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Evidence for Gravitational Theories



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a cura di C. MøLLER
Direttore del Corso

VARENNA SUL LAGO DI COMO
VILLA MONASTERO

19 GIUGNO - 1 LUGLIO 1961

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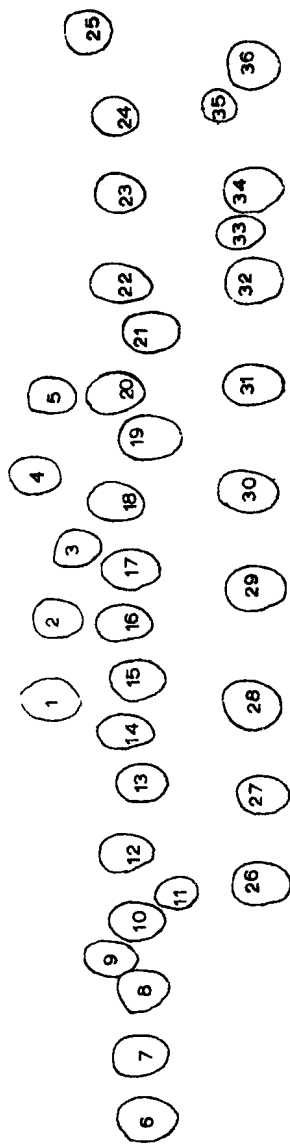
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Mach's Principle and Equivalence.

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1. — Experimental research on gravitation.

It is strange that the gravitational-inertial field which is fundamental to all physics, particularly in its inertial aspects, is but poorly known from an experimental or observational view point. There has been very little significant experimental work done on gravitation in the past fifty years.

As has been remarked many times, this seems to be due to a variety of causes [1]. First it should be noted that because of the weakness of the gravitational interaction it is extremely difficult to perform interesting experiments in gravitation. Second, because of the weakness of the interaction, it seems to have very little to do with the inner working of atoms and nuclei, subjects of interest to most physicists. Third, it must be recognized that the elegance and perfection of Einstein's General Relativity has led most physicists to conclude that it must represent an established part of physics.

Of course, those parts of physics which are understood are never as fascinating as the parts which are not, and it is to the latter that the real effort is directed. If one considers the history of General Relativity, one sees that there was a great deal of interest in this theory in the ten or fifteen years following the conception by EINSTEIN of this beautiful formalism, but that after this, most of the activity died and it was only in the past five, or ten years that people began again to become interested in gravitation as a field of physics.

Anyone who would do experimental research on gravitation or relativity soon changes his ideas about the nature of an experiment. With the training that most physicists receive, they immediately think of an experiment as carried out in the laboratory. It should be recognized however that because of the great weakness of gravitation, the large velocities often desired for experiments in gravitation, and the long time scales required for certain experiments, the best apparatus may not be in the laboratory, but may already exist in nature in the form of stars or planets. In order to have a source of

a strong gravitational field, a very large body is required. It is clear therefore, that experimental research on gravitation becomes hopelessly intertwined with observational physics and for this reason we do not make a distinction between experimental and observational research on gravitation. Occasionally one can find in the data already accumulated by geophysicists and astrophysicists the type of numbers needed to answer a fundamental question in physics. It would be rather foolish to think only in terms of laboratory experiments under these circumstances. It may be possible for an experimental physicist to devise new types of instrumental techniques for astrophysics and geophysics and hence to help obtain the improved astrophysical and geophysical data necessary to answer certain fundamental questions.

To summarize, our first conclusion is that for research on gravitation the physicist should change completely his way of thinking about the nature of an experiment and that he should include available observational data in tackling experimental problems.

As a second important point, in thinking about a program of research of an experimental or observational type on General Relativity, it should be noted that the only significant experiments to be performed are not necessarily those experiments which lead to positive results. Thus, for example, we hear frequently about the three experimental checks of General Relativity, the gravitational red shift, the gravitational deflection of light, and the relativistic perihelion rotation of the planet Mercury. These are regarded as checks of General Relativity because they lead to definite positive results associated with the theory. On the other hand, if we consider the origins of the formalism and its logical structure, we find that these are very dependent on certain null experiments. For example, General Relativity is more dependent in a fundamental and direct way on the Eötvös experiment, which established that the gravitational acceleration of different kinds of bodies are equal to a very high accuracy, than it is on the famous three tests. None of the three experimental or observational checks of the theory approach the accuracy and importance of the Eötvös experiment.

Two of the fundamental tests, that of the gravitational deflection of light and the gravitational red shift, as it was obtained classically from the observations on the sun and white dwarfs are of very poor accuracy. Only the perihelion rotation of Mercury associated with the relativistic effect could claim any significant accuracy. More recently and due to the very nice work with the Mössbauer effect, a comparable accuracy has been obtained on the gravitational red shift [2]. However, neither of these two important experimental observational results can be said to be particularly accurate. A wide range of possible modifications of General Relativity might be conceived that would not lead to a violation of these experimental results. On the other hand, the extremely precise result of the Eötvös experiment [4] with an accuracy

of 5 parts in 10^9 , sets some very precise limits to the type of gravitational theories which one can construct.

There are many other possible null experiments which have not yet been sufficiently explored. To give an example, one could determine if the active gravitational mass of a body, and by active gravitational mass here we mean the source strength of a body for the production of a gravitational field [3], is independent of the velocity of motion of this body relative to distant matter in the universe [1]. A possible null experiment which could be used to obtain an indication of the validity of this notion would be to simply observe the gravitational acceleration experienced at the surface of the earth as the earth revolves about the sun.

As the earth orbits the sun, its velocity relative to distant matter in the universe changes and it is conceivable that this change in velocity might be associated with a change in the active gravitational mass of the earth. An experiment could be carried out by placing an artificial satellite in an orbit around the earth and then investigating its time-keeping characteristics. One would ask whether its period was dependent upon the season of the year.

In conclusion, a physicist is severely limited if he considers only those experiments which would be expected to give a positive result. On the other hand, if he allows experiments of the null type, it is possible to increase greatly the range of significant investigations.

One distressing thing about the structure of General Relativity, from the first inception of the theory up to the present day, is the lack of contact of the theory with observational and experimental facts. This may be the reason that the theory is so strongly based upon philosophical arguments. The rather surprising thing is that a theory having this type of origin should become so firmly established. Certainly physicists must all agree that the primary basis for a theory should be observations, not philosophical arguments. As time has past, General Relativity has become even more remote from observations. A large number of theoretical papers are published every year on General Relativity and most of them are motivated by formal questions, having little to say about any experimental or observational data.

Experiments on gravitation are admittedly difficult and it is important that one should concentrate on those experiments which show the most promise. For this reason it is important that one should not attempt investigations for which there are already clear indications that an uninteresting negative result would be obtained. We make a distinction between null results which are important and fundamental and lead to an understanding of the basic structure of the theory and a negative result which contributes nothing to the structure of the theory itself.

Once again an almost unique difficulty appears. In other fields of physics, one can take the established theory at the moment as representing a synthesis,

probably even the best synthesis, of the observations up to that point. Hence an experiment which is done within the framework of the accepted theory is an experiment which already admits the validity of all the other experiments which were previously performed. With gravitation this is not true, at least not to the same extent, for as we have seen, the origins of General Relativity are mainly philosophical rather than observational. On the other hand, it should be recognized that at least certain aspects of General Relativity do represent the synthesis of earlier experiments of great accuracy. For example, the null result of the Eötvös experiment is incorporated in General Relativity; so also are the notions of the isotropy of space and the invariance of electrical charge, and the local Lorentz invariance of physical laws, all of which are supported by experiments.

Thus one should admit that while a certain healthy scepticism is warranted and necessary for the construction of important experiments on gravitation, care should be exercised that these experiments be constructed with due regard for the significance of earlier experimental results. While one might well be skeptical about theories, one should take careful cognizance of the experimental results which form the underlying basis of the theories.

Having criticized the role of philosophical arguments in the construction of theories it is now necessary to say something for such arguments. First, it has been noted that the experimental basis for theories of gravitation is extremely thin and in lieu of experimental and observational facts, it is necessary to have some sort of a foundation for an understanding of gravitation.

Second, it has been found by experience that certain philosophies make better platforms for the construction of successful theories than others. Such a general argument as is embodied in the statement: «Space devoid of all matter should be devoid of physical structure and the concept of the structure of a physical space should have meaning only when the space contains matter», clearly has as its basis a logical positivist philosophy. It is founded on the notion that physical concepts should be associated with things which are physically observable and these observables should be given an operational meaning. It is presumed that an empty space has no observable properties. Such a philosophical statement is sweeping in its theoretical implications and can narrowly delineate the class of permissible theories.

One way, and perhaps the most interesting way of developing a program of research on relativity, is to attempt to find certain inadequacies in the observational basis of relativity and then to attempt to explore in an observational and theoretical way these inadequacies. Here we merely give a few examples, some of which will be explored in greater depth later. The principle of equivalence, as it enters into General Relativity, states a great deal more than the fact that the gravitational acceleration is independent of the structure of the matter. On the other hand, the precise result of the Eötvös

experiment relates only to the constancy of the gravitational acceleration. One could attempt to construct experiments which would attempt to probe into the other assertions of the equivalence principle.

Another possible inadequacy of the theory is connected with a matter of philosophical principle. It should be noted that although Mach's Principle and considerations of the physical significance of inertial reactions were essential in Einstein's thinking when he constructed the theory of General Relativity, the resulting theory did not incorporate the principle in a satisfactory manner. For this reason a central problem requiring exploration concerns the significance of Mach's Principle in physics.

As a final example one can consider the mysteries associated with the large dimensionless numbers in physics. These numbers which have been discussed many times are still without rational basis. There is no obvious fundamental reason why the gravitational coupling constant for elementary particles should be so small. It might be thought that such a dimensionless number should be either derived by a theory or else be related to other large numbers characterizing the structure of the Universe.

These examples have served to illustrate the type of questions which can be raised about the structure of General Relativity and serve to suggest interesting fields of inquiry on the experimental-observational side. As a final remark it must be said that a program of experimental research in relativity should not be attempted without a parallel theoretical program of some strength. While this remark can be made about any experimental program, one has the feeling that for research in this particular field, the necessity for a parallel theoretical development is even more acute than normal. The observational facts are so few that it is necessary to squeeze the ultimate in understanding and comprehension from the few facts which are available.

2. — Established theory and precedent.

It has often been argued, in the interest of conservatism and thrift, that we consider only established theories, such as General Relativity, until they are proved wrong by experiment. This is usually a good principle. However, as was discussed in Section 1, the basis for acceptance of General Relativity is primarily esthetics rather than observations and there is a danger that an incorrect theory is propagated because of a precedent established by a historical accident. In order to illustrate what is meant by historical accident, we tell a little story. We claim no literary merit for the story and little historical accuracy. We do, however, assert that under the assumed conditions, it is likely that theoretical physics, under Lorentz's leadership, would have developed in a way similar to that outlined here.

It could have happened: a short story

Prologue. Let us imagine that the year is 1906, that Lorentz's electron theory has been exhibiting its phenomenal series of successes, that EINSTEIN is 15 years younger than he actually was, and that two remarkable observations are made. First it is discovered that Lorentz's formula for the velocity dependence of the electron's mass is correct to considerable accuracy. Second, it is observed that a star image in the vicinity of that of the eclipsed sun is shifted from its normal position.

Imagine, if you will, a warm spring day in the year 1906. The place is Columbia University, New York City, and H. A. LORENTZ is about to give the last lecture of the famous series later to be published under the title *Theory of Electrons*. LORENTZ had skillfully woven an intricate theoretical fabric in which seemingly all parts of physics were related to each other. Optics, heat, the dielectric properties of matter, electrical conduction, all had been related to the electron and its motion. The null result of the Michelson-Morley experiment was related to the length contraction which resulted from the motion of electrically charged matter through the ether. LORENTZ, following earlier suggestions of J. J. THOMSON and M. ABRAHAM, was even able to account for the inertial mass of the electron as an electromagnetic effect. This led to the prediction that the transverse mass of the electron should increase with speed in accordance with the relation:

$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}}.$$

ABRAHAM had earlier obtained a different relation.

LORENTZ quickly enters the lecture hall and announces that he has received word from Germany of two important observational results. First, BUCHERER has found an error in the experimental results of KAUFMANN, and Lorentz's formula for the mass variation of the electron is substantiated with considerable precision. (Actually this did not occur until 1908 and Kaufmann's experiments of 1906 and earlier may have done considerable harm to the development of Lorentz's ideas.) Second, photographs of the 1905 eclipse of the sun show that light is deflected by the sun and that the deflection is twice that one would expect naively from Newton's corpuscular picture of light (It is hardly necessary to remark that historically the observation of the deflection of light did not occur until 1919 and then with no great accuracy.)

LORENTZ had first obtained this news in a telegram the preceeding evening and had spent the whole night thinking about the implications of the observation of the deflection of light. It had been immediately clear to him that these observations required the assumption of a concentration of the ether,

about gravitating bodies. He had earlier given some thought to the possibility of such a condensation in connection with the problem of stellar aberration and had considered it unnecessary. [See H. A. LORENTZ: *Theory of Electrons*, note 67, Dover Publications, 1952. I am indebted to V. BARGMANN for pointing this out to me.]

Now he had considered more carefully the implications of a greater ether density in the vicinity of a gravitating body. He noted that a charged particle would be attracted toward regions of the ether with a higher dielectric constant. Finally it becomes completely clear, gravitation, the one physical phenomenon which he had been unable to incorporate into his theory, was also electromagnetic. A body falls toward the earth because the charged particles, of which the body is composed, tend to move into a region where the dielectric constant of the ether is greater.

LORENTZ spent the remainder of the night constructing the obvious theoretical formalism which would incorporate these ideas. He found that there was an almost unique theoretical path through the formalistic jungle and by morning he had a theory, complete in its principle features.

Now after his sleepless night, Lorentz proceeds to develop his new electromagnetic theory of gravitation in this his last lecture. He first writes his well-known equations of motion for a charged particle:

$$(1) \quad \frac{d}{dt} \left(\frac{m\mathbf{v}}{\sqrt{1 - (v/c)^2}} \right) = e(\mathbf{E} + (\mathbf{v}/c) \times \mathbf{B}).$$

He reminds his listeners that he had previously shown that these equations, together with Maxwell's equations for the vacuum, are obtained from the variational principle

$$(2) \quad 0 = \delta \int \left\{ \sum [mc^2 \sqrt{1 - (v/c)^2} - e(\mathbf{v}/c) \cdot \mathbf{A} + e\varphi] \delta^3(x - \bar{x}) + \frac{1}{8\pi} (B^2 - E^2) \right\} d^4x,$$

where $\mathbf{A}$ and φ are the vector and the scalar potentials of the electromagnetic field. The sum is over all particles. The particle positions are designated as $\bar{x}$, functions of t . Equation (1) is obtained by varying $\bar{x}(t)$ and Maxwell's equations by varying $\mathbf{A}$ and φ .

To generalize this action principle, ϵ and μ , dielectric constant and permeability of space respectively, are introduced. Also because of the electromagnetic structure of the charged particle, its mass would be a function of ϵ and μ . This suggests as an action principle.

$$(3) \quad 0 = \delta \int \left\{ \sum \left[m \frac{c^2}{\epsilon\mu} \sqrt{1 - \epsilon\mu(v/c)^2} - e\mathbf{A} \cdot (\mathbf{v}/c) + e\varphi \right] \delta^3(x - \bar{x}) + \frac{1}{8\pi} \left(\frac{1}{\mu} B^2 - \epsilon E^2 \right) \right\} d^4x,$$

with

$$(4) \quad m = m_0 f(\epsilon, \mu)$$

and m_0 constant.

This action principle leads to the following equation

$$(5) \quad \frac{d}{dt} \frac{mv}{\sqrt{1 - \epsilon\mu(v/c)^2}} = e(\mathbf{E} + (\mathbf{v}/c) \times \mathbf{B}) - \nabla \left(m \frac{c^2}{\epsilon\mu} \sqrt{1 - \epsilon\mu(v/c)^2} \right),$$

together with the usual Maxwell equations for a polarizable medium.

It is noted by LORENTZ that this new equation of motion contains a new force term, the last term on the right of eq. (5). This new force has a simple physical explanation. It is closely related to the well known force acting on a charged rigid body in a dielectric medium possessing a spatially varying dielectric constant.

The first term on the right of eq. (5) is the Lorentz force and is equal to the negative of the rate of change with respect to time of the momentum carried by the electromagnetic field. Consider an electrically neutral bound system such as an atom, consisting of a number of charged particles. Assuming only internal electromagnetic fields, the time derivative of its total momentum can be written, by summing eq. (5) over all the particles comprising the system, as

$$(6) \quad \frac{d}{dt} \mathbf{P} = \sum \left\{ \frac{d}{dt} \left[\frac{mv}{\sqrt{1 - (v/c)^2 \epsilon\mu}} \right] - e \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \right\} = \mathbf{F},$$

with

$$(7) \quad \mathbf{F} = -\nabla \sum \left(m \frac{c^2}{\epsilon\mu} \sqrt{1 - \epsilon\mu(v/c)^2} \right) = -\nabla W.$$

By the virial theorem [5], W represents for a stationary bound system, when averaged over a sufficiently long time, what we now call the «total energy» of the system, *i.e.*, the energy with particle self energies included. It should be noted that this «total energy» concept was not known to Lorentz but probably would have been recognized from the form of eq. (7).

The sum in eq. (7) must be time-averaged to represent energy. However, the averaging time for an atom need be only 10^{-13} s. Thus if the atom is transported slowly from rest at point (1) to another point (2) at rest, the work required is

$$(8) \quad -\int_1^2 \mathbf{F} \cdot d\mathbf{x} = \Delta \left[\sum \left(m \frac{c^2}{\epsilon\mu} \sqrt{1 - \epsilon\mu(v/c)^2} \right) \right] = \Delta \bar{W} = \text{change in internal energy.}$$

This relation would have suggested to Lorentz the interpretation of $\bar{W}$ as the total energy of an atom at rest.

On the other hand, the acceleration (time averaged) of an atom at rest is given by the expression

$$(9) \quad \frac{U}{c^2} \varepsilon \mu \frac{d}{dt} \bar{v} = \frac{d}{dt} \bar{P} = \bar{F} = -\nabla \bar{W}.$$

Here U is the total energy of the atom at rest and is defined by the equation

$$(10) \quad U = \sum \frac{mc^2}{\varepsilon \mu \sqrt{1 - \varepsilon \mu (v/c)^2}} + \frac{1}{8\pi} \int \left(\varepsilon E^2 + \frac{1}{\mu} B^2 \right) d^3x.$$

As was discussed above, when W is time-averaged for an atom at rest, it can be written as

$$(10a) \quad \bar{W} = U.$$

Having constructed these equations, Lorentz remarks that the force $\bar{F}$ is nothing but the gravitational force acting on the atom. In order that the constancy of the gravitational acceleration, as it was shown by the Eötvös experiment, should be accounted for, it is necessary to assume that the ratio

$$(11) \quad \frac{d\bar{v}}{dt} = -\frac{c^2}{\varepsilon \mu \bar{W}} \nabla \bar{W},$$

should be the same at any point, for all atoms, independent of the details of the atomic structure. This requirement imposes two conditions upon the theory:

1) The function $f(\varepsilon, \mu)$ must be the same for all particles that enter into the structure of the atom.

2) It must be assumed that $\varepsilon = \mu$. This is not an obvious requirement, but results from the necessity for the various types of energy which contribute to the total energy of the atom, changing fractionally with position by the same amount. Perhaps the easiest way to see this is to note that the fine structure constant

$$(12) \quad \frac{e^2}{\varepsilon \hbar c} \sqrt{\varepsilon \mu},$$

is invariant under these conditions, $\varepsilon = \mu$. But one should not suppose that this is necessarily a quantum mechanical result foreign to 1906, for, treated

classically, the adiabatic invariants of the classical motion in the atoms should be properly invariant under a transport of the atom from one point to another, ε and μ being different at the two points. This requirement leads to $\varepsilon = \mu$, if all types of energy are to change fractionally by the same amount.

We stop our narrative at this point to remark parenthetically that with these two conditions imposed, the variational principle, eq. (3), may be transformed into the exact principle used in General Relativity to describe the motion of charged particles and electromagnetic fields in an isotropic co-ordinate system, the metric tensor being presumed known and given and such that

$$\bullet \quad g_{\alpha\alpha} = -\left(\frac{f}{\varepsilon}\right)^2, \quad \text{and} \quad g_{44} = \left(\frac{f}{\varepsilon^2}\right)^2 \quad \alpha = 1, 2, 3.$$

Thus, the strong experimental conditions to be met, velocity independence of charge, charge conservation, space isotropy, and constancy of gravitational acceleration, are satisfied for this theory as well as they are for General Relativity.

To resume our narrative, Lorentz notes that to account for the observed value of the light deflection and the observed gravitational acceleration of matter by the sun, it is necessary to assume that

$$(13) \quad \varepsilon \simeq 1 + \frac{2GM}{rc^2},$$

and that

$$(14) \quad f = \varepsilon^{\frac{1}{2}},$$

with M the mass of the sun. Furthermore Lorentz finds that he can obtain the result given in eq. (13) if he treats ε as a scalar field and includes the obvious Lagrangian density in his action principle.

The question of the proper velocity to include as a propagation velocity for these new ε waves is easily answered. The necessity for the velocity being high, in order that the planetary perturbation be correct, is resolved by taking the velocity to be that of light, completely reasonable for an electromagnetic theory. Also, Lorentz notes that with the gravitation propagation velocity taken as the light propagation velocity, $c\varepsilon^{-1}$, Newton's laws of gravitation are obtained to a good approximation even if the sun is moving, relative to the ether.

The Lagrangian density assumed by Lorentz is

$$(15) \quad \lambda \varepsilon^{2n} \left[(\nabla \varepsilon)^2 - (\varepsilon^2/c^2) \left(\frac{\partial \varepsilon}{\partial t} \right)^2 \right],$$

where λ and n are constants. If this term is included in the equation for the action principle, and variations with respect to ε are taken, one obtains the wave equation

$$(16) \quad \frac{4\lambda\varepsilon^{n-\frac{1}{2}}}{2n+3} \left[\nabla^2 \varepsilon^{n+\frac{1}{2}} - (\varepsilon^2/c^2) \frac{\partial^2}{\partial t^2} \varepsilon^{n+\frac{1}{2}} \right] = \\ = -\varepsilon^{-\frac{1}{2}} \sum m_0 c^2 \left[\frac{1}{2} \sqrt{1 - \varepsilon^2(v/c)^2} + \frac{\varepsilon^2(v/c)^2}{\sqrt{1 - \varepsilon^2(v/c)^2}} \right] \delta^3(x - \bar{x}) - \\ - \frac{1}{8\pi\varepsilon} \left[\frac{1}{\varepsilon} B^2 + \varepsilon E^2 \right] + \lambda \varepsilon^{2n-1} \left[(\nabla\varepsilon)^2 + (\varepsilon^2/c^2) \left(\frac{\partial\varepsilon}{\partial t} \right)^2 \right].$$

The first three terms of the right-hand side of eq. (16) can be written as

$$(17) \quad -\frac{1}{\varepsilon} \left[\sum \frac{m_0 c^2}{\sqrt{\varepsilon}} \frac{1}{\sqrt{1 - \varepsilon^2(v/c)^2}} \delta^3(x - \bar{x}) + \frac{1}{8\pi} \left(\frac{1}{\varepsilon} B^2 + \varepsilon E^2 \right) \right] + \\ + \frac{1}{2\varepsilon} \sum m_0 \frac{c^2}{\sqrt{\varepsilon}} \sqrt{1 - \varepsilon^2(v/c)^2} \delta^3(x - \bar{x}).$$

The term in brackets is just the energy density of the particles plus field. If matter is assumed to consist of atoms comprising charged particles occupying a fixed volume and interacting electromagnetically only, the time average of the integral over this volume of the 2nd term of eq. (17) is,

$$(18) \quad \frac{1}{2\varepsilon} \sum m_0 \frac{c^2}{\sqrt{\varepsilon}} \sqrt{1 - \varepsilon^2(v/c)^2},$$

and, as was mentioned previously, by the virial theorem is $1/2\varepsilon$ times the « total energy » of the system. It is assumed, in forming these integrations, that the time and space variations of ε are negligible. If matter is stationary, but consists of many atoms containing rapidly moving particles interacting with each other electromagnetically, then averaged over dimensions greater than the atom diameter and averaged over times greater than a characteristic atom period, eq. (17) becomes $(-1/2\varepsilon)$ times the average energy density.

$$(19) \quad -\frac{1}{2} \frac{c^2}{\varepsilon^{\frac{1}{2}}} \varrho_0 = -\frac{1}{2} \frac{c^2}{\varepsilon^{\frac{1}{2}}} \varrho,$$

where ϱ_0 is the density of the matter, based on m_0 being a measure of mass, and ϱ is the mass density, equal to energy density times $(\varepsilon/c)^2$. Thus stationary matter is found to generate a gravitational field with a source strength proportional to its inertial mass, independent of the details of its structure. Note, however, that a light wave would generate just twice the gravitational