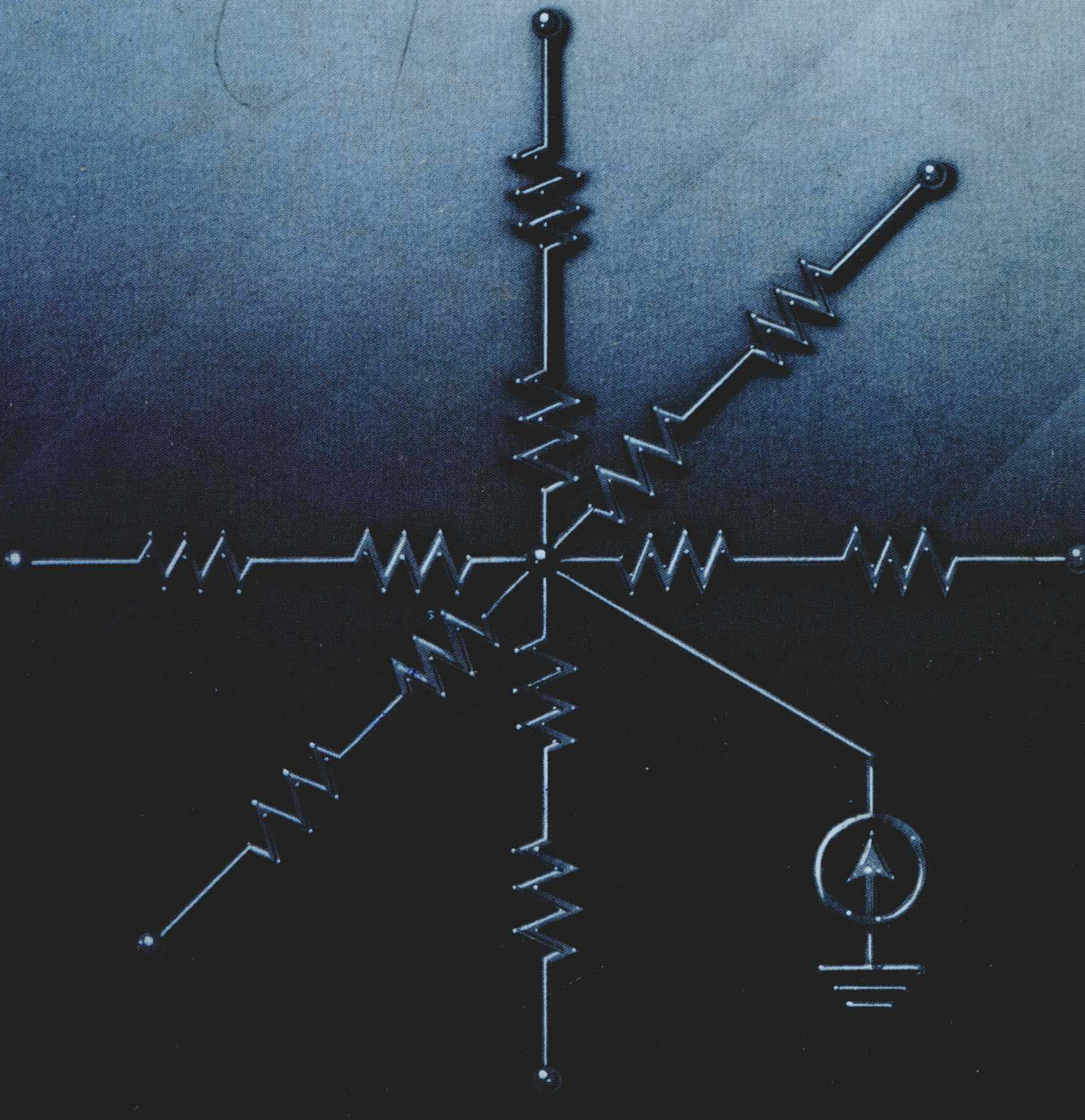
# BASIC ELECTROMAGNETIC FIELDS

HERBERT P. NEFF, JR.



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*To Barbara and the kids*

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# Preface

It is assumed in *Basic Electromagnetic Fields* that the student is familiar with vector addition and the scalar and vector products. Other, more advanced, vector operations are introduced and explained as they are needed. This approach is entirely consistent with the desire for a modern textbook, even though it may be a break with tradition. A rather complete summary of vector relations is included in Appendix A for reference.

Although it is not necessary, it is very helpful if the student has also had a basic course in electric circuit theory in the sophomore year. This is quite likely the case for most engineering colleges. The concept and use of the Dirac delta function is now being taught in circuit theory at this level in many institutions. The Dirac delta function proves convenient for use in supplying the proofs of many integral relations encountered in field theory. These proofs are not supplied in the body of this text, but, rather, are included in Appendix G for the interested reader.

Since one can find the response of a linear system to arbitrary excitation if its response to impulse (or Dirac delta function) excitation is known, and since the field theory in this text is almost exclusively "linear," impulse responses will be identified and used wherever possible. This proves to be not only conceptually effective, but quite often avoids the expenditure of much time and labor in pursuing classical methods of deriving superposition type integrals of field theory.

This text has been designed primarily for use at the junior or senior level in electrical engineering. From an applications viewpoint, it is by no means complete, but is heavily slanted toward guiding systems. For this reason, some mention of the cylindrical waveguide and cavity and the spherical cavity is included in Appendix H. A detailed study of these systems requires the introduction of certain special mathematical functions with which the undergraduate student may not be familiar. In order to obtain a qualitative and quantitative understanding of the behavior of these systems at this level of exposure, it is not necessary for the student to have a detailed knowledge of these functions. It is very easy, however, for the instructor to simply omit this material, if so desired, with no loss in continuity.

One of the most frustrating experiences the beginner can encounter in problems involving integration with several variables is the lack of a consistent notation. Even if the student understands the statement of a fundamental equation or law, he or she may have considerable difficulty in solving a problem involving the use of such a law. Many times this difficulty can be overcome if a precise and consistent notation is followed. This does not, of course, guarantee that the student will be able to solve the problem, but at least the solution can be started properly. One of the goals of this text has been that of following a precise and consistent notation.

Even experienced engineers occasionally find themselves in an apparent dilemma when working in an area as mathematically oriented as is field theory. Quite often the difficulty arises because of naive attitudes developed as an undergraduate. For example, in certain mathematical formulations, we sometimes overlook built-in constraints in plunging headlong toward our final result. It is worthwhile to stop along the way occasionally and ask if the last step was valid, and if so, under what conditions? Are the results reasonable on physical grounds? An attempt has been made in this textbook to point out explicitly these pitfalls when they arise.

The procedure followed in *Basic Electromagnetic Fields* is that of adopting experimental laws as the need arises, maintaining a perhaps slow, but steady, pace toward Maxwell's equations. We could start with Maxwell's equations, if desired, and show that the experimental laws are correct, but that they are merely special cases. The latter procedure is often followed at the graduate level. The former procedure has the distinct advantage of allowing the student to gradually accept the fundamental laws and the mathematics, instead of being engulfed by them.

There are many example problems that are worked out in the text when a particularly important, or perhaps difficult, point is being made. A modest number of problems has been included at the end of each chapter. These problems refer, with few exceptions, explicitly to the material just covered, and should, therefore, be considered an intrinsic part of the chapter.

A reference list is included at the end of each chapter and is referred to occasionally. The books that are mentioned explicitly are those that have had the most influence on this author, but it is certainly not implied that these are the only useful books. Many discussions of historical interest can be found in the literature and the student is invited to look into these. This material, often called "bedtime reading," is usually very interesting and informative, but, at the same time, represents one more time-consuming task in the already crowded schedule of the undergraduate engineering student. For this reason, outside reading is recommended but not demanded.

Listed below are three possible outlines assuming that three hours per week are allowed for the course.

## TWO QUARTERS

| FIRST QUARTER | SECOND QUARTER |
|---|---|
| (STATICS) | (DYNAMICS) |
| Chapter 1 | Chapter 9[a] |
| Chapter 2 | Chapter 10[b] |
| Chapter 3 | Chapter 11 |
| Chapter 4 | Chapter 12 |
| Chapter 5[a] | Chapter 13[b] |
| Chapter 6[a] | Chapter 14[a] |
| Chapter 7 | Chapter 15[b] |
| Chapter 8[a] | Chapter 16[b,c] |

## THREE QUARTERS

| FIRST QUARTER | SECOND QUARTER | THIRD QUARTER |
|---|---|---|
| (ELECTROSTATICS) | (MAGNETOSTATICS AND DYNAMICS) | (GUIDING SYSTEMS) |
| Chapter 1 | Chapter 7 | Chapter 11 |
| Chapter 2 | Chapter 8 | Chapter 12 |
| Chapter 3 | Chapter 9 | Chapter 13[b] |
| Chapter 4 | Chapter 10[a] | Chapter 14 |
| Chapter 5 | | Chapter 15[b] |
| Chapter 6 | | Chapter 16[a] |

[a] Selected topics only.
[b] Can be omitted entirely with no loss in continuity.
[c] If time allows.

## TWO SEMESTERS

| FIRST SEMESTER | SECOND SEMESTER |
|---|---|
| Chapter 1 | Chapter 9 |
| Chapter 2 | Chapter 10 |
| Chapter 3 | Chapter 11 |
| Chapter 4 | Chapter 12 |
| Chapter 5 | Chapter 13[b] |
| Chapter 6 | Chapter 14 |
| Chapter 7 | Chapter 15[b] |
| Chapter 8 | Chapter 16[b,c] |

[a] Selected topics only.
[b] Can be omitted entirely with no loss in continuity.
[c] If time allows.

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Herbert P. Neff, Jr.

# Symbols and Units

The International System of Units is an expanded form of the rationalized meter-kilogram-second-ampere (MKSA) system of units, and is used throughout this textbook. It is based on six basic units: the meter (length), the kilogram (mass), the second (time), the ampere (electric current), the kelvin degree (temperature), and the candela (luminous intensity). All other units may be derived from these six. Definitions of these fundamental units may be found in the references. See Hayt in the references at the end of Chapter 2, for example.

The symbols used in this text are, more or less, conventional. That is, they are usually the same as those for the same quantities found in other textbooks. Occasionally, it is convenient, because of inference, to use the same symbol for two different quantities. For example, $\omega_c$ is used to represent the angular cutoff frequency of a waveguide, and it may also be used to represent the cyclotron frequency. In a situation such as this, these quantities will not appear together in the discussion, and certainly not in the same equations. There will never be any confusion about what is meant. The following table lists the quantities that are encountered in the text. It also includes the symbol used for the quantity, the unit of the quantity, and the abbreviation of the unit.

## NAMES AND UNITS OF THE QUANTITIES FOUND IN THE TEXT IN THE INTERNATIONAL SYSTEM

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| ac capacitivity | $\varepsilon'(\omega)$ | farads/meter | F/m |
| ac inductivity | $\mu'(\omega)$ | henrys/meter | H/m |
| Acceleration | $\mathbf{a}$ | meters/second$^2$ | m/s$^2$ |
| Admittance | $Y$ | mhos | ℧ |
| Admittivity | $\hat{y}$ | mhos/meter | ℧/m |
| Angular frequency | $\omega$ | radians/second | s$^{-1}$ |
| Angular velocity | $\boldsymbol{\omega}$ | radians/second | s$^{-1}$ |
| Antenna aperture | $A$ | meters$^2$ | m$^2$ |
| Antenna gain | $G$ | — | — |
| Associated Legendre function, first kind | $P_n^m(x)$ | — | — |
| Associated Legendre function, second kind | $Q_n^m(x)$ | — | — |
| Attenuation constant | $\alpha$ | nepers/meter | m$^{-1}$ |
| Bessel function, first kind, order $n$ | $J_n(x)$ | — | — |
| Bessel function, second kind, order $n$ | $N_n(x)$ | — | — |
| Bound charge density | $\rho_v^b$ | coulombs/meter$^3$ | C/m$^3$ |
| Bound current density | $\mathbf{J}^b$ | amperes/meter$^2$ | A/m$^2$ |
| Capacitance | $C$ | farads | F |
| Characteristic impedance | $Z_0$ | ohms | Ω |
| Characteristic resistance | $R_0$ | ohms | Ω |
| Charge | $Q, q$ | coulombs | C |
| Coefficient of magnetic coupling | $k_{mc}$ | — | — |

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| Coefficient of reflection | $\Gamma$ | — | — |
| Coefficient of transmission | $T$ | — | — |
| Complex propagation constant | $\gamma$ | meter$^{-1}$ | m$^{-1}$ |
| Conductance | $G$ | mhos | ℧ |
| Conductivity | $\sigma$ | mhos/meter | ℧/m |
| Current | $i(t)$, $I$ | amperes | A |
| Current density | $\mathbf{J}$ | amperes/meter$^2$ | A/m$^2$ |
| Cutoff angular frequency | $\omega_c$ | radians/second | s$^{-1}$ |
| Cutoff frequency | $f_c$ | hertz | Hz |
| Del operator | $\nabla$ | — | — |
| Dirac delta function | $\delta(x)$ ($x$, meters) | meter$^{-1}$ | m$^{-1}$ |
| Dielectric loss angle | $\delta_d$ | — | — |
| Dielectric loss factor | $\varepsilon''(\omega)$ | farads/meter | F/m |
| Drift velocity | $\mathbf{u}_d$ | meters/second | m/s |
| Effective length | $l_E$ | meters | m |
| Electric dipole moment | $\mathbf{p}$ | coulomb-meter | C·m |
| Electric energy | $W_E$ | joules | J |
| Electric energy density | $w_E$ | joules/meter$^3$ | J/m$^3$ |
| Electric field intensity | $\mathbf{E}$ | volts/meter | V/m |
| Electric flux | $\Psi_E$ | coulombs | C |
| Electric flux density | $\mathbf{D}$ | coulombs/meter$^2$ | C/m$^2$ |
| Electric potential difference | $\Phi_{ab}$ | volts | V |
| Electric scalar potential | $\Phi$ | volts | V |
| Electric susceptibility | $\chi_E$ | — | — |

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| Electric vector potential | **F** | volts | V |
| Electromotive force | emf | volts | V |
| Electron charge | $-e$ | coulombs | C |
| Electron mobility | $\mu_e$ | meter²/volt-second | $m^2/V \cdot s$ |
| Electron rest mass | $m_e$ | kilograms | kg |
| Energy (work) | $W$ | joules | J |
| Force | **F** | newtons | N |
| Frequency | $f$ | hertz | Hz |
| Group velocity | $u_g$ | meters/second | m/s |
| Hankel function, first kind, order $n$ | $H_n^{(1)}(x)$ | — | — |
| Hankel function, second kind, order $n$ | $H_n^{(2)}(x)$ | — | — |
| Heat flux rector | **s** | watts/meter² | $W/m^2$ |
| Hole mobility | $\mu_h$ | meter²/volt-second | $m^2/V \cdot s$ |
| Impedance | $Z$ | ohms | Ω |
| Inductance | $L$ | henrys | H |
| Intrinsic impedance | $\eta$ | ohms | Ω |
| Length | $R$, $l$, $d$, etc. | meters | m |
| Line charge density | $\rho_l$ | coulombs/meter | C/m |
| Mass | $M$, $m$ | kilograms | kg |
| Magnetic dipole moment | **m** | ampere-meter² | $A \cdot m^2$ |
| Magnetic energy | $W_H$ | joules | J |
| Magnetic energy density | $w_H$ | joules/meter³ | $J/m^3$ |
| Magnetic field intensity | **H** | amperes/meter | A/m |
| Magnetic flux | $\psi_m$ | webers | Wb |
| Magnetic flux density | **B** | webers/meter² (tesla) | $Wb/m^2$ |
| Magnetic loss angle | $\delta_m$ | — | — |

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| Magnetic loss factor | $\mu''(\omega)$ | henrys/meter | H/m |
| Magnetic scalar potential | $\Phi_m, \Phi_f$ | amperes | A |
| Magnetic susceptibility | $\chi_m$ | — | — |
| Magnetic vector potential | **A** | webers/meter | Wb/m |
| Magnetization | **M** | amperes/meter | A/m |
| Magnetomotive force | mmf | ampere-turns | A·t |
| Modified Bessel function, first kind, order *n* | $I_n(x)$ | — | — |
| Permeability | $\mu$ | henrys/meter | H/m |
| Permeability of vacuua | $\mu_0$ | henrys/meter | H/m |
| Permittivity | $\varepsilon$ | farads/meter | F/m |
| Permittivity of vacuua | $\varepsilon_0$ | farads/meter | F/m |
| Phase constant | $\beta$ | radians/meter | $m^{-1}$ |
| Phase velocity | $u_p$ | meters/second | m/s |
| Polarization | **P** | coulombs/meter$^2$ | C/m$^2$ |
| Power | $P$ | watts | W |
| Power per unit solid angle | $P_s$ | watts/steradian | W |
| Power radiated | $P_r$ | watts | W |
| Poynting vector (power density) | **S** | watts/meter$^2$ | W/m$^2$ |
| Proton rest mass | $m_p$ | kilograms | kg |
| Quality factor | $Q$ | — | — |
| Radiation impedance | $Z_r$ | ohms | Ω |
| Radiation resistance | $R_r$ | ohms | Ω |
| Relative permeability | $\mu_R$ | — | — |

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| Relative permittivity | $\varepsilon_R$ | — | — |
| Reluctance | $\mathcal{R}$ | henrys$^{-1}$ | $H^{-1}$ |
| Resistance | $R$ | ohms | Ω |
| Resonant frequency | $f_r$ | hertz | Hz |
| Schelkunoff's Bessel function, first kind | $\hat{J}_n(x)$ | — | — |
| Schelkunoff's Bessel function, second kind | $\hat{N}_n(x)$ | — | — |
| Spherical Bessel function, first kind | $j_n(x)$ | — | — |
| Spherical Bessel function, second kind | $n_n(x)$ | — | — |
| Skin depth (depth of penetration) | $\delta$ | meters | m |
| Standing wave ratio | SWR | — | — |
| Surface area | $s$ | meter$^2$ | $m^2$ |
| Surface charge density | $\rho_s$ | coulombs/meter$^2$ | $C/m^2$ |
| Surface current density | $\mathbf{J}_s$ | amperes/meter | A/m |
| Surface impedance | $\eta$ | ohms | Ω |
| System operator | $H\{-\}$ | — | — |
| Temperature | $T$ | degrees kelvin | °K |
| Time delay | $\tau_D$ | seconds | s |
| Torque | $\mathbf{T}$ | newton-meters | N·m |
| Transfer function | H, **H**, H($\omega$) | varies | varies |
| Transverse wave impedance | $\eta^{\pm}$ | ohms | Ω |
| Unit impulse response | h, **h** | varies | varies |
| Unit step function | $u(x)$ | — | — |

| QUANTITY | SYMBOL | UNIT | ABBREVIATION |
|---|---|---|---|
| Vectors | $\mathbf{v}$ | — | — |
| Velocity | $\mathbf{u}$ | meters/second | m/s |
| Velocity of energy flow | $u_e$ | meters/second | m/s |
| Velocity of light | $c$ | meters/second | m/s |
| Voltage | $v(t)$, $V_0$, $V_{21}$, etc. | volts | V |
| Voltage standing wave ratio | VSWR, $s$ | — | — |
| Volume | $v$, vol | $\text{meter}^3$ | $m^3$ |
| Volume charge density | $\rho_v$ | $\text{coulombs/meter}^3$ | $C/m^3$ |
| Waveguide wavelength | $\lambda_g$ | meters | m |
| Wavelength | $\lambda$ | meters | m |
| Wave number (intrinsic phase constant) | $k$ | radians/meter | $m^{-1}$ |

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