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Optical Testing and Metrology III: Recent Advances in Industrial Optical Inspection

C. P. Grover
Chair/Editor

8–13 July 1990
San Diego, California

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Part Two of Two Parts

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Effects of the non-vanishing tip size in mechanical profile measurements

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ABSTRACT

The high-spatial-frequency behavior of mechanical-profiling instruments is determined principally by the non-linear geometrical interaction between the stylus tip and the surface irregularities. There is considerable interest in the nature of this interaction and its effects in practical applications. This paper explores the these issues analytically and presents a variety of results with emphasis on the frequency-domain description. Smooth surfaces are treated by perturbation theory and are found to be roughened by tip-size effects, while rough surfaces are smoothed. In lieu of closed form results for rough surfaces, we offer a simple conjecture regarding the nature of tip-size distortions and the maximum useable spatial frequency.

1. INTRODUCTION

All profile-measuring techniques are band-width limited; that is, they are sensitive to only a limited range of surface frequencies f. Optical measurements are generally linear over their operating range, and it is straight forward to determine their extreme band-width limits in terms of system parameters. In the case of Wyko-like measurements [1-4]:

$$\frac{1}{L} < f_{OPT} < \frac{1}{\lambda} \qquad (1)$$

where L is the trace length and λ is the wavelength of the radiation used (HeNe or thereabouts). This last means that the highest accessible spatial frequency is of the order of 1 $\mu m(-1)$.

The corresponding length parameters which determine the band-width limits of mechanical-profiling instruments are the trace length L and the radius of the stylus tip, R. In contrast with optical techniques, mechanical measurements are inherently non-linear at high spatial frequencies and the upper frequency limit of their bandpass is not simply $1/R$, but some more complicated function of both R and surface-finish parameters, which we denote by the collective symbol α :

$$\frac{1}{L} < f_{MECH} < F(R,\alpha) \qquad (2)$$

Since tip dimensions can be made as small as $R \sim 0.1$ μm there is the possibility of breaking the visible-light barrier using mechanical-stylus measurements; but how well and by how much depends on the nature of the non-linear geometrical interaction between the stylus tip and the surface contour.

Tip-size effects are easily calculated for specific surface shapes. For a sinusoidal profile of amplitude A, for example, this leads to the rule of thumb [5-7]:

$$F(R,\alpha) \approx \frac{1}{2\pi\sqrt{AR}} \tag{3}$$

Note that this result varies as $R(-1/2)$ rather than $R(-1)$, which reflects the non-linear nature of the measurement process. Later results involve even more complicated dependences on the tip size.

In this paper we address the more general problem of determining the effects of the tip size on the finish parameters of randomly rough surfaces, and, in particular, their power spectral densities $S(f)$. Little, if any, work appears to have been done in this area [8,9].

As a first step we consider the canonical problem of a circle of radius R rolling over a one-dimensional randomly-rough surface with the profile $Z(x)$. The measured profile is then defined as the locus of the lowest point on the circle.

2. SURFACE STATISTICS

The discussion of tip-size effects involves three surface parameters: σ, μ and γ; the rms (root-mean-square) values of the profile height, slope and curvature:

$$\begin{pmatrix} \sigma^2 \\ \mu^2 \\ \gamma^2 \end{pmatrix} = \begin{pmatrix} \langle z^2 \rangle \\ \langle (dz/dx)^2 \rangle \\ \langle (d^2z/dx^2)^2 \rangle \end{pmatrix} \tag{4}$$

These may be written in terms of the moments of the profile power spectrum, $S(f)$:

$$\begin{pmatrix} \sigma^2 \\ \mu^2 \\ \gamma^2 \end{pmatrix} = \int_0^\infty df\ S(f) \cdot \begin{pmatrix} (2\pi f)^0 \\ (2\pi f)^2 \\ (2\pi f)^4 \end{pmatrix} \tag{5}$$

or equivalently, in terms of the zeroth, second and fourth derivatives of the profile covariance function at zero lag [10,11].

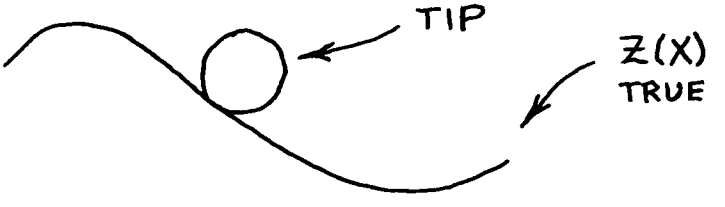
We also define a new quantity, the "rms radius of curvature" $\rho \triangleq 1/\gamma$, for comparison with the tip radius. This enables us to distinguish two limiting cases: $R \ll \rho$ which we call the sharp-tip limit, and $R \gg \rho$ which we call the blunt-tip limit.

3. SHARP-TIP LIMIT

In this case

$$R \ll \rho_{TRUE} \tag{6}$$

and the tip rides over the surface so:



(7)

That is, the bottom of the tip tracks the surface smoothly but experiences an offset which depends on the surface slope [3,4]:

$$Z(X)_{MEASURED} = Z(X)_{TRUE} + \frac{R}{2}\left(\frac{dz}{dx}\right)^2 + \cdots \tag{8}$$

This shows that tip-size effects roughen the measured profile in this limit.

Equation 8 can be rewritten in the frequency domain as

$$S(f)_{MEASURED} = S(f)_{TRUE} + \frac{1}{4}(2\pi)^4 R^2 \int_{-\infty}^{+\infty} dg\, g^2 (g-f)^2 S(|g|)_{TRUE}\, S(|g-f|)_{TRUE} + \cdots \tag{9}$$

where the second term on the right is the tip-size correction. Using this, we can calculate the effects on the finish parameters using Eq 5. For example,

$$\sigma^2_{MEASURED} = \sigma^2_{TRUE} + \frac{1}{2} R^2 \mu^4_{TRUE} + \cdots \tag{10}$$

If we require that the second term be small relative to the first, we find

$$R \ll \sqrt{2}\left(\frac{\sigma}{\mu^2}\right)_{TRUE} \tag{11}$$

which gives an additional requirement on the tip radius in terms of the moment parameters (cf Eq 6).

The most important use of Eq 9, however, lies in the fact that it provides a means for determining the maximum measurable frequency (Eq 2) by imposing conditions on the relative importance of the second term relative to the first. This procedure is illustrated below.

3.1 Sinusoidal profile

For the sinusoid

$$z(x)_{TRUE} = A \sin\left(2\pi \frac{x}{d}\right) \tag{12}$$

we have

$$\rho_{TRUE} = \frac{\sqrt{2}}{(2\pi)^2} \cdot \frac{d^2}{A} \tag{13}$$

Eq 9 then gives

$$S(f)_{MEASURED} = \frac{1}{2} A^2 \delta\left(f - \frac{1}{d}\right) + \frac{1}{8}\left(\frac{R}{\rho}\right)^2 \delta\left(f - \frac{2}{d}\right) + \cdots \tag{14}$$

The first term is the spectrum of the true profile and the second, which appears at twice the original frequency, is the first-order tip correction. If we define the maximum allowable spectral distortion to occur when the second term is 1/16-th the first, we obtain a condition on the radius which is precisely that in Eq 3.

3.2 Rectangular spectrum

Here we consider the random profile whose spectrum is band-width-limited white noise; that is

$$S(f) = K \cdot U\left(\frac{f}{f_0}\right) \tag{15}$$

where K and f_0 are constants and $u(x)$ is a step function: $u(x) = 1$ for $0 \leq x \leq 1$ and zero otherwise. In this case

$$\rho_{TRUE} = \frac{1}{(2\pi)^2}\sqrt{\frac{5}{K f_0^5}} \tag{16}$$

and

$$S(f)_{MEASURED} = K\left[U\left(\frac{f}{f_0}\right) + \left(\frac{R}{\rho}\right)^2\left\{\frac{12-30x+20x^2-x^5}{24}\right\}U\left(\frac{f}{2f_0}\right) + \cdots\right] \tag{17}$$

where $x = f/f_0$. Note that the second term extends to twice the frequency of the true spectrum.

The the curly bracket has a maximum of 0.5 at $x = 0$ and reaches a secondary maximum of 0.1972 at $x = 1.628$ before vanishing at $x = 2$. This means that the sharp-tip condition, Eq 6, is sufficient in itself to ensure the smallness of the correction in this example.

3.3 Gaussian spectrum

Here

$$S(f)_{TRUE} = 2\sqrt{\pi}\sigma^2 \ell\, e^{-(\pi \ell f)^2} \tag{18}$$

where ℓ is the 1/e correlation length. In this case

$$\rho_{TRUE} = \frac{1}{\sqrt{12}}\frac{\ell^2}{\sigma} \tag{19}$$

and the measured spectrum is

$$S(f)_{MEASURED} = S(f)_{TRUE}\left[1 + \left(\frac{R}{\rho}\right)^2\left\{\frac{3-2x^2+x^4}{24\sqrt{2}}\, e^{+x^2/2}\right\} + \cdots\right] \tag{20}$$

where $x = \pi \ell f$.

This curly bracket increases monotonically at high frequencies, and the tip correction eventually dominates the true spectrum. In this case, then, the sharp-tip

condition alone is not sufficient to ensure the smallness of the correction and we must impose an additional constraint on the maximum allowable frequency.

For example, if $R = 0.1\rho$ and we require that the second term in Eq 20 be less than 0.1 the first, f must be less than $0.745/\ell$; a result which is in the form of Eq 2, where the R dependence is implicit in the numerical factor.

4. BLUNT-TIP LIMIT

This means

$$R \gg \rho_{TRUE} \tag{21}$$

where ρ may be non vanishing or zero [11]. In this limit the tip may follow the low-frequency contour of the surface but rides over the high-frequency variations:

(22)

The measured profile then has an asymmetric, cloud-like appearance; with rounded peaks having a radius of the order of that of the tip separated by cusp-shaped valleys:

(23)

In this limit the tip smooths the surface by "filtering-out" high frequencies. At the same time it adds an $f(-4)$ component to the measured spectrum due to the discontinuities in slope at the tips of the cusps [10,12]. In the limit one might expect the spectrum to be dominated by this $f(-4)$ component.

5. COMMENTS AND CONJECTURE

5.1 Comments

The discussion above exploits physical idea that we can distinguish two extreme forms of behavior by comparing the stylus tip size with the rms curvature of the surface. In either regime one can, in principle at least, determine an allowable upper frequency based on a spectrum-based criterion of acceptability. We end up then, with two conditions: a configuration-space one on the tip radius and a

frequency-space one on the maximum frequency. From this point of view our hoped-for limit in Eq 2 turns out to be double barrelled. This is inconvenient.

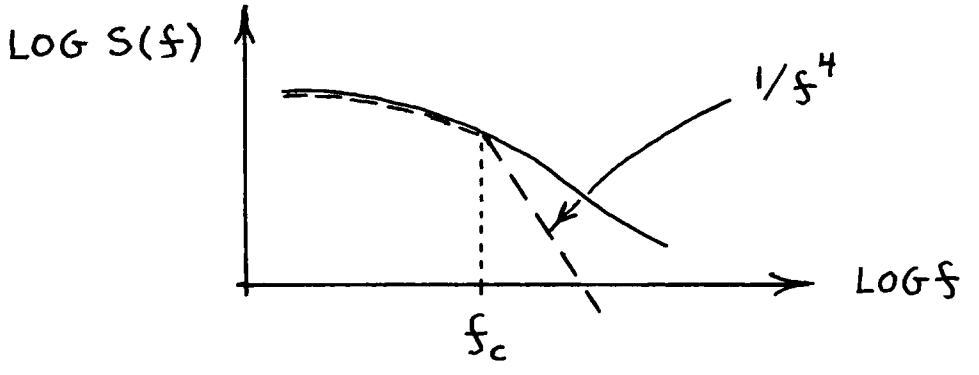
Another disadvantage of this approach is that it requires a-priori knowledge of the true spectrum which may not be available. Also, at this point at least, we have no detailed way of handling the problem except in the sharp-tip limit, where we have used a perturbation approach.

We would like to reformulate the problem in a way that does not involve the surface curvature explicitly, and determine the conditions for the useable frequency range directly in terms of the spectral parameters rather than its moments. For this we consider an ad-hoc procedure based on the above ideas which we state below in the form of a conjecture [3,4]. We offer this as an interim operating scheme and a target for future improvements.

5.2 Conjecture

"The spectrum of the measured profile is the same as that of the original profile up to a critical frequency, f_c, above which it falls off as f(-4). This frequency is determined by evaluating the moment integral of the mean-square profile curvature (Eq 5) between zero and f_c, multiplying the result by a constant of order unity, and setting the result equal to the mean-square curvature of the tip, $1/R^2$ "

Graphically, this means:



(24)

where the solid line is the true spectrum, the dashed line is the measured spectrum, and f_c is the critical frequency.

Algebraically:

$$S(f)_{MEASURED} = S(f)_{MEASURED} \cdot U\left(\frac{f}{f_c}\right) + S(f_c)_{MEASURED} \left(\frac{f_c}{f}\right)^4 \cdot \left[1 - U\left(\frac{f}{f_c}\right)\right] \qquad (25)$$

and f_c is defined implicitly by the equation

$$\frac{1}{R^2} = C \cdot (2\pi)^4 \int_0^{f_c} df\, S(f)_{MEASURED} \cdot f^4 \qquad (26)$$

where C is a constant of order unity.

This conjecture applies over the whole range of tip sizes. To establish the connection with the sharp-tip limit, note that in that case the right-hand side of Eq 26 approaches a finite value of C/ρ^2 as $f_c \to \infty$, and the conjecture then states that the measured spectrum will be totally unaffected providing

$$R < \rho/\sqrt{C} \tag{27}$$

as compared with Eq 6. If this inequality is not satisfied, Eq 26 leads to a finite value of the critical frequency, which may be taken as the upper limit of the undistorted spectrum; that is

$$F(R,\alpha) = f_c \tag{28}$$

To illustrate the application of Eqs 25 and 26 we evaluate them below for the spectral shapes considered in the previous section. Further examples are given in the Appendix.

5.3.1 Sinusoidal profile

If the tip radius satisfies Eq 27, f_c will be infinite and Eq 25 says that the measured spectrum will be unaffected. If Eq 27 is not satisfied, $f_c = 1/d$, and Eq 25, when reinterpreted for this special case of a deterministic profile, predicts that the measured spectrum will be a series of inverse-fourth-power harmonics corresponding to a regular series of parabolic cusps of depth $d^2/8R$ [12]. The dividing line between these two regimes lies at

$$f_c = \left[\frac{2}{C}\right]^{1/4} \cdot \frac{1}{2\pi\sqrt{AR}} \tag{29}$$

which is the same as Eq 3 except for the fourth-root factor, which is expected to be of order unity.

5.3.2 Rectangular spectrum

Here the spectrum will be unaffected up to

$$f_c = \frac{1}{2\pi}\left[\frac{10\pi}{CKR^2}\right]^{1/5} \tag{30}$$

If this is less than f_o, Eq 25 says that a portion the high-frequency end of the measured spectrum will be cut off and replaced by an inverse quartic tail, but if it is greater than f_o the measured profile will be unaffected. In contrast, perturbation theory predicts a small high-frequency contribution out to $2f_o$ (Eq 17).