A microscopic view of semiconductor device structures, showing curved lines and circular features, likely representing a high-speed optoelectronic device. The image is in shades of blue and purple, with a dark background and bright highlights.

Semiconductor Devices for High-Speed Optoelectronics

GIOVANNI GHIONE

CAMBRIDGE

TN2
G424

Semiconductor Devices for High-Speed Optoelectronics

GIOVANNI GHIONE

Politecnico di Torino, Italy



E2010000776



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521763448

© Cambridge University Press 2009

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-76344-8 hardback

Additional resources for this publication at www.cambridge.org/Ghione

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Preface

The development of high-speed fiber-based optical communication systems that has taken place since the early 1970s can be really considered as a technological wonder. In a few years, key components were devised (such as the semiconductor laser) with the help of novel technological processes (such as epitaxial growth) and found immediate application thanks to the development of low-loss optical fibers. New compound semiconductor alloys (namely, InGaAsP) were ready to provide their potential to emit the right wavelengths needed for long-haul fiber propagation. When electronic repeaters seemed unable to provide a solution to long-haul propagation, fiber amplifiers were developed that allowed for all-optical signal regeneration. And the list could be continued. A miracle of ingenuity from a host of researchers made it possible to assemble this complex puzzle in a few years, thus bringing optoelectronic technology to a consumer electronics level.

Increasing the system capacity by increasing the transmission speed was, of course, a main concern from the early stages of optical system development. While optoelectronic devices behave, on the electronic side, in a rather conventional way up to speeds of the order of 1 Gbps, for larger speeds (up to 40 Gbps and beyond) RF wave propagation has to be accounted for in designing and modeling optoelectronic devices. When speed increases, the distributed interaction between RF and optical waves becomes a useful, sometimes indispensable, ingredient in many optoelectronic devices, like modulators and (to a lesser extent) detectors. Similarly, the electronic circuits that interface light sources, modulators, and detectors should provide broadband operation up to microwave or millimeter-wave frequencies, thus making it mandatory to exploit compound semiconductor electronics (GaAs- or InP-based) or advanced Si-based solutions (like SiGe HBT integrated circuits or nanometer MOS processes).

Increasing speed beyond the 10 Gbps limit by improving device performance, however interesting it is from the research and development side, may in practice be less appealing from the market standpoint. The ultimate destiny of optoelectronic devices (such as sources, modulators, and detectors) optimized for 40 Gbps (or even faster) systems after the post-2000 market downturn still is uncertain, and research in the field has followed alternative paths to the increase of system capacity. At the same time, new application fields have been developed, for instance in the area of integrated all-Si optical signal processing systems, and also for integrated circuit level high-capacity communications. However, the development of high-speed optoelectronic devices has raised a number of stimulating (and probably lasting) design issues. An example is the

principle of the distributed interaction between optical and RF waves, which is common to a variety of high-speed components. Another relevant theme is the co-design and the (possibly monolithic) integration of the electronic and optoelectronic components of a system, not to mention the critical aspects concerning device packaging and interconnection in systems operating at 40 Gbps and beyond.

Taking the above into account, it is not surprising that the main purpose of the present book is to provide a kind of unified (or, perhaps, not too widely separated) treatment of high-speed electronics and optoelectronics, starting from compound semiconductor basics, down to high-speed transistors, ICs, detectors, sources and modulators. Part of the material was originally developed for a number of postgraduate and Master courses, and therefore has the ambition (but also the limitation) of providing a treatment starting from the very basics. It is hoped that this justifies both the presence of introductory material on semiconductors and semiconductor optical properties, and a treatment of high-speed electronics starting from a review of transmission lines and scattering parameters. From this standpoint, the text attempts to be as self-contained as possible. Of course, the choice of subjects is somewhat influenced by the author's personal tastes and previous research experience (not to mention the need to keep the page count below 500): some emphasis has been put on noise, again with an attempt to present a self-contained treatment of this rather difficult topic, and many important optoelectronic components have not been included (to mention one, semiconductor optical amplifiers). Yet another innovative subject that is missing is microwave photonics, where of course the RF and microwave and optoelectronic worlds meet. Nevertheless, the text is (in the author's opinion, at least) different enough from the many excellent textbooks on optoelectronics available on the market to justify the attempt to write it.

I wish to thank a number of colleagues (from Politecnico di Torino, unless otherwise stated) for their direct or indirect contribution to this book. Ivo Montrosset provided many useful suggestions on the treatment of optical sources. Incidentally, it was under the guidance of Ivo Montrosset and Carlo Naldi that (then an undergraduate student) I was introduced to the basics of passive and active optoelectronic devices, respectively; this happened, alas, almost 30 years ago. Helpful discussions with Gian Paolo Bava and Pierluigi Debernardi (Consiglio Nazionale delle Ricerche) on laser noise, with Simona Donati Guerrieri on the semiconductor optical properties and with Fabrizio Bonani and Marco Pirola on active and passive high-speed semiconductor electronic devices and circuits are gratefully acknowledged. Michele Goano kindly revised the sections on compound semiconductors and the numerical problems, and provided useful suggestions on III-N semiconductors. Federica Cappelluti prepared many figures (in particular in the section on photodetectors), initially exploited in lecture slides. Finally, Claudio Coriasso (Avago Turin Technology Center, Torino) kindly provided material on integrated electroabsorption modulators (EAM), including some figures. Additionally, I am indebted to a number of ME students who cooperated in research, mainly on lithium niobate modulators; among those, special mention goes to F. Carbonera, D. Frassati, G. Giarola, A. Mela, G. Omegna, L. Terlevich, P. Zandano. A number of PhD students also worked on subjects relevant to the present book: Francesco Bertazzi (now with Politecnico di Torino) on EM modeling of distributed electrooptic structures; Pietro Bianco,

on high-speed modulator drivers; Federica Cappelluti, on electroabsorption modulator modeling; Gloria Carvalho, on EAL modeling; Antonello Nespola (now with Istituto Superiore Mario Boella), on the modeling of distributed high-speed photodetectors. Part of the thesis work of Antonello Nespola and Federica Cappelluti was carried out within the framework of a cooperation with UCLA (Professor Ming Wu, now at University of California, Berkeley). Finally, I gratefully recall many helpful discussions with colleagues from the industry: among those, Marina Meliga, Roberto Paoletti, Marco Romagnoli, and Luciano Socci.

Giovanni Ghione
January 2009

Contents

Preface

page xiii

1	Semiconductors, alloys, heterostructures	1
1.1	Introducing semiconductors	1
1.2	Semiconductor crystal structure	2
1.2.1	The Miller index notation	3
1.2.2	The diamond, zinc-blende, and wurtzite semiconductor cells	5
1.2.3	Ferroelectric crystals	6
1.2.4	Crystal defects	7
1.3	Semiconductor electronic properties	8
1.3.1	The energy–momentum dispersion relation	8
1.3.2	The conduction and valence band wavefunctions	12
1.3.3	Direct- and indirect-bandgap semiconductors	13
1.4	Carrier densities in a semiconductor	17
1.4.1	Equilibrium electron and hole densities	17
1.4.2	Electron and hole densities in doped semiconductors	20
1.4.3	Nonequilibrium electron and hole densities	21
1.5	Heterostructures	24
1.6	Semiconductor alloys	25
1.6.1	The substrate issue	27
1.6.2	Important compound semiconductor alloys	28
1.7	Bandstructure engineering: heterojunctions and quantum wells	29
1.7.1	Carrier density and density of states in a quantum well	33
1.7.2	Carrier density and density of states in a quantum wire	38
1.7.3	Superlattices	40
1.7.4	Effect of strain on bandstructure	40
1.8	Semiconductor transport and generation–recombination	42
1.8.1	Drift and diffusion	42
1.8.2	Generation and recombination	43
1.8.3	Trap-assisted (Shockley–Read–Hall) recombination	44
1.8.4	Auger recombination and generation by impact ionization	46

1.9	Questions and problems	48
1.9.1	Questions	48
1.9.2	Problems	50
2	Semiconductor optical properties	52
2.1	Modeling the interaction between EM waves and the semiconductor	52
2.2	The macroscopic view: permittivities and permeabilities	53
2.2.1	Isotropic vs. anisotropic media	58
2.3	The microscopic view: EM wave–semiconductor interaction	59
2.3.1	Energy and momentum conservation	61
2.3.2	Perturbation theory and selection rules	68
2.3.3	Total scattering rates	74
2.4	The macroscopic view: the EM wave standpoint	78
2.4.1	The semiconductor gain energy profile	80
2.4.2	The semiconductor absorption energy profile	83
2.4.3	The QW absorption profile	84
2.4.4	Spontaneous emission spectrum	89
2.4.5	Spontaneous emission, gain, and absorption spectra	91
2.5	The macroscopic view: the semiconductor standpoint	93
2.5.1	Carrier radiative lifetimes	95
2.6	Questions and problems	101
2.6.1	Questions	101
2.6.2	Problems	102
3	High-speed semiconductor devices and circuits	104
3.1	Electronic circuits in optical communication systems	104
3.2	Transmission lines	104
3.2.1	RG , RC , and high-frequency regimes	109
3.2.2	The reflection coefficient and the loaded line	111
3.2.3	Planar integrated quasi-TEM transmission lines	113
3.2.4	Microstrip lines	114
3.2.5	Coplanar lines	115
3.3	The scattering parameters	117
3.3.1	Power and impedance matching	119
3.4	Passive concentrated components	121
3.4.1	Bias Ts	124
3.5	Active components	126
3.5.1	Field-effect transistors (FETs)	126
3.5.2	FET DC model	128
3.5.3	FET small-signal model and equivalent circuit	130
3.5.4	High-speed FETs: the HEMT family	133
3.5.5	High-speed heterojunction bipolar transistors	141

3.5.6	HBT equivalent circuit	143
3.5.7	HBT choices and material systems	145
3.6	Noise in electron devices	147
3.6.1	Equivalent circuit of noisy N -ports	148
3.6.2	Noise models of active and passive devices	149
3.7	Monolithic and hybrid microwave integrated circuits and optoelectronic integrated circuits	151
3.8	Questions and problems	155
3.8.1	Questions	155
3.8.2	Problems	157
4	Detectors	158
4.1	Photodetector basics	158
4.2	Photodetector structures	159
4.3	Photodetector materials	161
4.3.1	Extrinsic and QW detectors	165
4.4	Photodetector parameters	165
4.4.1	PD constitutive relation	165
4.4.2	Responsivity and quantum efficiency	167
4.4.3	PD electrical bandwidth and equivalent circuit	171
4.4.4	Photodetector gain	174
4.5	Photodetector noise	174
4.6	Photodiodes	178
4.7	The pn photodiode	179
4.7.1	Analysis of the pn photodiode response	180
4.8	The pin photodiode	184
4.8.1	The pin photocurrent, responsivity, and efficiency	185
4.8.2	Conventional pin photodetector structures	188
4.9	The pin frequency response	189
4.9.1	Carrier diffusion and heterojunction charge trapping	190
4.9.2	Dynamic pin model and space-charge effects	191
4.9.3	Transit time analysis and transit time-limited bandwidth	193
4.9.4	Capacitance-limited bandwidth	197
4.9.5	Bandwidth–efficiency trade-off	199
4.10	Advanced pin photodiodes	200
4.10.1	Waveguide photodiodes	201
4.10.2	Traveling-wave photodetectors	203
4.10.3	Velocity-matched traveling-wave photodetectors	209
4.10.4	Uni-traveling carrier photodiodes	210
4.11	Avalanche photodiodes	211
4.11.1	Analysis of APD responsivity	213
4.12	Noise in APDs and $pins$	220
4.12.1	Analysis of APD noise	222

4.13	The APD frequency response	228
4.14	Advanced APD structures	231
4.15	Concluding remarks on high-speed PDs	232
4.16	The photodiode front end	233
4.16.1	Photodetector and front-end signal and noise model	234
4.16.2	High- and low-impedance front ends	234
4.16.3	Transimpedance amplifier front ends	236
4.16.4	High-speed transimpedance stages	240
4.17	Front-end SNR analysis and <i>pin</i> -APD comparison	242
4.18	Front-end examples	247
4.18.1	Hybrid and monolithic front-end solutions	250
4.19	Questions and problems	251
4.19.1	Questions	251
4.19.2	Problems	253
5	Sources	255
5.1	Optical source choices	255
5.2	Light-emitting diodes	255
5.2.1	LED structures	256
5.2.2	Homojunction LED power-current characteristics	257
5.2.3	Charge control model and modulation bandwidth	260
5.2.4	Heterojunction LED analysis	261
5.2.5	LED emission spectrum	262
5.2.6	LED materials	264
5.3	From LED to laser	265
5.4	The Fabry-Perot cavity resonant modes	268
5.4.1	Analysis of the TE slab waveguide fundamental mode	269
5.4.2	Longitudinal and transversal cavity resonances	272
5.5	Material and cavity gain	275
5.5.1	Analysis of the overlap integral	275
5.6	The FP laser from below to above threshold	278
5.6.1	The threshold condition	279
5.6.2	The emission spectrum	281
5.6.3	The electron density and optical power	282
5.6.4	The power-current characteristics	283
5.6.5	The photon lifetimes	283
5.6.6	Power-current characteristics from photon lifetimes	284
5.7	The laser evolution: tailoring the active region	285
5.7.1	Quantum-well lasers	286
5.7.2	Laser material systems	290
5.8	The laser evolution: improving the spectral purity and stability	290
5.8.1	Conventional Fabry-Perot lasers	291
5.8.2	Gain-guided FP lasers	291

5.8.3	Index-guided FP lasers	292
5.8.4	Distributed-feedback (DFB and DBR) lasers	294
5.8.5	DBR and tunable DBR lasers	299
5.8.6	Vertical cavity lasers	300
5.8.7	Quantum dot lasers	302
5.9	The laser temperature behavior	303
5.10	Laser linewidth	304
5.10.1	Linewidth broadening analysis	306
5.11	Laser dynamics and modulation response	315
5.12	Dynamic large-signal and small-signal laser modeling	321
5.12.1	Steady-state (DC) solution	323
5.12.2	Small-signal model	325
5.12.3	Chirp analysis	329
5.13	Laser relative intensity noise	330
5.13.1	Analysis of Langevin sources	331
5.13.2	Carrier and photon population fluctuations	338
5.13.3	Output power fluctuations	340
5.13.4	Relative intensity noise	343
5.13.5	Phase noise and linewidth from the Langevin approach	346
5.14	Questions and problems	352
5.14.1	Questions	352
5.14.2	Problems	353
6	Modulators	356
6.1	Light modulation and modulator choices	356
6.2	Modulator parameters	358
6.2.1	Electrooptic (static) response	358
6.2.2	Dynamic response	360
6.2.3	Small-signal frequency response	360
6.2.4	Optical and electrical modulation bandwidth	362
6.2.5	Chirp	363
6.2.6	Optical bandwidth	363
6.2.7	Electrical or RF input matching	363
6.2.8	Linearity and distortion	363
6.3	Electrooptic modulators	364
6.3.1	Lithium niobate electrooptic modulators	365
6.3.2	Semiconductor electrooptic modulators	372
6.3.3	Polymer modulators	374
6.4	The Mach–Zehnder electrooptic modulator	375
6.4.1	The lumped Mach–Zehnder modulator	376
6.4.2	Static electrooptic response	377
6.4.3	Lumped modulator dynamic response	378
6.4.4	Efficiency–bandwidth trade-off in lumped MZ modulators	380

6.5	The traveling-wave Mach–Zehnder modulator	382
6.5.1	Mach–Zehnder traveling-wave modulator dynamic response	383
6.5.2	Analysis of the TW Mach–Zehnder modulator response	387
6.5.3	The Mach–Zehnder modulator chirp	391
6.6	High-speed electrooptic modulator design	394
6.6.1	Lithium niobate modulators	396
6.6.2	Compound semiconductor, polymer, and silicon modulators	399
6.7	Electroabsorption modulator physics	402
6.7.1	The Franz–Keldysh effect (FKE)	403
6.7.2	The quantum confined Stark effect (QCSE)	404
6.8	Electroabsorption modulator structures and parameters	409
6.8.1	EAM static response	410
6.8.2	Lumped EAM dynamic response	412
6.8.3	EAM chirp	414
6.9	The distributed electroabsorption modulator	415
6.10	Electroabsorption modulator examples	420
6.10.1	Integrated EAMs (EALs)	423
6.11	Modulator and laser biasing	425
6.12	Modulator and laser drivers	427
6.12.1	The high-speed driver amplifier	430
6.13	Questions and problems	436
6.13.1	Questions	436
6.13.2	Problems	438
	<i>List of Symbols</i>	440
	<i>References</i>	450
	<i>Index</i>	457

1 Semiconductors, alloys, heterostructures

1.1 Introducing semiconductors

Single-crystal semiconductors have a particularly important place in optoelectronics, since they are the starting material for high-quality sources, receivers and amplifiers. Other materials, however, can be relevant to some device classes: polycrystalline or amorphous semiconductors can be exploited in light-emitting diodes (LEDs) and solar cells; dielectrics (also amorphous) are the basis for passive devices (e.g., waveguides and optical fibers); and piezoelectric (ferroelectric) crystals such as lithium niobate are the enabling material for a class of electrooptic (EO) modulators. Moreover, polymers have been recently exploited in the development of active and passive optoelectronic devices, such as emitters, detectors, and waveguides (e.g., fibers). Nevertheless, the peculiar role of single-crystal semiconductors justifies the greater attention paid here to this material class with respect to other optoelectronic materials.

From the standpoint of electron properties, semiconductors are an intermediate step between insulators and conductors. The electronic structure of crystals generally includes a set of allowed energy bands, that electrons populate according to the rules of quantum mechanics. The two topmost energy bands are the *valence* and *conduction* band, respectively, see Fig. 1.1. At some energy above the conduction band, we find the *vacuum level*, i.e., the energy of an electron free to leave the crystal. In *insulators*, the valence band (which hosts the electrons participating to the chemical bonds) is separated from the conduction band by a large energy gap E_g , of the order of a few electronvolts (eV). Due to the large gap, an extremely small number of electrons have enough energy to be promoted to the conduction band, where they could take part into electrical conduction. In insulators, therefore, the conductivity is extremely small. In *metals*, on the other hand, the valence and conduction bands overlap (or the energy gap is *negative*), so that all carriers already belong to the conduction band, independent of their energy. Metals therefore have a large conductivity. In *semiconductors*, the energy gap is of the order of 1–2 eV, so that some electrons have enough energy to reach the conduction band, leaving *holes* in the valence band. Holes are pseudo-particles with positive charge, reacting to an external applied electric field and contributing, together with the electrons in the conduction band, to current conduction. In pure (*intrinsic*) semiconductors, therefore, charge transport is *bipolar* (through electrons and holes), and the conductivity is low, exponentially dependent on the gap (the larger the gap, the lower the conductivity). However, impurities can be added (*dopants*) to provide large numbers of electrons to

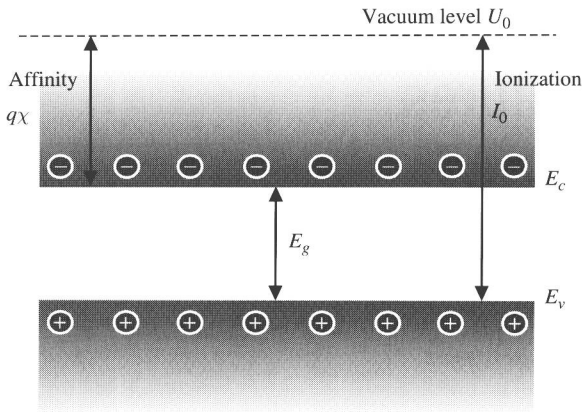


Figure 1.1 Main features of semiconductor bandstructure. E_g is the energy gap; E_c is the conduction band edge; E_v is the valence band edge.

the conduction band (*donors*) or of holes to the valence band (*acceptors*). The resulting doped semiconductors are denoted as *n*-type and *p*-type, respectively; their conductivity can be artificially modulated by changing the amount of dopants; moreover, the dual doping option allows for the development of *pn* junctions, one of the basic building blocks of electronic and optoelectronic devices.

1.2 Semiconductor crystal structure

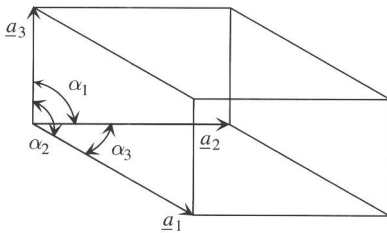
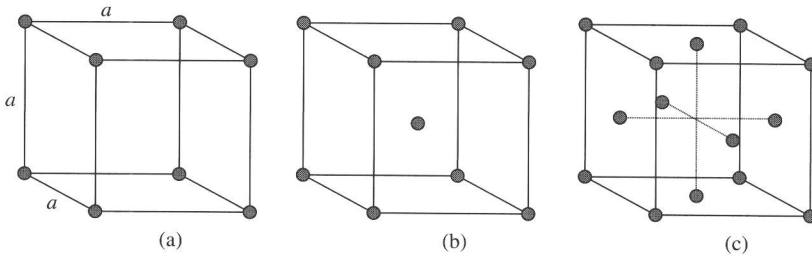
Crystals are regular, periodic arrangements of atoms in three dimensions. The point set \underline{r} defining the crystal nodes, corresponding to the atomic positions (Bravais lattice) satisfies the condition $\underline{r} = k\underline{a}_1 + l\underline{a}_2 + m\underline{a}_3$, where k, l, m are integer numbers and $\underline{a}_1, \underline{a}_2, \underline{a}_3$ are the *primitive vectors* denoting the *primitive cell*, see Fig. 1.2. Bravais lattices can be formed so as to fill the entire space only if the angles $\alpha_1, \alpha_2, \alpha_3$ assume values from a discrete set ($60^\circ, 90^\circ, 120^\circ$, or the complementary value to 360°). According to the relative magnitudes of a_1, a_2, a_3 and to the angles $\alpha_1, \alpha_2, \alpha_3$, 14 basic lattices can be shown to exist, as in Table 1.1. In semiconductors, only two lattices are technologically important at present, i.e. the *cubic* and the *hexagonal*. Most semiconductors are cubic (examples are Si, Ge, GaAs, InP...), but some are hexagonal (SiC, GaN). Both the cubic and the hexagonal structure can be found in carbon (C), where they are the diamond and graphite crystal structures, respectively.

Three kinds of Bravais cubic lattices exist, the simple cubic (sc), the face-centered cubic (fcc) and the body-centered cubic (bcc), see Fig. 1.3. The cubic semiconductor crystal structure can be interpreted as two *shifted* and *compenetrated* fcc Bravais lattices.

Let us consider first an elementary semiconductor (e.g., Si) where all atoms are equal. The relevant cubic lattice is the *diamond lattice*, consisting of two interpenetrating

Table 1.1 The 14 Bravais lattices.

Name	Bravais lattices	Conditions on primitive vectors
Triclinic	1	$a_1 \neq a_2 \neq a_3, \alpha_1 \neq \alpha_2 \neq \alpha_3$
Monoclinic	2	$a_1 \neq a_2 \neq a_3, \alpha_1 = \alpha_2 = 90^\circ \neq \alpha_3$
Orthorhombic	4	$a_1 \neq a_2 \neq a_3, \alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$
Tetragonal	2	$a_1 = a_2 \neq a_3, \alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$
Cubic	3	$a_1 = a_2 = a_3, \alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$
Trigonal	1	$a_1 = a_2 = a_3, \alpha_1 = \alpha_2 = \alpha_3 < 120^\circ \neq 90^\circ$
Hexagonal	1	$a_1 = a_2 \neq a_3, \alpha_1 = \alpha_2 = 90^\circ, \alpha_3 = 120^\circ$

**Figure 1.2** Semiconductor crystal structure: definition of the primitive cell.**Figure 1.3** Cubic Bravais lattices: (a) simple, (b) body-centered, (c) face-centered.

fcc Bravais lattices, displaced along the body diagonal of the cubic cell by one-quarter the length of the diagonal, see Fig. 1.4. Since the length of the diagonal is $d = a |\hat{x} + \hat{y} + \hat{z}| = a\sqrt{3}$, the displacement of the second lattice is described by the vector

$$\underline{s} = \frac{a\sqrt{3}}{4} \frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}} = \frac{a}{4} (\hat{x} + \hat{y} + \hat{z}).$$

1.2.1 The Miller index notation

The Miller indices are a useful notation to denote planes and reference directions within a lattice. The notation (h, k, l) , where h, k, l are integers, denotes the set of parallel planes that intercepts the three points $\underline{a}_1/h, \underline{a}_2/k$ and \underline{a}_3/l , or some multiple thereof, while $[h, k, l]$ in square brackets is the direction orthogonal to plane (h, k, l) .

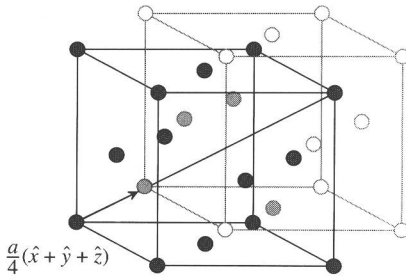


Figure 1.4 The diamond lattice as two cubic face-centered interpenetrating lattices. The pale and dark gray points represent the atoms falling in the basic cell.

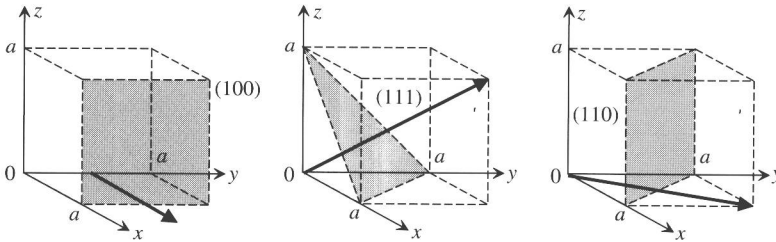


Figure 1.5 Examples of planes and directions according to the Miller notation.

Additionally, $\{h, k, l\}$ is a family of planes with symmetries and $\langle h, k, l \rangle$ is the related direction set. In cubic lattices, the primitive vectors coincide with the Cartesian axes and $a_1 = a_2 = a_3 = a$, where a is the lattice constant; in this case, we simply have $[h, k, l] \equiv h\hat{x} + k\hat{y} + l\hat{z}$ where \hat{x} , \hat{y} and \hat{z} are the Cartesian unit vectors.

To derive the Miller indices from the plane intercepts in a cubic lattice, we normalize with respect to the lattice constant (thus obtaining a set of integers (H, K, L)), take the reciprocal (H^{-1}, K^{-1}, L^{-1}) and finally multiply by a minimum common multiplier so as to obtain a set (h, k, l) such as $h : k : l = H^{-1} : K^{-1} : L^{-1}$. Notice that a zero index corresponds to an intercept point at infinity. Examples of important planes and directions are shown in Fig. 1.5.

Example 1.1: Identify the Miller indices of the following planes, intersecting the coordinate axes in points (normalized to the lattice constant): (a) $x = 4, y = 2, z = 1$; (b) $x = 10, y = 5, z = \infty$; (c) $x = 3.5, y = \infty, z = \infty$; (d) $x = -4, y = -2, z = 1$.

We take the reciprocal of the intercept, and then we multiply by the minimum common multiplier, so as to obtain an integer set with minimum module. In case (a), the reciprocal set is $(1/4, 1/2, 1)$, with minimum common multiplier 4, leading to the Miller indices $(1, 2, 4)$. In case (b), the reciprocals are $(1/10, 1/5, 0)$ with Miller indices $(1, 2, 0)$. In case (c), the plane is orthogonal to the z axis, and the Miller indices simply are $(1, 0, 0)$. Finally, case (d) is similar to case (a) but with negative intercepts; according to the Miller notation we overline the indices rather than using a minus sign; we thus have $(\bar{1}, \bar{2}, 4)$.

1.2.2 The diamond, zinc-blende, and wurtzite semiconductor cells

The cubic diamond cell includes 8 atoms; in fact, if we consider Fig. 1.6, the corner atoms each contribute to eight adjacent cells, so that only $8/8 = 1$ atom belongs to the main cell. The atoms lying on the faces belong half to the main cell, half to the nearby ones, so that only $6/2 = 3$ atoms belong to the main cell. Finally, the other (internal) 4 atoms belong entirely to the cell. Therefore, the total number of atoms in a cell is $1 + 3 + 4 = 8$. In the diamond cell, each atom is connected to the neighbours through a tetrahedral bond. All atoms are the same (C, Si, Ge...) in the diamond lattice, while in the so-called *zinc-blende lattice* the atoms in the two fcc constituent lattices are different (GaAs, InP, SiC...). In particular, the corner and face atoms are metals (e.g., Ga) and the internal atoms are nonmetals (e.g., As), or vice versa.

In the diamond or zinc-blende lattices the Miller indices are conventionally defined with respect to the cubic cell of side a . Due to the symmetry of the tetrahedral atom bonds, planes (100) and (110), etc. have two bonds per side, while planes (111) have three bonds on the one side, two on the other. Moreover, the surface atom density is different, leading, for example, to different etch velocities.

Some semiconductors, such as SiC and GaN, have the hexagonal *wurtzite* crystal structure. Hexagonal lattices admit many *polytypes* according to the stacking of successive atom layers; a large number of polytypes exists, but only a few have interesting semiconductor properties (e.g. 4H and 6H for SiC). The wurtzite cell is shown in Fig. 1.7, including 12 equivalent atoms. In the ideal lattice, one has

$$|a_3| = c, \quad |a_1| = |a_2| = a, \quad \frac{c}{a} = \sqrt{\frac{8}{3}} \approx 1.633.$$

Some properties of semiconductor lattices are shown in Table 1.2.¹ It can be noted that wurtzite-based semiconductors are often anisotropic (uniaxial) and have two dielectric constants, one parallel to the c -axis, the other orthogonal to it.

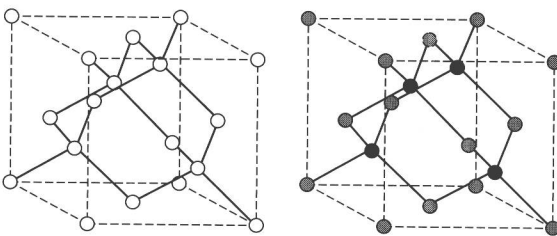


Figure 1.6 The diamond (left) and zinc-blende (right) lattices.

¹ Semiconductor properties are well documented in many textbooks; an excellent online resource is provided by the Ioffe Institute of the Russian Academy of Sciences at the web site [1].