

**PROGRÈS RÉCENTS
DANS LA MESURE
DE L'INTENSITÉ ACOUSTIQUE**

***RECENT DEVELOPMENTS
IN ACOUSTIC INTENSITY
MEASUREMENT***

Senlis (France) ~~30~~ septembre - 2 octobre 1981



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Olson a été le premier à décrire, dès 1930, le principe de la mesure de l'intensité acoustique vectorielle.

Il a fallu attendre près d'un quart de siècle pour que la formulation du gradient de pression utilisée en particulier par Schultz ouvre la voie aux premières applications.

On peut s'étonner que ce nouveau pas n'ait pas suffi à donner l'impulsion nécessaire à la diffusion dans tous les laboratoires d'acoustique d'une technique a priori si séduisante.

Un autre quart de siècle se sera donc écoulé avant de voir la communauté acousticienne internationale faire un premier bilan sur sa mise au point et son utilisation.

Nul doute que l'absence relative d'enseignement d'acoustique vectorielle ait conduit les praticiens à utiliser les seuls concepts scalaires, lesquels suffisent d'ailleurs à résoudre la plupart des problèmes d'acoustique architecturale ou industrielle.

De plus, la mesure de pression acoustique était simple et parfaitement contrôlable par l'oreille, alors, pourquoi chercher à lui substituer une mesure d'intensité beaucoup plus délicate et sans liaison directe avec l'impression auditive.

La nécessité de résoudre des problèmes très difficiles à traiter par les mesures classiques de pression tels que localisation de sources, détermination des flux d'énergie, extraction de la puissance acoustique d'une machine dans un environnement très perturbé... a sans conteste été l'élément moteur des développements de ces 10 dernières années.

Pourtant, cela n'aurait pu s'accomplir sans les progrès considérables réalisés dans les domaines de l'électronique et du traitement du signal.

Le terrain est encore peu défriché et il faut sans doute s'en réjouir mais déjà un intérêt considérable se fait sentir au niveau des utilisateurs potentiels dans différentes branches de l'acoustique et en particulier celles qui touchent aux industries mécaniques.

La quarantaine de communications émanant aussi bien de chercheurs que de praticiens de 12 pays permettra, j'en suis convaincu de préciser aux représentants de 20 nations les possibilités de cette technique, mais aussi d'en mieux cerner les limites.

Je vous souhaite un congrès agréable et enrichissant.

Jean TOURRET

Chef du Département Acoustique

Centre Technique des Industries Mécaniques

Olson was the first to define, as early as 1930, the principles of vectorial acoustic intensity measurement.

A quarter of a century had to elapse before the formulation of the pressure gradient, used especially by Schultz, could bring about the first applications.

One can wonder that this new step forward was not sufficient to give the necessary impulse for spreading such an a priori attractive technique in the acoustic laboratories all over. We had to wait for another quarter of a century to have the international acoustic community make a first evaluation of its development and use.

There is no doubt that a more or less absence of teaching in vectorial acoustics led the users to apply the scalar concepts only which are sufficient for the resolution of most problems in architectural or industrial acoustics.

Besides, the measurement of acoustic pressure was simple and perfectly checked by hearing, and there was no reason to find a substitute in the form of a more delicate intensity measurement having no direct connection with the operator's hearing.

The need to deal with problems such as spotting of sources, energy flux determination, acoustic power emission of a machine in noisy surroundings, which could be difficult to solve by ordinary pressure measurements, was the incentive of the last ten years progress. However, the necessary developments would not have been achieved without the tremendous progress in electronics and signal processing.

There is quite a lot of work to be done, and we may be happy to have to do it. A tremendous interest is already arising among the potential users in different branches of acoustics and especially those dealing with the mechanical industries. I am convinced that the forty papers issued by research workers as well as users from 12 countries will enable the attending representatives of twenty countries to get better acquainted with this technique, to learn its possibilities and limitations.

Let me wish you a successful and enjoyable meeting.

Jean TOURRET

Head of the Department of Acoustics
Centre Technique des Industries Mécaniques

TABLE DES MATIÈRES

CONTENTS

★ Communication faite sur invitation
★ *Invited paper*

★★ Texte non parvenu pour la date d'impression
★★ *Paper not received for the date of impression*

I - PRINCIPES DE MESURE ET ERREURS INHÉRENTES : *MEASUREMENT PRINCIPLES, INHERENT ERRORS :*

★ Fundamental aspects of the cross-spectral method of measuring acoustic intensity J.Y. Chung	1
Mesure de l'intensité active et réactive dans différents champs acoustiques J.C. Pascal	11
Confidence limits and random error for acoustic intensity measurements A.F. Seybert	21
Some effects of microphone environment on intensity measurements J. Tichy	25
Intensity vector field mapping with nearfield holography E.G. Williams, J.D. Maynard	31

II - APPAREILS DE MESURE : *INSTRUMENTATION :*

★ Practical considerations in the choice of transducers and signal processing techniques for sound intensity measurements F.J. Fahy, S.J. Elliott	37
Basic features of signal processing applied to intensity meters G. Pavic	45
Real time sound intensity measurements performed with an analog and portable instrument D. Pleeck, E.C. Petersen	53
An analogue intensity meter based on the gradient principle H. Kutter	61
A sound intensity real-time analyzer O. Roth	69
Un appareil de mesure de la densité de l'énergie acoustique dans l'air R. Riedlinger	75
Acoustic intensity measurement probe G. Rasmussen, M. Brock	81
Surface intensity measurements using a fiber optic-pressure probe D.E. Boone, T.H. Hodgson	89

III - EXPLORATION DES CHAMPS ACOUSTIQUES : ANALYSIS OF SOUND FIELDS :

★ Notion d'ondes appliquée à la description du rayonnement acoustique. Caractérisation en champ proche L. Gaudriot	★ ★
Intensité acoustique en champ proche et lointain, fonction de transfert d'une antenne. Qu'est-ce qu'une situation asymptotique ? P. Flandrin, B. Escudie	95
Describing acoustic energy flow in two dimensions by the use of intensity vectors O.K. Pettersen, U.R. Kristiansen	103
Flux d'énergie proche d'une source ponctuelle au-dessus d'un plan réfléchissant : comparaison avec la pression acoustique M. Bockhoff	111
Visualisation and estimation of the near field acoustic intensity P. Sas, R. Snoeys	119

IV - ETUDE DES SOURCES SONORES CHARACTERISATION OF SOUND SOURCES :

★ The use of surface and acoustic intensity techniques to identify noise sources on machines M.J. Crocker	127
--	-----

IV. 1 - SOURCES INDUSTRIELLES : INDUSTRIAL NOISE SOURCES :

Investigation of diamond drilling equipment noise by acoustic intensity method G. Krishnappa	137
L'intensité acoustique appliquée à l'analyse et au contrôle des bruiteurs industriels M. Mercusot	★ ★
Laboratory and field measurements with an analog intensity meter H.M.M. van der Wal, B.G. van Zijl	145

IV. 2 - BÂTIMENT : BUILDING STRUCTURES :

Measurement of sound powers radiated by individual room surfaces using the acoustic intensity method M. Villot, J. Roland	153
Application of acoustic intensity measurement for the evaluation of transmission loss of structures M.J. Crocker, B. Forssen, P.K. Raju, Y.S. Wang	161

V - DÉTERMINATION DE LA PUISSANCE ACOUSTIQUE : SOUND POWER DETERMINATION :

★ Higher accuracy in sound power determination of machines under in situ conditions by using sound intensity meter G. Hübner	171
Estimation de la puissance acoustique à l'aide d'intensimètres J.C. Pascal	179

La normalisation de la détermination intensimétrique de la puissance acoustique	
F. Laville	187
The application of acoustic intensity to engine noise reduction	
M.D. Croker, R.J. Tyrrell, M.P. North	193
Measuring sound power of earthmoving machinery using acoustic intensity	
E.M. Sweeney	201

VI - PROPAGATION DANS LES STRUCTURES ET LES FLUIDES : **PROPAGATION IN STRUCTURES AND FLUIDS :**

VI. 1 - STRUCTURES : **STRUCTURES :**

★ Determination of sound power-flow in structures : principles and problems of realisation	
G. Pavic	209
Flux d'ondes vibratoires dans les structures mécaniques	
L. Gaudriot	★ ★
A method to compute the acoustic intensity of a closed vibrating structure	
G.H. Koopmann, J.P. Perraud, H. Benner	217
Mutual transformation of transversal and longitudinal vibrations in structures of finite size and its influence on the experimental determination of vibrational power	
G. Rosenhouse, F.P. Mechel	225
Methods for structural power transmission measurement	
R.J. Pinnington, W. Redman-White, K.T. Brown	229

VI. 2 - FLUIDES : **FLUIDS :**

★ Measurement of fan sound power in ducts using the acoustic intensity technique	
J. Roland, M.J. Crocker, M. Sandbakken	237
Mesure de l'intensité sonore dans les conduits	
S. Lewy, J. Lambourion	245
Mesure des caractéristiques d'une ligne hydraulique haute pression par traitement des signaux de deux capteurs décalés	
L. Martin, A. Calmard	★ ★
Application de l'intensimétrie acoustique à l'identification des sources de pulsation de pression dans les circuits	
A. Badie-Cassagnet, M. Bockhoff, J.M. Lambert	253
Using transfer function measurements to determine energy propagation in fluid lines , with applications to centrifugal pump systems	
U. Bolleter	261
Problèmes posés par la mesure des impédances acoustiques en basse fréquence à l'aide d'un intensimètre	
J.C. Guilloud, R. Perret, J. Coudrieau	267



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FUNDAMENTAL ASPECTS OF THE CROSS-SPECTRAL METHOD OF MEASURING ACOUSTIC INTENSITY

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INTRODUCTION

During the last several decades, different methods have been developed for measuring acoustic intensity [1-13]. Due to their analog nature and other circuit requirements, these methods are not suitable for narrow band acoustic intensity spectrum (AIS) analysis. In recent years, this narrow band AIS analysis has been shown to be very useful in noise-source identification and noise-source ranking for engines and other vehicle components [14-18]. It is expected that a digital AIS analysis will have a wide variety of applications in the area of noise control engineering and other general acoustic measurements.

The major factor that makes the digital AIS analysis possible is the discovery of the relation between the acoustic intensity and the cross-spectrum [19] of acoustic pressures at two closely spaced microphones. Strictly, the term cross-spectrum is applicable only to stochastic signals which are commonly encountered in practice. Hence a cross-spectral formulation must be based on estimates of stochastic spectral quantities. In general, the derivations of formulae based on a stochastic signal is much more involved than that based on a deterministic signal. The formulation of acoustic intensity based on a deterministic signal was developed by Alfredson [20] and Lambert et al. [21]. Fahy, in his letter-to-the-editor [22] described an expression of acoustic intensity for stochastic signals without presenting any derivation. Presumably, Fahy simply used the deterministic formula to interpret the expression for stochastic signals. This is based on the fact that in his later paper [23], a deterministic model was used to derive an intensity formula for a non-stationary stochastic process of a transient signal [24]. In fact, a rigorous derivation of the cross-spectral formula using an ordinary Fourier analysis has not appeared in the literature to date.

Parallel to the deterministic formulation mentioned above, Chung [25], in his effort to use digital Fourier analysis for engine noise-source-ranking, developed a cross-spectral formula using the digital finite Fourier transform (DFT) procedure [26]. Although the formulation was based on a stochastic signal (which is generally the case for engine noise), it is strictly valid only when the digital finite Fourier transform is considered. Hence, from a theoretical point of view, a rigorous derivation using the classical correlation function approach (such as the one used to derive the intensity expression for a plane wave [27]) is yet to be shown. It is interesting to note that before such a rigorous formulation was obtained, researchers have already conducted laborious error analysis for the formula and even expressed their interpretations of its origin [28,31].

The cross-spectral formula derived by Chung had been carefully verified with laboratory tests by Chung and Pope [14] using DFT procedure, before it was introduced at INTER-NOISE 78. In this experiment, a large rotatable baffle was used to determine both the near-field and the far-field sound power radiated from a sound source located at the middle of the baffle. A stationary random signal was used throughout the tests. Three significant findings were reported: (a) The cross-spectral method can be used to measure the resistive sound power in the acoustic near-field. (b) The inherent finite difference approximation error associated with the finite microphone spacing can be predicted

precisely for a simple source. (c) The far-field result obtained by the cross-spectral method agrees well with that of the conventional SPL measurement within the frequency range where the finite difference approximation error is negligible.

The above experiment led to the conclusion that the cross-spectral method is valid for a stochastic signal so long as the DFT procedure employed is valid. It is well established, however, that the DFT provides a meaningful estimate of spectral quantities of stationary random signals provided that inherent problems such as the leakage and aliasing errors and the effect of a finite time window on the cross-spectral estimate are taken into consideration. Hence although a rigorous derivation for the cross-spectral formula is yet to be sought, considerable amount of experimental evidence has supported both the validity and the accuracy of the cross-spectral method. In fact, so long as the digital finite Fourier analysis is employed, the derivation presented by Chung [25,30] is complete.

BASIC ASSUMPTIONS AND LIMITATIONS

Three-Dimensional Wave

The theory behind the cross-spectral method is developed directly from the well known Navier-Stokes equations with simplifications. The Navier-Stokes equations are quite general requiring only the fluid in question to be isotropic and Newtonian. These requirements are generally satisfied in the case of linear acoustics regardless of the acoustic field involved (i.e., near-field, far-field, free-field or reverberant-field). The restriction imposed on the cross-spectral method, therefore, can be found clearly from the simplifications made to the Navier-Stokes equations in order to arrive at the cross-spectral formula.

For small perturbations about the mean quantities, the x-component Navier-Stokes equations in Cartesian coordinate, can be written as [29],

$$\rho \left[\frac{\partial(u + u')}{\partial t} + (u + u') \frac{\partial(u + u')}{\partial x} + (v + v') \frac{\partial(v + v')}{\partial y} + (w + w') \frac{\partial(w + w')}{\partial z} \right] = B - \frac{\partial(P + P')}{\partial x} + \phi(\mu) \quad (1)$$

where the prime denotes perturbations. u, v, w are the x-, y-, z- component velocities respectively, p is the pressure and the function $\phi(\mu)$ represents the viscous terms. Also B and ρ are the body force and mean density of the fluid respectively. Equation 1 can be simplified into,

$$\rho \frac{\partial u'}{\partial t} = - \frac{\partial P'}{\partial x} \quad (2)$$

assuming,

(a) No mean-flow and no body force.

(b) The higher order quasi-linear terms such as $u'(\partial u'/\partial x)$ are small relative to the term, $\partial u'/\partial t$.

(c) The viscous terms $\phi(\mu)$ can be ignored.

In fact, Equation 2 is the familiar general relation between the acoustic pressure and the x-component acoustic velocity in linear acoustics. For a three-dimensional wave propagation, the y- and z- component acoustic velocities are related to the acoustic pressure in similar expressions as,

$$\rho \frac{\partial v'}{\partial t} = - \frac{\partial P'}{\partial y} \quad (3)$$

$$\rho \frac{\partial w'}{\partial t} = - \frac{\partial P'}{\partial z} \quad (4)$$

The acoustic intensity vector \bar{I} for a stochastic signal can then be defined as

$$\bar{I} = E [p'(u'\hat{i} + v'\hat{j} + w'\hat{k})] = I_x \hat{i} + I_y \hat{j} + I_z \hat{k} \quad , \quad (5)$$

where \hat{i}, \hat{j} , and \hat{k} are unit vectors and I_x, I_y, I_z are the component magnitudes of the intensity vector in the x-, y- and z- directions respectively while E denotes the expected value [24].

Consider a pair of closely spaced pressure sensors aligned in the x-direction with Δr being the separation distance and let P'_1 and P'_2 be acoustic pressures detected at the first and the second microphone, respectively. Using finite difference approximation in Equation 2, the pressure p' and the velocity u' at the mid-point between the microphones can be approximated as

$$p' = (p'_2 + p'_1)/2 \quad (6)$$

$$u' = -\frac{1}{\rho \Delta r} \int (p'_2 - p'_1) dt \quad (7)$$

Using finite Fourier transforms of p' and u' , it was shown in Refs. 25 and 30 that the acoustic intensity spectrum, I_x is related to the cross-spectrum of P'_1 and P'_2 as

$$I_x(\omega) = \text{Im}[G_{12}]/\rho\omega\Delta r, \quad k\Delta r \ll 1 \quad (8)$$

where $\text{Im}[G_{12}]$ is the imaginary part of the cross-spectrum of P'_1 and P'_2 , ω is the angular frequency and k is the wave number. The other component intensities, i.e., I_y and I_z can be measured using the same formula given by Equation 8 when the two microphones are aligned in the y- and z- directions, respectively. The total intensity vector \bar{I} can then be determined from Equation 5.

From the assumptions made in the Navier-Stokes equations in order to obtain Equation 2, it is seen that the cross-spectral method can be used in any acoustic field so long as the acoustic medium in question is linear, isotropic, Newtonian and with no mean flow. However, there are limitations to the method due to the expected value used in Equation 5 and the finite difference approximation used in Equations 6 and 7. These limitations are,

- (a) The stochastic signal is stationary and ergodic.
- (b) No source exists between the sensors.
- (c) The microphone separation distance Δr must be small relative to the wave length, or $k\Delta r \ll 1$.

Item (c) will be discussed in more detail in a later section.

In the derivation of Equation 8, it was assumed that the reactive sound power which dominates in the acoustic near-field vanishes by means of a proper ensemble averaging. Thus the acoustic intensity measured in the near-field based on Equation 8 is only the resistive sound power which propagates towards the far-field. Hence for an accurate measurement of intensity a sufficient number of ensemble averages is required.

Plane Wave

In the case of one-dimensional wave propagation such as a plane wave in a duct system, a similar two-microphone approach can be used without a finite difference approximation. The intensity expression takes the following form [27],

$$I_p(\omega) = \text{Im}(G_{12})/\rho c \sin(k\Delta r) \quad (9)$$

where the subscript p indicates the plane wave intensity. It is interesting to note that in this plane wave formula no restriction is imposed on the value of $k\Delta r$. Equation 9 has also been verified experimentally [27].

ERROR ANALYSIS

Misconception in Error Analysis Using Coherence Function

To analyze error in acoustic intensity measurement using the ordinary coherence function (which was found in a published literature) can be misleading.

It is well known that the ordinary coherence function between the input and the output of a dynamic system can be used to determine the linearity of the system in question and to examine the output extraneous noise [32] such as the instrument noise. For a linear system without extraneous noise the value of the ordinary coherence function is at its highest value of unity at least theoretically. But when an ordinary coherence function is used at two system outputs which resulted from multiple partially correlated inputs (corresponds to the two microphone signals in a normal acoustic intensity measurement), it should be interpreted differently. In this case, the value of the coherence function does not have to be high even for a perfectly linear system without any extraneous noise. This phenomenon was described in detail in Reference 33. In fact, there is no direct relation between the value of the coherence function and the accuracy in the cross-spectral method. In many cases, better accuracy is even associated with a lower coherence function value. A typical example will be given in the later section entitled Signal-to-Noise in Acoustic Intensity Measurement.

Error Due to Finite Difference Approximation

To examine the error due to finite difference approximation, one could refer back to the original governing equations which are simplified versions of the Navier-Stokes equations. The error associated with the cross-spectral method is similar to that of any other numerical finite difference approximation involving the Navier-Stokes equations.

From Equations 6 and 7 it is seen that the only requirement for a good approximation is

$$\begin{cases} p_1' \sim p_2' \\ \frac{\partial p'}{\partial r} \sim \frac{p_2' - p_1'}{\Delta r} \end{cases} \quad (10)$$

Equation 10 indicates that within the region between the two microphones, the smaller the acoustic pressure gradient the better the approximation. This is true in a near-field, a far-field, a free-field or in a reverberant-field. Hence for a given microphone spacing, the error is more severe at higher frequency since the pressure gradient is generally inversely proportional to the wave length.

In practice, the sound field is quite different from that of some simplified sound sources such as a monopole, a dipole or a quadrupole radiating in a free-field. Hence, the error cannot be estimated until the actual pressure gradient is given. It is useful, however, to examine the error associated with a simple source. In this case, it can be shown easily that

$$\frac{|I|}{|I_a|} = \text{sinc}(k\Delta r) / [1 - (\Delta r/2R)^2] \quad (11)$$

where $\text{sinc}(k\Delta r) \equiv \sin(k\Delta r)/k\Delta r$ and $|I_a|$ represents the actual intensity magnitude. Eqn. 11 indicates that in the acoustic far-field where $(\Delta r/2R)^2 \rightarrow 0$, the finite difference error is simply a sinc function with the non-dimensional microphone spacing, $k\Delta r$ being its argument. This far-field finite difference error agrees well with the experimental data [18] shown in Fig. 1. The error in the near-field is not only affected by $\text{sinc}(k\Delta r)$ but also by $(\Delta r/2R)^2$. This error is shown in Fig. 2 for values of $k\Delta r$ being 0.1, 0.5 and 1.0. It is seen that less than 1 dB finite difference error occurs when $\Delta r/R \leq 2/3$ and $k\Delta r \leq 1$. This is the condition that the distance between the measurement surface and the nearest microphone is equal to or larger than the microphone spacing while the microphone spacing is less than about 1/6 of the wave length.

In theory, the finite difference error becomes smaller as the frequency becomes lower, hence there is no lower bound for an accurate measurement of the acoustic intensity. It should be noted that a theoretical analysis presented by Thompson and Tree [28] concluded that such a lower bound exists. But their conclusion appears to be due to errors in their analysis [34]. In practice, however, measurements at the low frequencies often become inaccurate due to reasons discussed below.

Error at Low Frequencies

There are mainly two reasons which cause inaccuracy in low frequency measurements:

(a) Due to a small microphone spacing relative to the wave length, the actual physical phase shift between the two microphone signals becomes the same order of magnitude as the instrument phase mismatch. In this case, the measurement error in decibels (assuming an over estimate over the actual level) can be expressed by [35]

$$E = 10 \log_{10} (1 + \theta_e / \theta_p) \quad (12)$$

where θ_e and θ_p are the instrument phase shift and the physical phase shifts in radians, respectively. Both phase angles are evaluated with reference to the same microphone. It is seen from Equation 12 that the measurement accuracy is very sensitive to the instrument phase error. In this condition, the most convenient way of reducing the error is to increase the microphone spacing and hence to increase the value of θ_p . Another effective way of reducing the error is to utilize the so-called microphone switching technique for eliminating the phase mismatch error [25,30]. This technique will be discussed further in a later section.

(b) Any digital electronic equipment such as an analog-to-digital converter, a spectrum analyzer, its controller or a large computer used in a batch system has a limitation in its computational accuracy. When the imaginary part of the cross-spectrum becomes too small to be computed accurately, erroneous intensity data is obtained even if no phase mismatch error exists between the two microphone systems. Since all digital equipment are not the same in their computational accuracy, it is difficult to provide a specific dynamic range for an accurate computation of the imaginary part of the cross-

spectrum. Such type of error related to small physical phase is discussed further in the next section.

Error Due to Small Physical Phase

Let θ be the physical phase shift between the two microphone signals. Fig. 3 illustrates the condition when θ is relatively small. Due to random error, bias error and possible instrument phase mismatch, there is uncertainty in the estimate of the magnitude of the cross-spectrum $|G_{12}|$. For convenience, a circle is used to show the range of the uncertainty. For small θ , the range of the uncertainty can be extended from the first quadrant to the fourth, thus changing the sign of the acoustic intensity measured. Therefore, a small error in estimating G_{12} could result in erroneous estimation of both the magnitude and the direction of the acoustic intensity vector. In general, the range of the uncertainty depends on the complexity of the sound field in question. From experience, for complex machinery noise, the real part should not exceed the imaginary part by as much as 12 to 15 dB in level. To be more precise, however, tests must be conducted for each individual case to determine the limitation.

Other Measurement Errors

Other measurement errors that could be encountered in practice are: (1) Bias and random errors in the cross-spectral estimate; (2) Error caused by interference of acoustic field by the microphones; (3) Error due to uncertainty of the acoustic center of the microphone diaphragm and (4) Error due to phase mismatch between two microphone channels.

Generally speaking, for a stationary random signal, a frequency resolution (associated with DFT) of 25 Hz or less with 100 or more ensemble averages should avoid significant bias and random error (item 1) in estimating the imaginary part of the cross-spectrum. A cross-spectral estimate is, in fact, much more "noise free" than an auto-spectral estimate such as the SPL measurement. When high background noise is present, the cross-spectral method provides as much as 10 dB to 15 dB noise rejection if proper ensemble averaging is performed.

The error due to the interference of acoustic field by the microphones (item 2) and due to the uncertainty of the acoustic center of the microphone diaphragm (item 3) are generally not severe for 6.35 mm (quarter inch) or 12.7 mm (half inch) microphones with two microphones parallel to each other and with the microphone spacing greater than 16 mm. Table 1 shows the standard deviation in the measurement of the directional intensity components using 6.35 mm (quarter inch) and 12.7 (half inch) microphones with two microphones parallel to each other and with 16 mm center-to-center microphone spacing [18]. Note that the standard deviation increases with both frequency and microphone diameter. This trend could indicate that the second microphone was in an acoustical shadow zone which was created by the first microphone. The uncertainty of the acoustic center of the microphone could also contribute to the trend, since the intensity was measured in the direction parallel to the microphone diaphragms. For the microphone configuration tested, however, the errors in terms of dB deviations are insignificant.

The most significant error encountered in practice is that due to the phase mismatch between two microphone channels. In general, the microphones, the electronic filters (such as anti-aliasing filters or weighting networks, etc.), tape recorders or voltage amplifiers could cause phase mismatch. In most cases, the mismatch is linearly proportional to the frequency. The mismatch caused by the tape recording, however, can be frequency independent [30]. An estimate of the effect of phase mismatch on measurement accuracy is shown in Fig. 4 where the curve represents the percentage phase error for a 0.5 dB measurement error in terms of the actual physical phase shift between the two microphone signals. The correction of phase mismatch error will be discussed later.

Signal-to-Noise in Acoustic Intensity Measurement [18]

One of the most important factors affecting the acoustic intensity measurement is the ratio of the magnitude of acoustic intensity to that of the power spectrum of the sound pressure. To demonstrate the effect of this "signal-to-noise" ratio in acoustic intensity measurements, a test with three different ratios was made. The test configuration is shown in Fig. 5 where two independent sound sources were used to generate conditions of different signal-to-noise ratios. The angle θ shown in Fig. 5 was set to be zero in this case; thus, the sound generated from source #1 (at 0°) was detected as the intensity while that from source #2 (at 90°) was recorded as background noise. The three test conditions A, B, and C, are tabulated in Table 2. The sound pressure levels indicated in Table 2 are those recorded at the intensity probe location. The ratio of the mean-square pressure spectra for source #1 and source #2 are shown in Fig. 6. Initially, the acoustic intensity generated by source #1 was determined with source #2 switched off. This intensity level was kept the same throughout the test and was plotted in Figs. 7 through 9 in a solid line as a reference intensity spectrum. Figure 7 shows the intensity level (dotted line) with source #1 and #2 being about equal in strength (test A). It is seen that the measurement was not significantly affected by source #2. When source #2 was 11 dB higher in level than source #1 (test B), the measurement was then affected (see dotted line in

Fig. 8), but the error in the overall intensity level was still less than 1 dB. In test C, source #2 was about 19 dB higher than source #1. Erroneous intensity was measured in this case (Fig. 9). Generally speaking, for incoherent noise, if source #2 is not more than 10 dB higher in level than source #1, an acoustic intensity measurement can be made with reasonably good accuracy. For coherent second source, however, no general rule can be established.

It should be noted that the ordinary coherence function measured with source #2 off and for test C (almost perfect coherence in both cases) were higher than that of tests A and B due to the domination of source #1 and #2 respectively. The accuracy in the acoustic intensity measurements, however, is good when source #2 was off and worst in test C. This is a typical example, illustrating the previous comment that the ordinary coherence function cannot be used to estimate the accuracy of the intensity measurement.

CORRECTION OF PHASE MISMATCH ERROR [18]

In general, to measure a cross spectrum correctly requires two channels of phase-match instruments. Since the cross-spectral method uses measurement of the imaginary part of the cross spectrum, phase matching is an important requirement for determining the acoustic intensity. Generally speaking, a phase-matched system can be obtained with a pair of high-quality microphones selected for phase-matched cartridges and preamplifiers along with a matched pair of amplifiers. In practice, however, tape recording and electronic filtering often cause significant phase mismatch between the channels, as mentioned previously.

Three approaches to correct for the phase mismatch are described as follows:

1. The Transfer Function Approach

This procedure was presented previously by Blaser and Chung [36]. The calibration of the microphone systems is accomplished by mounting both microphones in a plate that can be rigidly attached to the end of a duct. The two microphones can then be assumed to be exposed to the same acoustic field, and the transfer function measured in this configuration represents the response of one microphone system relative to the other. If microphone #1 is used as a reference, the phase and gain correction can be made according to the following formula:

$$[G_{12}]_{\text{calibrated}} = G_{12} / (H_{12} \cdot |H_1|^2), \quad (13)$$

where H_{12} is the calibration transfer function and $|H_1|$ is the gain factor of the first microphone system.

2. The Microphone Switching Approach

The switching procedure can be represented by [25,30],

$$I_x(\omega) = \text{Im} \{ [G_{12} \cdot G_{12}^s]^{1/2} \} / \rho \omega \Delta r |H_1| |H_2| \quad (14)$$

where G_{12}^s is the cross-spectrum measured with the two microphone sensing locations interchanged and $|H_1|$ and $|H_2|$ are the gain factors of each microphone. It is seen from Equation 14 that the phase-corrected cross spectrum is obtained from the geometric mean of the original cross-spectrum and the switched cross spectrum G_{12}^s , or

$$[G_{12}]_{\text{calibrated}} = [G_{12} \cdot G_{12}^s]^{1/2} / |H_1| |H_2| \quad (15)$$

If the microphone cartridges and other consecutive electronics are phase matched, then the switching can be made right behind these phase-matched elements.

3. The Modified Microphone Switching Approach

This procedure was reported previously by Chung and Blaser [37]. In this procedure, the relative phase and gain factors of the two microphone systems are obtained from the square root of the ratio of the original and the switched transfer functions. In determining these transfer functions, any broad-band signal can be used and the two microphones need not be exposed to the same acoustic field. Mathematically, this procedure can be represented by

$$[G_{12}]_{\text{calibrated}} = G_{12} / (K_{12} \cdot |H_1|^2) \quad (16)$$

where

$$K_{12} = [H_{12}/H_{12}^S]^{\frac{1}{2}} \quad (17)$$

and H_{12} and H_{12}^S are the original and switched transfer functions mentioned above.

The advantage of the first approach, i.e., the transfer function approach, is that there is no need to evaluate G_{12}^S and the square root of a complex variable. Hence it does not require a relatively sophisticated spectrum analyzer; also, it saves time in ensemble averaging. The disadvantage of this approach is that it is not easy to determine an accurate H_{12} over a broad frequency range. Generally speaking, to determine H_{12} in a free field is not practical since this procedure requires a rather good free-field condition which is not readily available in most practical situations. Thus, a duct system such as that mentioned above is normally used to determine H_{12} . Due to resonances of the duct system, however, it is difficult to generate a broad-band signal (say 100 Hz to 4 kHz) at any location within the duct system without a wide signal dynamic range. Hence, the H_{12} so obtained often contains significant error at frequencies where the signal level is relatively low. Another disadvantage of this approach is that error due to drifting of the phase factors cannot be corrected, hence the calibration must be repeated at some unknown time interval.

The advantage and the disadvantage of the second method, i.e., the microphone switching approach, are just opposite to that of the transfer function approach. It can be said, however, that there is less chance of inaccurate measurements using the microphone switching approach.

The third method mentioned above, i.e., the modified microphone switching approach is a compromise between the first two approaches. In this approach, a good free-field condition is not needed; thus, it does not have the major disadvantage of the transfer function approach. It also has the advantage of cutting down the ensemble averaging time since it does not involve the determination of G_{12}^S for each measurement. However, it does require computation of the square root of a complex variable.

MEASUREMENT IN ENVIRONMENT WITH FLOW

Three-Dimensional Wave

In deriving the basic governing equation, the mean flow was neglected in the Navier-Stokes equations. Hence, strictly, the cross-spectral method does not apply to the sound field with flow. For a low flow condition (say Mach number of 0.001) the method can be used without significant error. In this case, a simple wind screen such as an open-celled foam can be used to suppress the air flow around the microphones. It should be noted that the flow not only induces disturbance to the sensors, but also complicates the definition of the acoustic intensity [38]. Fig. 10 illustrates that, for a flow Mach number of 0.05, as much as 3 dB error can be induced [18] when the measurement is made in the direction of flow.

Plane Wave

The use of the two microphone method can be extended to a case of plane wave propagation in a uniform flow field, provided additional terms are added to the expression for the intensity. Chung and Blaser [27] developed the expression for the acoustic intensity in a uniform flow with Mach number M as,

$$I_x(\omega) = \frac{G_{11}}{4\rho c \sin^2[k\Delta r/(1-M^2)]} \left\{ (1+M)^2 \left| \exp\left[j\left(\frac{k\Delta r}{1-M}\right)\right] - H_{12} \right|^2 - (1-M)^2 |H_{12} - \exp\left[-j\left(\frac{k\Delta r}{1-M}\right)\right]|^2 \right\} \quad (18)$$

where, G_{11} is the auto spectrum of the microphone signal, k is the wave number, c is the speed of sound and H_{12} is the transfer function between the two microphone signals. It is seen that due to flow, the cross-spectrum alone cannot completely describe the acoustic intensity. Equation 18 has been verified experimentally [27].

SPACE-TIME AVERAGED ACOUSTIC INTENSITY

So far, the cross spectral method has been shown to be effective in evaluating sound radiation from a machine surface. In actual application of the method, a space-time-averaged acoustic intensity is often determined by scanning the intensity probe over the surface area of interest. The scanning can be performed mechanically or manually. In theory, a scanning motion will cause error in data acquisition. In

practice, however, when the scanning speed is of the order of 30 mm per second or slower, no significant error will be induced. In one of the experiments conducted by Chung and Blaser [27], the acoustic intensity determined by scanning was compared with a non-scanning result. Fig. 11 shows the experimental set-up with a plane wave of stationary random signal propagating in a duct system. Microphones #1 and #2 were used to measure the net acoustic intensity in the direction towards the duct opening, thus determining the total sound power radiated from the duct. Microphones #3 and #4 were then used to determine the space-time averaged acoustic intensity on a hypothetical surface enclosing the duct opening. The total radiated sound power was determined by the product of the surface area and the space-time averaged acoustic intensity. The results obtained from both inside and outside of the duct are compared in Fig. 12. The agreement is excellent, thus concluding the accuracy of the scanning procedure as well as the cross-spectral method.

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Table 1 Standard Deviations in the Measurement of the Directional Intensity Components

Microphones	Frequency Ranges (kHz)			
	0.6-1.4	1.4-2.2	2.2-3.0	0.6-3.0
Bruel and Kjaer Quarter-inch Condenser	1.2%	1.4%	2.1%	1.3%
Bruel and Kjaer Half-inch Condenser	2.8%	3.7%	4.2%	3.5%

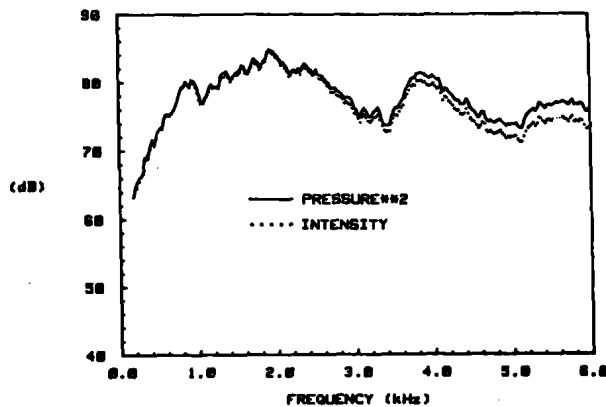


Fig. 1 Finite difference approximation error in the far-field--solid curve represents the standard intensity spectrum.

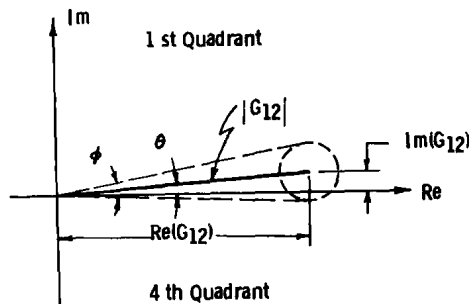


Fig. 3 Inaccuracy in acoustic intensity measurement for small phase angle.

Table 2 Sound Pressure Levels Generated by the Two Sound Sources

Test	Sound Source #1 (dB)	Sound Source #2 (dB)	#1 and #2 Combined (dB)
A	82.0	82.5	85.2
B	82.0	93.0	93.2
C	82.0	100.3	100.4

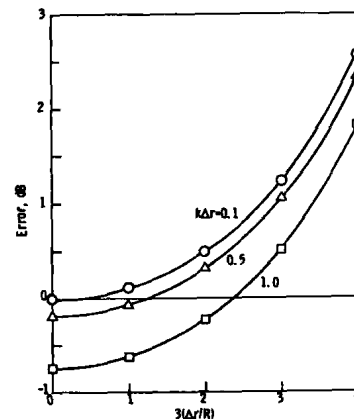


Fig. 2 Finite difference approximation error in the near-field of a simple source.

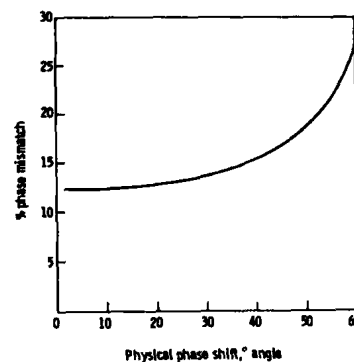


Fig. 4 Percentage phase mismatch for 0.5 dB error in acoustic intensity measurement.