.5) has some serious limitations, since the optimal solution $\mathbf{h}^*$ or $\mathbf{u}^*$ may not exist for the stated design limitations, even though there exists a minimal value $J^*$ for the cost functional. Moreover, we may not be particularly interested in finding $\mathbf{h}^*$ or $\mathbf{u}^*$. It may suffice for our purposes to uncover the general trends while searching for an optimal solution and to find the sensitivity of functionals that are describing important characteristics of a structure with respect to changes in the values of the parameters. Therefore, it is of interest to broaden the formulation of the design problems, and to introduce mathematical rigor so as to derive the entire process of obtaining a solution by means of techniques of modern sensitivity analysis.

This generalized setting would include a construction of a minimizing sequence, where for each fixed superscript "$i$" the elements of that sequence satisfy (1.1) to (1.4) and the condition

$$\lim_{i \to \infty} (J^i) = J^*, \tag{1.6}$$

where $J^i = J(\mathbf{u}^i, \mathbf{h}^i, \mathbf{q})$.

In the case when a classical solution exists for the problem represented by (1.1) to (1.4), the limits: $\mathbf{h}^i \to \mathbf{h}^*$, and $\mathbf{u}^i \to \mathbf{u}^*$ (as $i \to \infty$) also exist.

## 1.3 Basic Functionals and Optimization Criteria

The choice of functionals in design optimization is the most important part of formulating optimization problems. Many factors enter into this choice, for example, the main purpose of the structure, working conditions, technology available for its construction, cost limitations, properties of the model used to describe the mechanical behaviour of the structure, and the a priori known properties of the optimal design problem.

In what follows we consider typical functionals that are most frequently considered in structural optimization:

1.3.1 One of the most important properties of a structure is its total weight, and consequently this functional is considered in most design optimization projects as either the quality criterion or as one of the assigned constraints. The weight of the structure determines both the quantity of materials used in the construction and some of its functional properties. For example, an increase in the weight of an aircraft results not only in the increase of the quantity of materials necessary for the construction, but will also produce higher fuel consumption and will no doubt worsen other important flight characteristics.

Weight is an integral property of the design. For homogeneous continua, weight is proportional to the volume occupied by the body: