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**APPLIED
OPERATIONS
RESEARCH:
A SURVEY**

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To our wives, Marian and
Bonnie, who eagerly
accepted a share of the
load that was more than
we had any right to ask of
them—an act for which we
shall be eternally grateful.

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preface

This textbook provides the advanced undergraduate and first-year graduate students, regardless of their major discipline, with an understandable text that covers the fundamental operations research/management science techniques and their computational aspects. The trend toward increased use of these techniques in business and the social services, as evidenced for example by the CPA examination that requires a knowledge of linear programming, must be satisfied. In achieving this objective we feel that the text does three things.

First, it presents the techniques and their computational aspects within a broad, real-world applications environment. In short, the book whets the students' appetite with application, application, and more application through demonstrated usefulness. Each chapter explains the techniques using illustrative examples, numerous application cases used and usable in the real world, and a generous helping of progressively difficult problems that build a student's confidence early. We feel we have succeeded in alleviating the need for supplemental texts or readings, which should delight instructors as well as students.

Second, the text covers a very significant number of the available operations research techniques, at least those that are generally considered fundamental to an introductory understanding of the discipline. We have presented as much as we could in this survey, but the broad topical coverage of the eight chapters presents more material than can be dealt with normally in a single, three-credit-hour course. However, selecting topics of primary interest to the student for a three-hour course will pose no problem to the instructor if such a limitation exists. Adequate coverage of all the topics in an extended course is desirable and feasible.

Last, but not least, the text departs from the heavy mathematical orientation frequently encountered in the treatment of the subject matter. The average student who has completed algebra, introductory probability and statistics, and has a general facility with numbers has an

adequate background for handling our treatment of the topics covered. Only a touch of calculus is used in a few selected topics where we felt it was indispensable. One of our colleagues, in reviewing the mathematical treatment, observed that it may distress those who are mathematically oriented but that it would be a joy to the average student.

No preface is complete without an acknowledgment. We would be remiss if we did not mention our colleagues, too numerous to name, both inside and outside of our discipline as well as our many students without whose encouragement we probably would not have been able to complete this book. Nor can we fail to acknowledge the understanding, encouragement, and assistance we were given by Professor George E. Kane, our department chairman. If the book achieves its objective, primary credit rests with their reinforcement of the spirit; if it falls short in any respect, primary responsibility rests with us.

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contents

1 Introduction

- 1.1 What is Operations Research? **1**
- 1.2 Systems Modeling and the Tools of Operations Research **5**
- 1.3 Management's Perspective of Quantitative Methods **6**
- 1.4 Application Studies **12**
 - 1.4.1 What is an Application Study? **12**
 - 1.4.2 Human Decision versus Math Model: An Experiment **12**
 - 1.4.3 When Should You Trade Your Car? **16**

2 Decision Analysis

- 2.1 Introduction to Decision Analysis **21**
- 2.2 Decision Making Under Assumed Certainty **24**
- 2.3 Decisions Under Risk **28**
- 2.4 Decisions Under Uncertainty **36**
- 2.5 Decision Tree Analysis **43**
- 2.6 Game Theory **51**
- 2.7 Application Studies **56**
 - 2.7.1 Insurance Decisions **56**
 - 2.7.2 A Competitive Bidding Model **57**
 - 2.7.3 Equipment Purchase **59**
 - 2.7.4 The Determination of an Optical Testing Sequence **61**

- 2.7.5 Decision Analysis in the National Football League **66**
- 2.7.6 Labor Negotiations **68**
- 2.8 Problems **68**

3 Linear Programming

- 3.1 Introduction **80**
- 3.2 Graphical Technique **80**
- 3.3 Algebraic Technique **84**
- 3.4 The Simplex Algorithm **85**
- 3.5 Conversion to Standard Form **96**
- 3.6 Duality and Shadow Prices **102**
- 3.7 Postoptimality Analysis **105**
- 3.8 Application Studies **111**
 - 3.8.1 Fertilizer Blending **111**
 - 3.8.2 The Optimal Use of Common Labor **113**
 - 3.8.3 Optimizing a Parts-Purchasing Decision **120**
 - 3.8.4 Effective Use of School Busing to Achieve Desegregation **125**
 - 3.8.5 Use of Linear Programming in Air-Cleaner Design **126**
 - 3.8.6 A Capital Budgeting Decision **129**
 - 3.8.7 Solution of A Two-Person Game **134**
- 3.9 Problems **135**

4 Specialized Mathematical Programming Topics

- 4.1 Introduction **147**
- 4.2 The Transportation Model **147**
- 4.3 The Assignment Model **161**
- 4.4 The Transshipment Model **166**
- 4.5 Branch and Bound **169**
- 4.6 Separable Programming **180**
- 4.7 Dynamic Programming **192**
- 4.8 Application Studies **197**

- 4.8.1 Personnel Assignment **197**
- 4.8.2 Production Line Scheduling **201**
- 4.8.3 Factory-Warehouse Distribution **205**
- 4.8.4 Shipment Loading **207**
- 4.8.5 Capital Budgeting **208**
- 4.9 Problems **212**

5 Network Techniques

- 5.1 Introduction **223**
- 5.2 Activity Networks: CPM and PERT **223**
- 5.3 Shortest Path Analysis **240**
- 5.4 Maximal Flow Methods **244**
- 5.5 Application Studies **248**
 - 5.5.1 Launching a New Product **248**
 - 5.5.2 Project Management for Construction of a Warehouse **256**
 - 5.5.3 Planning to Move a Hospital to New Location **261**
 - 5.5.4 Equipment Replacement Problem **266**
 - 5.5.5 Maximizing the Rate of Return on Investments **267**
 - 5.5.6 Maximizing the Minimum Level of Efficiency in an Assignment Problem **268**
 - 5.5.7 Assignment of Families to Low-Income Housing **270**
- 5.6 Problems **270**

6 Inventory Models

- 6.1 Structure of the Inventory Models **285**
- 6.2 Deterministic Models (Economic-Lot-Size Models) **287**
- 6.3 Single-Period Probabilistic Models **294**
- 6.4 Multiperiod Probabilistic Model **304**
- 6.5 Application Studies **309**
 - 6.5.1 Cash Investments Through a Brokerage House **309**

- 6.5.2 Quantity Discount Model **311**
- 6.5.3 The Value of Forecasting in Inventory Management **313**
- 6.5.4 A Multistage Perishable Fruit Inventory Model **314**
- 6.5.5 Optimizing Control of Hospital Inventories **317**
- 6.6 Problems **323**

7 Waiting-Line Analysis

- 7.1 Introduction and Definition of Terms **329**
- 7.2 Poisson Arrivals and Exponential Service Waiting-Line Models **333**
- 7.3 Nonexponential Service Models **347**
- 7.4 Economic Models Using Waiting-Line Analysis **352**
- 7.5 Application Studies **354**
 - 7.5.1 In-Process Storage in a Plant Layout **354**
 - 7.5.2 Employee Service Facilities **355**
 - 7.5.3 Assignment of Operators to Automatic Machines **356**
 - 7.5.4 Military Car Pool **362**
 - 7.5.5 Queueing Model of a Hospital Emergency Room **364**
- 7.6 Problems **371**

8 Simulation

- 8.1 Introduction **377**
- 8.2 Model Formulation **379**
- 8.3 Random Number Generation **381**
- 8.4 Process Generation **383**
- 8.5 Event-Oriented Simulation **388**
- 8.6 Length of Simulation Runs **392**
- 8.7 Simulation Languages **398**
- 8.8 Application Studies **402**
 - 8.8.1 Cutting Quality Control Costs **402**

8.8.2 A Corporate Financial Model **404**

8.8.3 Developing Manning Standards **413**

8.9 Problems **419**

Appendices

A—Random Numbers **427**

B—Cumulative Normal Distribution **429**

Index 431

introduction 1

Operations research is a relatively new field of study. It first surfaced in Britain during World War II. The British were initially interested in the use of quantitative methods in the employment of radar. They named the approach *operational research* because they had used scientists to analyze *operational* problems. During the war the approach was very successful in dealing with problems in convoy operations, antisubmarine operations, strategic bombing operations, and mining operations. This application of *operational research* leads to our favorite definition of the approach:

“The art of winning wars without actually fighting.”

After the war early practitioners of operations research concentrated on formalizing the approaches that they had developed during the war and seeking uses for these techniques in the industrial sector. Actually some work had been started in the industrial sector by such men as Frederick W. Taylor and his successors, which had led to the new discipline of industrial engineering. Taylor realized that before the Industrial Revolution most businesses were small one-man operations. But, with mechanization and growth, management and specializations developed. Automation and decentralization of operations caused further management problems. Many new disciplines developed in addition to industrial engineering such as marketing research, finance, various branches of engineering, and the like. With these specializations splitting of objectives developed. The various branches of the organization began solving problems in a way that was not necessarily in the interest of overall organization. Consider the production-inventory decision facing most organizations; the various elements of the organization would like to handle this problem differently:

1.1 WHAT IS OPERATIONS RESEARCH AND WHERE IS IT USED?*

* Material in this section is adapted from *Careers in Operations Research*, Operations Research Society, 1975, and is used with the permission of ORSA.

PRODUCTION wants long uninterrupted runs to reduce setup costs that would lead to high inventories.

MARKETING wants a high inventory of lots of products so customers can receive whatever they want immediately.

FINANCE wants no inventory.

PERSONNEL wants to stabilize employment throughout the year, which would lead to high inventory at times to smooth out the cyclic demands.

The executive must decide “What is the best policy for the whole organization?” The executive must find the overall optimum solution. The individual optimum solutions are generally easy to find but the overall optimum is difficult. The role of *operations research* or *management science* (the popular names for operational research used in business and industry) is to help management solve these problems involving interactions of objectives. Operations research tries to find the best decisions relative to as large a portion of the *total system* as possible. The Operations Research Society of America has suggested the following definition of operations research [5] which is probably more appropriate to the industrial sector:

“Operations Research is concerned with scientifically deciding how to best design and operate man-machine systems usually under conditions requiring the allocation of scarce resources.”

The goals of Taylor’s industrial engineering and those of operations research do not differ. Operations research just adds mathematical sophistication and “tools” that didn’t exist previously. Some industrial decisions such as inventory, maintenance, and scheduling are similar to those encountered by the military, so these were the first industrial problems analyzed after World War II. Major acceptance of operations research in the industrial sector did not develop until the late 1950s with advent of computers and specialized techniques to handle problems such as marketing and capital budgeting. As knowledge of operations research increased in management, operations research groups were formed in hundreds of companies, especially large corporations such as oil companies.

Some of the industrial problems analyzed by operations research analysts are:

Finance, Budgeting, and Investments

1. Cash flow analysis, long range capital requirements, dividend policies, investment portfolios.
2. Credit policies, credit risks, and delinquent account procedures.
3. Claim and complaint procedures.

Purchasing, Procurement, and Exploration

1. Rules for buying supplies with stable or varying prices.
2. Determination of quantities and timing of purchases.

3. Bidding policies.
4. Strategies for exploration and exploitation of raw material sources.
5. Replacement policies.

Physical Distribution

1. Location and size of warehouses, distribution centers, and retail outlets.
2. Distribution policy.
3. Company-owned outlets versus franchised outlets.
4. Worldwide logistics and supply systems (military and industrial).

Facilities Planning

1. Numbers and location of factories, warehouses, hospitals, and the like, sizes and interactions.
2. Loading and unloading facilities for railroads, trucks, and boats.

Manufacturing

1. Production scheduling.
2. Stabilization of production and employment, the effects of instability, and the cost of hiring, training, lay-offs, and firing.

Construction Maintenance, Project Scheduling

1. Maintenance policies, preventive maintenance.
2. Maintenance crew sizes.
3. Project scheduling, allocation of resources.

Marketing

1. Product selection, timing, competitive actions.
2. Number of salesmen, frequency of calling on accounts, percent of time spent on prospectus.
3. Advertising strategies.

Personnel

1. Selection of personnel, mixes of age and skills.
2. Recruiting policies, assignments of jobs.

Research and Development

1. Determination of areas of concentration.
2. Reliability.
3. Control of development projects.

The applications of operations research have also had a strong impact in the study of societal problems and public affairs. Public administrators and government officials have become increasingly aware of how operations research can assist in the everyday decision-making activities. Significant applications have been made in the study of public health, city and regional planning, and educational systems. Recent advances in areas such as criminal justice, population control, meteorology, and ecological systems makes us aware that this is just the beginning.

Consider the following applications suggested by the Operations Research Society of America in their excellent booklet *Careers in Operations Research* [5]:

EXAMPLE *Operations Research Management*
Emergency Energy Capacity Study

The objective of this study is to determine feasibility, costs, and benefits associated with alternative methods of assuring energy supplies to meet essential needs in an emergency due to a cutoff of imports. The methodology is a critical review of U.S. ability (using a regional refining, storage, and distribution model) to withstand future disruptions in foreign supply in relation to surplus producing capacity, stored crude and products available for emergency use, and transportation capacity and refinery capacity. The model consists of a linear programming transportation storage, distribution model with sub-models representing refinery operation flexibility.

EXAMPLE *Operations Research in Society*
Operations Research and the Drug Abuse Problem

Current evaluation and operations research activities within the federal and state government are discussed. Practicality of applying operations research techniques to this problem is considered. Ongoing efforts in determining treatment effectiveness, treatment costs, ancillary support requirements, population descriptions, and patient flows are described. A model is developed that optimizes treatment and rehabilitation cost-effectiveness from a total system viewpoint. Model utilization is illustrated with select data gathered by Johns Hopkins University. Various measures of effectiveness, such as pre- versus posttreatment arrest rates, median time of client retention versus programmed treatment cycle time, and the like, are examined in detail. A scheduling model, using queueing concepts, is developed and manipulated with representative data.

EXAMPLE *Medical Manpower Modeling*
Modeling Physician Career Disposition

The problem to be solved by a comprehensive model of medical manpower supply and demand is how to match the geographic, speciality, and worksetting distributions of needs for health care personnel with corresponding multivariate distributions of graduates from health science institutions. The supply side of such a model requires social-psychological submodels that represent a number of application/selection and career disposition modification processes. The structure of such a submodel for physician manpower, currently being developed, will be presented, and preliminary insights from longitudinal data being collected to fit the model will be discussed.

Advanced Concepts for Highway Systems

The Effects of Driver Control and Display Aids on Simulated Platoon Traffic

One approach to improving the current transportation system is through augmentation of the driver's sensory, decision-making, and control capabilities. Various control and display aids were developed and tested under highway conditions for their effects on the individual driver. The effects of driver aiding in a series of closely following cars were also investigated through simulation experiments. Previously developed experimental display and control aids were tested in this simulation by altering the number of cars possessing the aid and finding subsequent effects on platoon safety and flow. Driver aiding resulted in improved traffic safety and flow and was significantly more important than altering vehicle or road characteristics.

EXAMPLE

The distinguishing feature of a system is the interaction among its various components. The structure of the system is determined by this interaction. The analyst must be concerned with these interactions when analyzing a given system. Insight into the cause-and-effect relationships and into the performance of the system as a whole can be gained through this knowledge. Quantitative analysis of systems is dependent on the structure of the system.

Morris[3] has suggested the following steps for constructing models of systems:

1. Identify and formulate the manager's decision in writing.
2. Identify the constants, parameters, and variables involved. Define them verbally and then introduce symbols to represent each one.
3. Select the variables that appear to be most influential so that the model may be kept as simple as possible. Distinguish between those that are controllable by the manager and those that are not.
4. State verbal relationships among the variables based on known principles, specially gathered data, intuition, and reflection. Make assumptions or predictions concerning the behavior of the noncontrollable variables.
5. Construct the model by combining all relationships into a system of symbolic relationships.
6. Perform symbolic manipulations.
7. Derive solutions from the model.

1.2 SYSTEMS MODELING AND THE TOOLS OF OPERATIONS RESEARCH*

* Material in this section is adapted from W. T. Morris, *The Analysis of Management Decisions*, Homewood, Ill., Richard D. Irwin, Inc., 1964. Used with the permission of Richard D. Irwin, Inc.

8. Test the model by making predictions from it and checking against real-world data.

9. Revise the model as necessary.

In this text we are concerned with a number of techniques that have become useful to the operations research analyst. We will also emphasize the applications of these techniques through the use of numerous examples and the use of *Application Studies* in each chapter. The Application Studies will demonstrate how people have used and misused the techniques discussed in business, government, and society.

We discuss what might be considered the basic tools of operations research. These are the ones that are being used the most by operations research analysts. There are, however, a number of “newer” techniques that, as you become more proficient in operations research, you may wish to consider. These techniques are reported in the more advanced texts of operations research and journals in the operations research area.

1.3 MANAGEMENT'S PERSPECTIVE OF QUANTITATIVE METHODS *

Managers over the years have had reservations about using operations research techniques because quantitative methods were not their “style” of management. Morris[2] has attempted through interviews with 320 middle managers and through his own extensive experience to assess 25 years of quantitative methods in decision making. Morris developed the following seven propositions that describe the majority views of the managers that he interviewed. We hope that Morris’s discussion will give you a good feel for how management views the area you are about to study. It also should give you a feel for the environment within which the operations research analyst must work.

Morris’s propositions are as follows:

1. *“Quantitative” is probably too narrow a word for the efficient development of one’s management style. Better to think of a broad range of approaches that have in common three basic attributes: they are explicit; they involve data, whether quantitative or qualitative; and they are to some degree analytical.*

This seemed to be a way of saying that the original questions about the role of quantitative methods in management were not very usefully stated. The numbers, measurements, and equations ordinarily associated with “quantitative” are *really only surface manifestations*. One should consider a broader class of techniques that goes beyond such things as mathematical programming and simulation. It might be useful, for example, to consider whatever fundamental similarities there may be

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among such approaches as management by objectives, organizational development, job enrichment, various participative strategies, sensitivity groups, T groups, risk analysis, zero defects programs, value analysis, cost-effectiveness analysis, and so on. All of these, in varying degrees, involve an explicit statement of a policy, an hypothesis, an objective, or a program statement. They all tend toward the generation of data on performance, effectiveness, progress, or output. Data generation involves all sorts of scaling and measurement problems employing judgments, ratings, evaluations, classifications, and so on. Yet, data are produced by some sort of measurement process. Also, all of these approaches tend to be analytical, if not in a formal mathematical sense, then in terms of some less rigorous logic. They tend to encourage a look into the structures or models that might be helpful in analyzing management phenomena.

The relevant question then for many of these managers seemed to be the degree to which approaches involving explicitness, measurement, and analysis were to become a part of their management styles.

2. *Management can be usefully viewed as an experimental undertaking in much the same sense that science is so regarded.*

To suggest that the ongoing operations of an organization be regarded as a series of experiments is to suggest a rather important management concept. This is the proposition that a firm should be run so as not only to produce its products or services at a profit but also to produce information on how to improve its own operations. Organizations ought deliberately to produce, among their various outputs, information relevant for moving further toward their objectives. In less general terms, a production manager should seek to produce not only the products but also information on how the products and the production operations themselves may be improved. Each management action should be treated as a test of the decision conceptualization on which it is based, a test of a hypothesis that has been produced out of assumptions and simplifications, the truth of which is necessarily a matter of doubt and uncertainty. Ideally, the setting in which managers decide and act should be something like that of a laboratory.

Based on the very casual evidence available from these discussions one could plausibly raise the hypothesis that the greater the tendency of a manager to view management as an experimental undertaking, the greater the tendency for him to incorporate quantitative methods in his personal style of managing. The use of explicit, analytical approaches to management problems appears to be, logically enough, consistent with the viewpoint that there is much in common between good management and good science.

3. *"Quantitative" methods (in the large sense) sometimes have life cycles that can be roughly modeled as a series of phases:*