

# STATISTICS FOR TEACHERS

BY

ERNEST W. TIEGS, PH.D.

DEAN OF UNIVERSITY COLLEGE  
UNIVERSITY OF SOUTHERN CALIFORNIA

AND

CLAUDE C. CRAWFORD, PH.D.

PROFESSOR OF EDUCATION  
UNIVERSITY OF SOUTHERN CALIFORNIA



HOUGHTON MIFFLIN COMPANY

• NEW YORK • CHICAGO • DALLAS  
ATLANTA • SAN FRANCISCO

The Riverside Press Cambridge

COPYRIGHT, 1930

BY ERNEST W. TIEGS AND CLAUDE C. CRAWFORD

ALL RIGHTS RESERVED INCLUDING THE RIGHT TO REPRODUCE  
THIS BOOK OR PARTS THEREOF IN ANY FORM

*The Riverside Press*

CAMBRIDGE MASSACHUSETTS

PRINTED IN U.S.A.

## EDITOR'S INTRODUCTION

ONE of the distinguishing characteristics of the great change which has taken place in educational work, within the past twenty years, has been the application of new statistical procedures to the study of educational problems. Whereas two decades ago we dealt with our problems in the light of accumulated experience, and by the aid of common sense and logical thinking tried to formulate the principles of action involved, to-day we attempt to determine principles from a study of concrete situations, and in turn to test the validity of the principles thus derived by a statistical treatment of the data from which the principles or conclusions were deduced. The concrete data for the study of the problem must be gathered accurately and after a carefully formulated plan, and the data must be organized and treated according to mathematical procedures. By such a plan we seek to eliminate errors, to reveal relationships, to clear up little-understood implications, to determine meanings and indicated trends which the data embody, and not only to deduce provable conclusions but in addition to be able to state the probability of error and its extent in the conclusions so deduced.

As a consequence of this change in the method of studying our educational problems, a new educational literature has been created, within recent years. New terms have been introduced into current educational thinking that are meaningful and need defining, new types of graphic display have been employed that need to be read intelligently, and new mathematical formulæ, which two decades ago were

almost unknown, have been derived and applied to the study of educational and social and psychological problems. As a result of this marked change in our methods of treating observational and recorded data, much of our educational literature of to-day is not intelligible to the reader who has no grasp of the newer methods for the statistical treatment of educational, social, and psychological data.

To meet the need for training in these newer procedures, courses in the statistical treatment of educational problems are now given in practically all schools of education, and the needs of the classroom teacher, who cannot take graduate courses in education, are being supplied through extension and summer-session courses in the elements of statistical methods as applied to the study of problems in educational practice.

It is this common need of teachers and beginning students of education that the authors of the present volume have tried to meet by preparing a textbook that will enable the teacher or student who studies it to follow intelligently the current educational literature. They have not tried to produce another comprehensive volume on statistical procedures as applied to education, or to education and psychology, but rather to present the elements of statistical science as applied to the one field of education, and to present these elements in such a simple manner that the beginner in the study, with little or no mathematical background, may grasp and be able to understand the fundamental principles of statistical procedures as now commonly employed in the educational literature of the day. Still more, the text has been constructed largely on a learning-by-doing plan, supplementing the usual statistical theory by adequate practical exercises, and with a view to preparing the teacher to apply simple statistical procedures in connection with the daily work of the school. These

purposes the authors have carried out very effectively, presenting in clear and simple language a volume which can be strongly recommended for classroom use as a beginning textbook in statistical methods for students of education.

ELLWOOD P. CUBBERLEY

## PREFACE

ANY one who has the courage to add another volume to the long and growing list of textbooks in the field of statistics must necessarily do so for good cause. The authors believe that this work will justify itself, and that it will meet a definite need in the training of teachers and other school workers. Below are a few brief statements of what they consider the essential characteristics of the work:

1. It aims to meet the needs of the very large number of unspecialized educational workers, rather than the needs of the much smaller number of highly specialized educational research experts. The former group is typified by the classroom teacher, who has given the work its name. The research expert may use the book as a first step in his training, but will need to follow it with work in more advanced treatises.

2. It is addressed to educational workers only, rather than to those in such fields as sociology, economics, or business. While principles and processes are rather similar in all fields, the topics and illustrations have been selected specifically to meet the needs of teachers. It has seemed best to serve teachers well, rather than to seek a wider audience at the expense of more direct application of what is taught.

3. It is constructed for use as an actual classroom textbook or manual. It is intended to be the major, if not the only, book to be used in the course, and is arranged in the order which the authors find it best to teach the various steps and processes. The aim has not been to present a complete compendium of statistical information, but rather

to organize a set of definite lessons in convenient and sound instructional order.

4. It is provided with a set of true-false tests at the ends of the chapters for the use of students in studying, and for the use of the teacher in developing an understanding of the theory or principles which are included. If these tests are used effectively, they should go far toward cultivating statistical insight, or toward preventing statistical work from being mere mechanical juggling.

5. It attempts to develop actual skills, rather than merely to impart knowledge or information about statistical operations. For this reason numerous problems and exercises have been placed at the ends of chapters, as well as examples within the chapters. The authors recommend that students be required to do all the exercises, and that, if necessary, some classes be asked to do additional exercises based on data supplied by the instructor, in order that an adequate degree of skill may be attained.

6. It approaches new concepts, principles, or processes by way of concrete cases, whenever possible, and also applies these concepts or principles to concrete situations or actual educational problems after they have been explained and illustrated. Most of the model computations are based on actual rather than imaginary data, in order that learning may be carried on under conditions as similar as possible to real life situations.

7. It correlates graphics and statistics by presenting them both throughout the work, instead of including a chapter on charts and graphs as a separate unit. The authors think of graphs not as "statistical luxuries," but rather as "statistical necessities."

8. It seeks to avoid highly technical or theoretical phases of mathematics. Only the common elements of arithmetic and occasional phases of algebra are taken for granted.

In actual practice the authors have found it useless to assume more than this.

9. It does not attempt to derive formulas nor to explain their mathematical foundations if they are at all complex or difficult; the same amount of time probably can be spent to better advantage in a beginning course if devoted to learning to use formulas and to interpret results. The mathematical proofs of formulas are recognized as being valuable, but are omitted because of the impossibility of accomplishing all desirable objectives in a single elementary course.

10. It omits entirely, or else mentions very briefly, many processes and concepts which are often treated in statistics texts. This is done from necessity rather than preference, lest the effort to teach everything result in failure to teach well the minimum essentials.

11. It has been written as an outgrowth of the experience of the authors in teaching statistics to a considerable number of classes, as well as in directing scores of graduate thesis investigations, many of which were of a statistical nature. The selection and the presentation of topics has been influenced by what seemed to be the real needs and the real difficulties in practical situations.

12. It has been tried out in mimeographed form with four classes, taught by three different instructors and has been revised in the light of these practical tests.

13. It has been read and criticized very constructively by Dr. Walter E. Eells, of Stanford University. A number of very important revisions and improvements have resulted from these criticisms.

The authors are indebted to a number of persons for helpful suggestions, criticisms, or special courtesies shown in the preparation of the manuscript. Among these are the following: (1) Dr. W. S. Ford and Dr. O. R. Hull, Professors

of Education in the University of Southern California, for numbers of practical suggestions. (2) Mr. C. H. Nettels, Assistant Director of the Division of Psychology and Educational Research, of the Los Angeles City Schools, for helpful comments growing out of his experience in teaching the mimeographed edition of the book in his summer school classes at the University of Southern California. (3) Dr. Walter C. Eells, Professor of Education at Stanford University, for courtesies already mentioned. (4) Dr. Ellwood P. Cubberley, Dean of the School of Education at Stanford University, for editorial criticisms and improvements in the manuscript. (5) The students who were in the classes taught by the writers and by Mr. Nettels, both before and after the writing of the book, for their frank reactions as well as encouraging interest in the work.

ERNEST W. TIEGS

CLAUDE C. CRAWFORD

UNIVERSITY OF SOUTHERN CALIFORNIA  
LOS ANGELES

# CONTENTS

I. WHY STUDY STATISTICS . . . . .	1
II. LABOR- <small>SAVING</small> DEVICES AND EQUIPMENT . .	12
III. TABULATING DATA . . . . .	28
IV. THE MEAN . . . . .	49
V. THE MEDIAN . . . . .	62
VI. PERCENTILES . . . . .	73
VII. VARIABILITY . . . . .	91
VIII. THE NORMAL CURVE . . . . .	106
IX. RELIABILITY . . . . .	129
X. CORRELATION . . . . .	153
XI. FURTHER ASPECTS OF CORRELATION . .	174
XII. PRINCIPLES OF STATISTICAL RESEARCH .	191
SELECTED REFERENCES . . . . .	205
INDEX . . . . .	207

## LIST OF TABLES

I. Illustration of a Simple Frequency Distribution, with Tabulation . . . . .	29
II. Illustration of a Frequency Table Grouped in Class Intervals . . . . .	31
III. Illustrations of Methods of Designating Class Intervals . . . . .	36
IV. Illustration of the Method of Finding the Mean from a Frequency Table, Using Zero as the Starting-Point . . . . .	52
V. Illustration of the Method of Finding the Mean when the Intervals are Larger than One, and Using an Assumed Mean . . . . .	54
VI. Illustration of the Method of Checking the Accuracy of the Computation of a Mean by Using a Different Assumed Mean . . . . .	56
VII. Illustration of the Method of Finding the Median of Data Grouped into a Frequency Table . . . . .	64
VIII. Illustration of the Cumulative Frequency Table . . . . .	77
IX. Illustration of the Process of Converting Cumulative Frequencies into Percentile Ranks of the Tops of Class Intervals . . . . .	78
X. Illustration of the Method of Finding the Percentile Ranks of the Midpoints of Class Intervals . . . . .	80
XI. Illustration of the Computation of Percentiles for Ungrouped Data . . . . .	88
XII. Illustration of the Computation of Quartile Deviation . . . . .	97
XIII. Illustration of the Computation of the Standard Deviation . . . . .	99
XIV. Illustration of the Computation of Quartile and Standard Deviation . . . . .	101
XV. Illustration of the Possible Combinations of Heads and Tails when Two Coins are Tossed . . . . .	107
XVI. Illustration of the Possible Combinations of Heads and Tails when Three Coins are Tossed . . . . .	108

XVII.	Percentages of the Normal Distribution Lying Between the Mean and Various $\sigma$ Unit Values . . .	115
XVIII.	Percentages of Students Assigned Each Mark Under Various Systems that Use the Normal Curve as a Basis for Marking . . . . .	117
XIX.	Chances that Measures will not Fall Below Various $\epsilon$ Unit Values . . . . .	137
XX.	Illustration of the Computation of Various Quantities as they are Involved in a Rotation Experiment	143
XXI.	Chances that Measures will not Fall Below Various P.E. Unit Values . . . . .	148
XXII.	A Scatter Diagram Showing Correlation of .79 . . .	157
XXIII.	Illustration of the Pearson Product-Moment Method of Computing the Coefficient of Correlation from a Correlation Table . . . . . <i>Between 158 and</i>	159
XXIV.	Illustration of the Computation of the Pearson Product-Movement Correlation Coefficient from Ungrouped Pairs of Data . . . . .	165
XXV.	Illustration of the Computation of the Coefficient of Correlation by the Spearman Rank Differences Method . . . . .	167
XXVI.	Data to be Used for Practice in Computing Coefficients of Correlation . . . . .	172
XXVII.	Data for Practice in Connection with Special Aspects of Correlation . . . . .	190

## LIST OF FIGURES

1. Graphic Illustration of a Class Interval . . . . .	33
2. Illustration of the Graphic Representation of a Frequency Distribution: Frequency Rectangles . . . . .	38
3. Illustration of the Graphic Representation of a Frequency Distribution: Histogram and Frequency Polygon . . . . .	40
4. Illustration of the Comparison of Groups by Means of Frequency Polygons . . . . .	41
5. Illustration of the Difficulty Involved in Comparing Groups of Unequal Size by Plotting Simple Frequencies . . . . .	42
6. Illustration of the Comparison of Unequal Groups by Plotting Percentage Frequencies . . . . .	44
7. Illustration of the Construction of a Percentile Graph . . . . .	83
8. Illustration of the Use of Percentile Graphs to Compare Two Groups . . . . .	86
9. Illustration of the Need of Taking Account of Variability when Comparing Two Distributions . . . . .	93
10. Illustration of the Curve Obtained from Plotting the Expanded Binomial . . . . .	110
11. Illustration of the Percentages of the Total Distribution Corresponding to Successive $\sigma$ Units from the Mean . . . . .	113
12. Illustration of the Relation Between $\sigma$ Unit Values and Fractional Parts of a Normal Distribution . . . . .	116
13. Illustration of the Method of Dividing the Normal Distribution Curve into Five Parts for the Assignment of Teachers' Marks . . . . .	119
14. Illustration of the Computation of Difficulty Values for Five Test Questions . . . . .	121
15. Illustration of the Basis for Converting an Observed Difference into a Statement of Chances that the True Difference is Above Zero . . . . .	141
16. Illustration of the Steps in Finding a Partial Correlation Coefficient . . . . .	181
17. Illustration of the Steps in Finding a Multiple Correlation Coefficient . . . . .	183
18. Illustration of the Meaning of Curvilinear Correlation . . . . .	187

# STATISTICS FOR TEACHERS

## CHAPTER I

### WHY STUDY STATISTICS?

**Need for a knowledge of statistical procedures.** The stories of scientists who became so absorbed in their researches that they forgot their dinners or their weddings have their counterparts in the lives of statisticians. Statistical work is extremely fascinating. Figures are not dry! Under the direction of a trained statistician, statistical procedures can perform feats quite as remarkable as the tricks of a Houdini. The solution of statistical problems yields a satisfaction as deep and as natural as that which a veteran chess player gets from directing his campaign, or an inventor derives from the creation of a new machine.

And yet the major reason for studying statistics is not that of interest. The most vital reason is utility, or practical value. A teacher must study statistics, because without it he cannot hope to get far in education, or even be a highly successful teacher in this generation.

This need for statistics on the part of educational workers manifests itself in three separate areas of their educational responsibilities and activities. A knowledge of statistics is necessary in order to understand and interpret educational science, in order to do the day's work most effectively,

and in order to contribute to our present knowledge about education. We shall consider each of these more fully.

**Need for statistics in the study of educational science.** The elementary concepts of statistics constitute a part of the common culture or heritage which integrates the teaching profession. A teacher who is not familiar with means, medians, percentile ranks, standard deviations, or correlations is in a situation comparable to that of the person who attends religious services conducted in a foreign tongue. He may watch his neighbors and imitate them, but he neither understands, participates, nor contributes in an intelligent or effective manner.

A considerable body of educational literature, increasingly more statistical in nature, has been published in recent years, and a growing number of new books and journals of this type are published every year. Some years ago, the writers of educational books and articles apologized to their readers if they were forced to use statistical terms, even of the most elementary kind; to-day they take a knowledge of statistics for granted and do not hesitate to publish material of a highly technical nature. The improved statistical training of the readers has made this possible, but at the same time this tendency on the part of educational writers has created the need for more adequate statistical training on the part of readers. Teachers to-day will be able to read professional literature intelligently and with profit in proportion as they gain understanding in this field.

A knowledge of statistics is essential to the teacher in order to use intelligently the findings of the educational

research experts. Profound theses embodying valuable work for the Ph.D. degree are often "buried" because those who should apply the findings in the educational activities of the schools cannot understand the research reports. If we are ever to succeed in bridging the chasm between the research experts and the practical educational workers, we shall have to build from both sides of the gulf which now separates them. The study of statistics by practical school people will enable them to meet the research workers halfway.

**Statistical training good protection against errors.** Education, like medicine, law, and politics, is infested with many "isms" and "ologies." We have numerous quacks, shysters, and false prophets, each recommending his pet theory, hobby, or scheme, and enlisting teachers in carrying out his program. With so many available leaders, whom shall the teacher follow? Our educational democracy desires increasing freedom for each teacher in making decisions and plans; this freedom must, therefore, be accompanied by a type of training which will enable him to use this freedom wisely. Statistical training is a good protection against the false and erroneous conclusions of others. It is said that figures can't lie but that liars can figure. A propagandist can present absolute falsehoods supported by misleading statistical evidence and make them appear to be the essence of truth — unless his audience knows enough about statistics to discover his tricks and statistical fallacies.

**Need for statistics in the conduct of educational activities.** A knowledge of statistics is useful in the daily work

of the teacher and the school administrator. Before school begins in the autumn there is need for an analysis of the available school-census data. Students have to be classified into grades, levels, and sections for instructional purposes, on the basis of quantitative data. The success of the year's work is very greatly increased if this phase of school organization and administration is carried out according to sound statistical principles, and by the aid of suitable statistical techniques.

Nor is this need for statistics confined to a limited few in the administrative offices. The classroom teachers usually bear a considerable burden in getting the school well started. One year's plans are made up in the light of the previous year's records, which are often prepared largely by classroom teachers. The task of organizing and administering a school is so great, and the interrelations between the various members of the school staff are so intricate, that each member, however insignificant he may seem to be, needs to know something about the larger scheme of things in order to coöperate more intelligently. An ignorance of statistical procedures on the part of the teachers tends to tie the hands of the best administrative staff, or the best school research department, just as an untrained army handicaps the best of generals.

Statistical procedures are valuable to the classroom teacher in handling daily routine. Teaching seeks to produce change in pupils, and this in turn calls for measurement of pupil achievement. Marks and grades occupy a very prominent place in the teacher's work, and without a