MODERN CONTROL PRINCIPLES AND APPLICATIONS

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To IRENE and ELISABETH

Preface

This book is intended for those who wish to obtain a reasonably detailed working knowledge of the pertinent modern theories of control without wading through the myriad of publications in the field. It also is intended for those who wish to apply these theories to concrete problems.

There is currently an impression that in order to appreciate some of the newer control theories, one must be steeped in certain modern branches of mathematics. This is true only to a degree. There is, generally speaking, no substitute for a thorough and deep mathematical grounding. Nevertheless, we feel that the essence of these theories can be imparted without wallowing in the jargon of the mathematics. We try to avoid lengthy chapters on set theory, linear vector spaces, and the like. Instead, we introduce most of the needed mathematics in the text proper, in order to indicate the exact part that the mathematics plays relative to the given problems. Thus our presentation is made in such a way that plausibility arguments generally accompany the mathematical detail. Where called for, we present the mathematics without undue compromise. The mathematical levels of the chapters are however graduated, such that the more difficult concepts are introduced in the later parts of the book.

This book is aimed at first-year graduate students and qualified undergraduates. It is also intended for the large class of working engineers who wish to keep abreast of the development in modern control theory. The subject matter evolved from a course given at the Bell Telephone Laboratories and later tried out at the Newark College of Engineering, as well as at Utah State University in a summer course sponsored by the National Science Foundation.

The book covers what to us are the two most important areas of automatic control: stability and performance. It is a universal fact that, if the loop gain is sufficiently high, a closed-loop system can become unstable even without input. When all the system components are essentially operating in their linear ranges, there is a relatively clear understanding of the conditions for instability. This is, however, not the case for nonlinear systems. The extent that modern theories can allow us to determine stability in certain classes of nonlinear systems is a chief topic of the text.

Even in the case of linear systems, how to specify good performance has always been a question that has nagged system designers. This frequently reduces to a question of specifying a suitable criterion of performance (in practice, a single criterion is frequently not enough). Modern optimal control theory skirts this issue and takes the opposite stand: given a suitable performance criterion, find a control system that is best for the purpose. While this is not completely satisfactory from the engineering viewpoint, it does represent an important step forward. Moreover, the problem has certain aesthetic appeal. We treat this fascinating topic in some depth in order to accentuate the power and pitfalls of the theories in this area, particularly in relation to applications.

One interesting aspect of our presentation is that it clearly shows the return to prominence of the frequency-domain techniques in nonlinear system analysis. Theoretical interest in the frequency domain was apparently eclipsed when engineers rediscovered the so-called state-space approach to differential systems. However, it was soon appreciated that most nonlinear systems about which we can develop some understanding boast a linear time-invariant part. By representing the linear part in the frequency domain, new results often can be derived and new insights can be gained. Thus, Popov and others obtained new stability results in the frequency domain. Even in the area of optimal control theory, some new and useful frequency-domain criteria have been derived.

One important benefit derived from the use of the frequency domain for linear time-invariant systems is the intuition it provides in determining the soundness of a system. We try to show how this intuition could be used to assess the workability of the system to be analyzed.

The prime prerequisites for digesting the contents of the text include a course in linear control systems theory, some understanding of differential equations, and the theory of functions of a complex variable. In addition, we assume a level of maturity equivalent to that of a first-year graduate student. An understanding of the theory of matrices, calculus of variations, and sampled-data systems will be helpful but not essential.

Whenever possible, we attempt to share with the reader our practical experience. To this end, in addition to many remarks in the text proper, we have incorporated a number of examples. They are backed up by exercises of varying degrees of difficulty.

For lack of space, two topics are regretfully omitted. One is discrete systems and the other is systems with uncertainties. The first omission is not judged to be too serious, for it is possible, with some effort, to extrapolate to the discrete case most of the results pertaining to the continuous systems. The second omission is more difficult to rationalize, as there is, in practice, no system without uncertainty. However, the problems related to systems with noise are extremely difficult to solve and generally speaking, a clear understanding of the noise-free case is prerequisite for appreciating problems with uncertainty. We feel that the present choice of topics will provide the necessary grounding for handling uncertainty later.

Even with the above omissions, it is not possible to give an adequate coverage of all the important topics in deterministic nonlinear automatic control theory. We try to indicate other relevant results by means of comments, footnotes, and exercises. Because of the nature of the topics covered and because of the manner in which the exposition is given, solving at least part of the exercises should be considered an integral part of the reading effort. Further, in order for the reader to participate actively in "thinking along" with the authors, the text is liberally sprinkled with small challenges ("why," "show this," etc.). These challenges are designed to test the reader's mastery of the text material and in most cases can be readily answered.

A bibliography is appended at the end of the book. All references in the text are made by alphamerics (enclosed in brackets), referring to specific entries in the bibliography. Our references fall into two categories. Those which are judged to be basic for the subject matter of a given chapter are given at the end of each chapter with appropriate annotations. More specialized references are cited in the footnotes. The above policy is in line with our intention to guide the reader through the field of modern control theory with a reasonable expenditure of time and effort. No effort is made to make the bibliography complete.

We are indebted to a number of individuals in the course of preparing the text. Special thanks to our colleagues G. A. Ford, W. C. Grimmell, H. Heffes, J. M. Holtzman, S. Horing, S. H. Kyong, Y. S. Lim, J. C. Lozier, V. O. Mowery, M. A. Murray-Lasso, W. L. Nelson, J. A. Norton, S. Pyati, F. A. Russell, I. W. Sandberg, J. A. Stiles, H. C. Torng, P. P. Wang, and H. S. Witsenhausen for their thorough comments in the course of preparation of the manuscript. We are grateful to G. S. Axelby and J. G. Truxal for their painstaking review of the book and for their valuable suggestions. Particular credit should go to our students at Bell Telephone Laboratories, Newark College of Engineering, and Utah State University; their incisive questions and suggestions did much to improve the constitution of the book. Our gratitude also is due Mrs. Gertrude Martin, who proofread our entire manuscript and lent invaluable editorial aid; Mrs. Nancy Campbell and her group of mathematical typists, particularly Miss Norma Lockwood, who typed and retyped our work with distinction; and C. W. Christ and his draftsmen, who rendered yeoman service with the illustrations.

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Symbols*

$\equiv$	identically equal to (p. 41)
$\simeq$	approximately equal to (p. 184)
$\triangleq$	defined as (p. 21)
$\lvert a \rvert$	the absolute value of a, where a is a scalar (p. 51)
$\lVert \mathrm{x} \rVert$	the norm of the vector x (p. 49)
$\lVert \mathrm{A} \rVert$	the norm of the matrix A (p. 177)
$\hat{x}$	the peak value of the sinusoidal function $x(t)$ (p. 184)
$\in$	belongs to (p. 364)
$\mathrm{sgn}\,[x(t)]$	(p. 93)
$\mathrm{sat}\,[x(t)]$	(p. 625)
$\mathrm{dez}\,[x(t)]$	(p. 577)
$\angle z$	the angle $\tan^{-1}(b/a)$ of a complex variable $a + jb$ (p. 207)
$\overline{x(t)}$	the average value of $x(t)$ (p. 210)
$\nabla V(\mathrm{x})$	the gradient of the function $V(\mathrm{x})$, also given as grad $V(\mathrm{x})$ (p. 322)
Σ	summation (p. 21)
Π	product (p. 31)
$\mathcal{R}e\,[z]$	the real part of the complex variable z (p. 189)
$\mathcal{I}m\,[z]$	the imaginary part of the complex variable z (p. 189)
$[x : x \in A]$	the set of all values of x that belong to A (p. 547)
$\dfrac{\partial \mathrm{f}}{\partial \mathrm{x}}$	the matrix whose ij entry is $\dfrac{\partial f_i}{\partial x_j}$ (p. 150)
$\dfrac{\partial H}{\partial x}$	the vector whose ith component is $\dfrac{\partial H}{\partial x_i}$ (p. 600)
$\min\,[a, b]$	the smaller of the two quantities a and b (p. 52)
$\max\limits_{t_1 \le t \le t_2}\,[x(t)]$	the largest value of $x(t)$ in the range $t_1 \le t \le t_2$ (p. 450)
$\mu_{-1}(t)$	the unit step input function applied at $t = 0$ (p. 282)
$\mu_0(t)$	the unit impulse function applied at $t = 0$ (p. 58)
◆◆	designation of the start and finish of theorems, lemmas, examples and definitions (p. 19)

*The page at which the symbol first appears is given in parentheses.

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PART III OPTIMUM SYSTEM PERFORMANCE ANALYSIS